

THE PROBABILISTIC CALCULATING OF FATIGUE CRACK PROPAGATION USING FCPRBCALC PROGRAM

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Abstract: *The paper gives examples of the probabilistic assessment of a steel cyclic loaded structure. Fatigue progression of the cracks from the edge and from the surface is used as a basis for proposing a system of inspections. The newly developed method Direct Optimized Probabilistic Calculation (DOProC method) was used for solution. The method was applied in FCProbCalc software.*

Keywords: *DOProC method, FCProbCalc, Fatigue Crack, Inspection of Structure, Safety Margin.*

1. Introduction

Reliability of bearing structures which are subject to load cycles is affected considerably by degradation processes being, in particular, the result of fatigue of the basic material. Methods are under development now which would detect failures and defects, if any, resulting from initiation cracks. Linear fracture mechanics is among alternative methods. Mechanical engineering experts have been dealing with such issues for many years. Results have been gradually taken over and implemented into designs of the loading structures in buildings. Because input variables include uncertainties and reliability should be taken into account, probabilistic methods should be used in investigation into the propagation rate of the fatigue crack.

This paper describes the use of the original method and method which is under development now: the Direct Optimized Probabilistic Calculation ("DOProC") can be used for a probabilistic design and assessment of reliability of structures with the specified designed probability of failure, without using any simulation techniques - Janas et al. (2009); Janas et al. (2009). The DOProC method deals, as the other probabilistic methods, with tasks where at least certain input quantities are of a random nature. In many cases, this calculation method is very efficient and provides accurate estimates of resulting probabilities. Only a calculation error and an error resulting from discretizing of input and output quantities are involved there.

DOProC method has proved to be a good solution, among others, in probabilistic analysis of fatigue crack propagation in constructions subject to cyclical loads. Detailed methods with examples of the probabilistic assessment for a construction subject to fatigue load are available, a particular attention being paid to cracks from the edge and those from the surface. Similarly to other probabilistic analysis, this information is used as a basis for designing a system of inspections of the cyclic load construction, e.g. Moan (2005), Straub (2009), Chen et al. (2011), Krejsa et al. (2011), the aim being to analyze real propagation of fatigue cracks in those structural details which tend most to be damaged by fatigue. If no fatigue cracks are found, the analysis of inspection results give conditional probability during occurrence.

In order to improve quality of probabilistic calculations, a special software - FCProbCalc - was developed. Using this software, it is possible to monitor effectively and flexibly development of fatigue damage in structures, to determine times for inspections and to ensure that the construction will be fit for operation in terms of fatigue safety. The methods and application can considerably improve estimation of maintenance costs for the structures and bridges subject to cyclical loads.

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2. Direct Optimized Probabilistic Calculation (DOProC)

The Direct Optimized Probabilistic Calculation ("DOProC") has been under development since 2002. Principles of DOProC method which can be used for various types of probabilistic tasks have been described in theory in many publications. The calculation procedure for a certain task in DOProC method is clearly determined by its algorithm, while Monte Carlo simulation methods generate calculation data for simulation on a random basis, e.g. Konecny et al. (2007), Krivy et al. (2007).

Similarly as with the other probabilistic methods, Kralik (2009), input random quantities in DOProC (such as the load, geometry, material properties, or imperfections) are described using the non-parametric (empirical) distribution in histograms. This technique can be also used for parametric divisions. The distribution is typically based on observations, being often long-lasting ones. A computational procedure is being developed now, the aim being to implement into DOProC method the statistic dependence of input parameters.

In DOProC method, each calculation of the failure probability, p_f , is, however, limited by the number of random input variables. If there are too many random variables, the application is extremely time demanding - even if high-performance computers are used. The number of necessary computational operations and time needed for the calculation is also influenced by the number of intervals in each input random variable. This influences, in turn, accuracy of final probabilities. Therefore, efforts have been made to optimize calculations in order to reduce the number of operations, keeping, at the same time, reliable calculation results. The goal of optimizing techniques is to minimize the computational time and to maintain correctness and high accuracy even in relatively demanding probabilistic tasks. This is the reason why the method includes in its name the word "Optimized".

DOProC method has been used, so far, in probabilistic assessment of combined load or reliability of cross-sections and systems consisting of statically determined or undetermined load carrying constructions, e.g. Janas et al. (2003), in probabilistic assessment of load carrying constructions which are subject to shocks in probabilistic analysis of steel-fibre reinforced concrete mixtures or in probabilistic assessment of reliability of anchored reinforcement or arc reinforcement in underground and long mine works with a special focus on anti-slipping properties.

It is possible to use ProbCalc in DOProC probabilistic calculations (see Fig. 1). ProbCalc is a software application which is still under development. It is rather easy and simple to implement quite a complicated analytical transformation model of a probabilistic task defined using a text-oriented editor. In more complex numerical calculation models, there is a chance to use the procedure programmed by the user as DLL (with a dynamic library extension). The optimizing techniques have been implemented into ProbCalc and can be combined now in the probabilistic calculation.

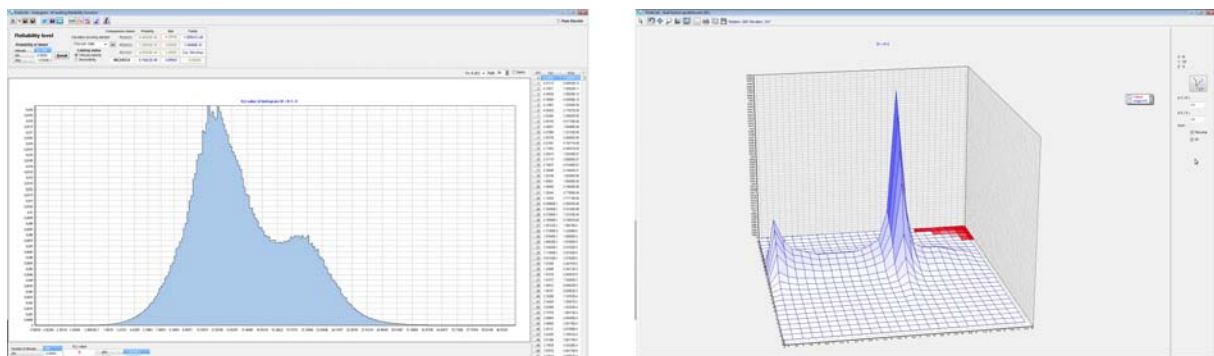


Fig. 1: Desktop of the ProbCalc software (left) and calculated 3D chart of reliability function (right).

More advanced user knowledge is required then to enter the probabilistic tasks in ProbCalc. It is essential to know, at least, general basics of algorithms because this influence the way of defining the computational model and selection of the best optimizing procedure. This weakness is removed if the application software is customized for a specific probabilistic task, this being, for instance, the case of FCProbCalc.

3. Probabilistic calculation of fatigue crack propagation

Occurrence of initiation cracks and crack propagation in structures subject to fatigue load has been known for a long time. The process is closely connected with fabrication of the steel structures, Kala (2005), and, in particular, with creation of details which tend to be damaged by fatigue. The key difference is between initiation of cracks resulting from steelmaking inclusions and those created during fabrication of structural details. Regarding the former, it takes a long time until it reaches the surface, while the latter is at the surface from the beginning of the loading. Standardized approaches of previous EC standards suppose that surface cracks were not present there. The acceptable damage method which is described in the new standard admits random occurrence of surface cracks. The major difference is that a fatigue crack might not be fragile, but could be ductile. In real components of steel structures and bridges, the latter is more frequent than the former which is used in experimental measurements in processed small test-pieces. This fact is not a new phenomenon. It has been known for a long time and has been mentioned, for instance, by Anderson (2005). During the designing, fabrication and processing of details, nobody, however, paid attention to random occurrence of initiation cracks from surface areas (from the surface or from the edge).

Three sizes are important for the characteristics of the propagation of fatigue cracks. These are the initiation size, the detectable size and the final acceptable size which occurs prior to failure caused by a fragile or ductile crack. The fatigue crack damage depends on a number of stress range cycles. This is a time factor in the course of reliability for the entire designed service life. In the course of time, the failure rate increases, while the reliability drops.

The topic is discussed in two levels that affect each other: the probabilistic solution to the propagation of the fatigue crack and uncertainties in determination of quantities used in the calculation. When investigating into the propagation, the fatigue crack that deteriorates a certain area of the structure components is described with one dimension only: a . In order to describe the propagation of the crack, the linear elastic fracture mechanics is typically used. It is based on the Paris-Erdogan law, e.g. Sanford (2003):

$$\frac{da}{dN} = C \cdot (\Delta K)^m, \quad (1)$$

where C , m are material constants Carpinteri et al. (2007), a is the crack size and N is the number of loading cycles.

The initial assumption is that the primary design should take into account the effects of the extreme loading resulting from the ultimate state of carrying capacity method. Then, the fatigue resistance should be assessed. This means, the reliability margin in the technical probability method is:

$$Z_{(R,S)} = RF = R - S, \quad (2)$$

where R is the random resistance of the element and S represents random variable effects of the extreme load.

When using (1), the condition for the acceptable crack length a_{ac} is:

$$N = \frac{1}{C} \int_{a_0}^{a_{ac}} \frac{da}{\Delta K^m} > N_{tot}, \quad (3)$$

where N is the number of cycles needed to increase the crack from the initiation size a_0 to the acceptable crack size a_{ac} , and N_{tot} is the number of cycles throughout the service life.

The equation for the propagation of the crack size (1) needs to be modified for this purpose. The state of stress near the crack face is described using ΔK (the stress intensity coefficient) which depends on the loading (bending, tension), size and shape of the fatigue crack, and geometry of the load-bearing component. If the $\Delta\sigma$ stress range and axial stress-load of the flange are constant, the following relation applies:

$$\Delta K = \Delta\sigma \cdot \sqrt{\pi a} \cdot F_{(a)}, \quad (4)$$

where $F_{(a)}$ is the calibration function which represents the course of propagation of the crack. After the change of the number of cycles from N_1 to N_2 , the crack will propagate from the length a_1 to a_2 . Having

modified (1) and using (4), the following formula will be achieved:

$$\int_{a_1}^{a_2} \frac{da}{(\sqrt{\pi a} \cdot F_{(a)})^m} = \int_{N_1}^{N_2} C \cdot (\Delta\sigma)^m dN. \quad (5)$$

If the length of the crack a_1 equals to the initial length a_0 (this is the assumed size of the initiation crack in the probabilistic approach) and if a_2 equals to the final acceptable crack length a_{ac} (this is the acceptable crack size which replaces the critical crack size a_{cr} if the crack results in a brittle fracture), the left-hand side of the equation (5) can be regarded as the resistance of the structure $R_{(a_{ac})}$:

$$R_{(a_{ac})} = \int_{a_0}^{a_{ac}} \frac{da}{(\sqrt{\pi a} \cdot F_{(a)})^m}. \quad (6)$$

If the upper integration limit a_d is used, the resistance of the structure $R_{(a_d)}$ can be specified similarly. Similarly, it is possible to define the cumulated effect of loads that is equal to the right side (randomly variable effects of the extreme load) (5):

$$S = \int_{N_0}^N C \cdot (\Delta\sigma)^m dN = C \cdot (\Delta\sigma)^m \cdot (N - N_0), \quad (7)$$

where N is the total number of oscillations of stress peaks ($\Delta\sigma$) for the change of the length from a_0 to a_{ac} , and N_0 is the number of oscillations in the time of initialization of the fatigue crack (typically, the number of oscillations is zero).

It is possible to define a reliability function RF :

$$RF(\mathbf{X}) = R_{(a_{ac})} - S_{(N)}. \quad (8)$$

where \mathbf{X} is a vector of random physical properties such as mechanical properties, geometry of the structure, load effects and dimensions of the fatigue crack.

The analysis of the reliability function (8) gives a failure probability p_f :

$$p_f = P(RF(\mathbf{X}) < 0) = P(R_{(a_{ac})} < S_{(N)}). \quad (9)$$

3.1. Probabilistic calculation of fatigue cracks propagating from the edge

A tension flange has been chosen for applications of the theoretical solution suggested in the studies Tomica et al. (2007). Depending on location of an initial crack, the crack may propagate from the edge or from the surface (see Fig. 2). Regarding the frequency, weight and stress concentration, those locations rank among those with the major hazard of fatigue cracks appearing in the steel structures and bridges.

A flange without stress concentration is used for confronting the both cases depending on the location of the crack initiation. The cases are different in calibration functions $F_{(a)}$ - and in weakened surfaces which are appearing during the crack propagation.

For the crack propagating from the edge, the calibration function is:

$$F_{(a)} = 1.12 - 1.39 \cdot \frac{a}{b} + 7.32 \cdot \left(\frac{a}{b}\right)^2 - 13.8 \cdot \left(\frac{a}{b}\right)^3 + 14.0 \cdot \left(\frac{a}{b}\right)^4, \quad (10)$$

where a is the length of the crack and b is the width of the flange Janssen et al. (2002); (see Fig.2).

The acceptable crack size a_{ac} can be described then by a formula resulting from the deduced weakening of the cross-section area of the flange:

$$a_{ac} = b \cdot \left(1 - \frac{\sigma_{\max}}{f_y}\right). \quad (11)$$

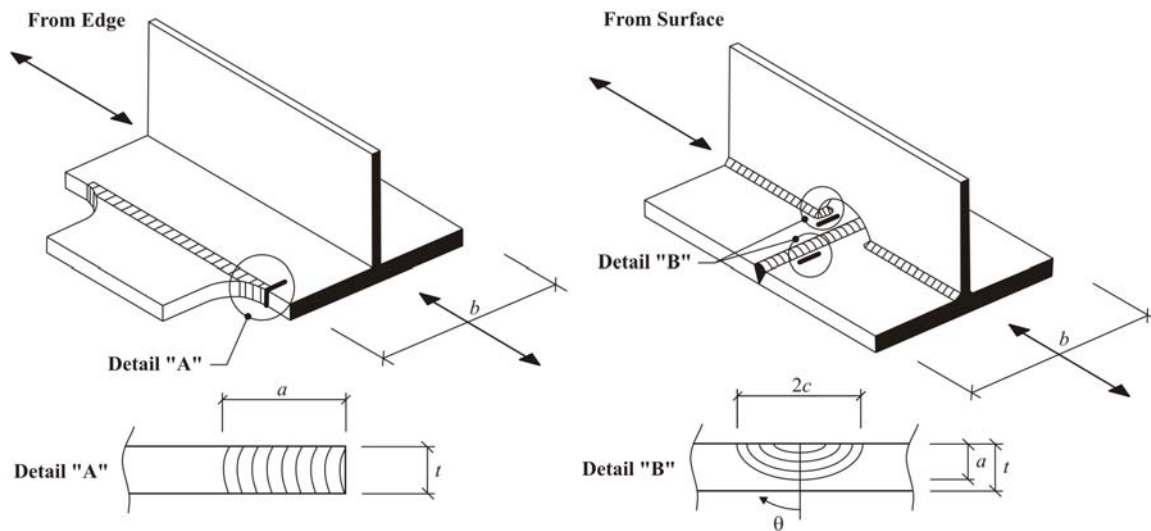


Fig. 2: Characteristic propagation of cracks from the outer edge (left) and from the surface (right).

3.2. Probabilistic calculation of fatigue cracks propagating from the surface

A similar approach can be used to determine the acceptable size of a crack propagating from the surface. The bending component can be neglected for welded steel two-axis symmetric I-profiles where the fatigue crack appears in the lower tension flange. The flange is loaded only by the normal stress resulting from the axial load - tension: $\sigma_m = \sigma$.

It is rather difficult to deduce analytically the acceptable size of the crack propagating from the surface. In accordance with Krejsa et al. (2010), the shape is replaced with a semi-elliptic curve where the ellipsis axes are a (the crack depth) and c (a half of the crack width) - see Fig. 2. The area of the surface crack depends on the number of N loading cycles and is described by the following formula:

$$A_{cr(N)} = \frac{1}{2} \cdot \pi \cdot a_N \cdot c_N . \quad (12)$$

During propagation of the fatigue crack from the surface, it is not enough to monitor only one crack size (which would be sufficient, for instance, for a crack propagating from the edge). In that case, the crack size needs to be analyzed for directions of the both semi-axes: a and c . The propagation of the fatigue crack from the surface in the a direction depends on the propagation in the c direction. Crack velocity propagation is described by (1). In Krejsa et al. (2010) there is a formula for calculation of the crack depth Δa as a result of an increased width of the Δc crack:

$$\Delta a = \left\{ \frac{1}{\left[1.1 + 0.35 \cdot \left(\frac{a}{t} \right)^2 \cdot \sqrt{\frac{a}{c}} \right]} \right\}^m \cdot \Delta c . \quad (13)$$

The crack sizes for a and c are during the propagation limited by upper limit values:

$$2 \cdot c \leq 0,4 \cdot b_f \quad a \leq 0,8 \cdot t_f , \quad (14)$$

If these upper limit values are exceeded, the fatigue crack propagates differently. Krejsa (2011) gives also the formula for the mutual dependence of the sizes in a and c :

$$c = 0.3027 \cdot \frac{a^2}{t} + 1.0202 \cdot a + 0.00699 \cdot t . \quad (15)$$

When determining the acceptable crack size, a modified relation (12) using (13) and (15), should be taken as a basis. After modification:

$$\sigma_{\max} \cdot \frac{b_f t_f}{b_f t_f - \frac{1}{2} \cdot \pi a \cdot \left(0.3027 \cdot \frac{a^2}{t_f} + 1.0202 \cdot a + 0.00699 \cdot t_f \right)} \leq f_y, \quad (16)$$

It is difficult to describe the a crack size directly explicitly. In order to calculate the acceptable crack size a_{ac} , it is necessary to use a numerical iteration approach where restrictions resulting from (16) should be taken as a basis.

3.3. Determination of inspections of structures subject to fatigue

Because it is not certain in the probabilistic calculation whether the initiation crack exists and what the initiation crack size is and because other inaccuracies influence the calculation of the crack propagation, a special inspection is necessary to check the size of the measurable crack in a specific period of time. The acceptable crack size influences the time of the inspection. If no fatigue cracks are found, the analysis of inspection results give conditional probability during occurrence.

While the fatigue crack is propagating, it is possible to define following random phenomena that are related to the growth of the fatigue crack and may occur in any time, t , during the service life of the structure. Then:

- **$U_{(t)}$ phenomenon:** No fatigue crack failure has not been revealed within the t -time and the fatigue crack size $a_{(t)}$ has not reached the detectable crack size a_d . This means:

$$a_{(t)} < a_d, \quad (17)$$

- **$D_{(t)}$ phenomenon:** A fatigue crack failure has been revealed within the t -time and the fatigue crack size $a_{(t)}$ is still below the acceptable crack size a_{ac} . This means:

$$a_d \leq a_{(t)} < a_{ac}, \quad (18)$$

- **$F_{(t)}$ phenomenon:** A failure has been revealed within the t -time and the fatigue crack size $a_{(t)}$ has reached the acceptable crack size a_{ac} . This means:

$$a_{ac} < a_{(t)}. \quad (19)$$

Using the phenomena above, it is possible to define probability for their occurrence in any t -time. Those three phenomena cover the complete spectrum of phenomena that might occur in the t -time. This means:

$$P(U_{(t)}) + P(D_{(t)}) + P(F_{(t)}) = 1. \quad (20)$$

The probabilities of random phenomena can be determined in any period of time, t , using, for instance, DOProC method. The probabilistic calculation is carried out in time steps where one step equals to one year of the service life of the construction. When the failure probability (for the F phenomenon) reaches the designed failure probability p_d , the inspection should be carried out in order to find out fatigue cracks, if any, in the construction element. The inspection provides information about real conditions of the construction. Such conditions can be taken into account when carrying out further probabilistic calculations. The inspection in the t time may result in any of the three mentioned phenomena. Using the inspection results for the t time, it is possible to define the probability of the mentioned phenomena in another times: $T > t_I$. For that purpose, the conditional probability should be taken into consideration.

3.4. Using the conditioned probability to determine times to inspect the construction

If the crack is not revealed within the t -time, this may mean that there is not any fatigue crack in the construction element. This might be an initiative phase of nucleation of the fatigue crack (when a crack appears in the material) and this phenomenon is not taken into account in the fracture mechanics.

Even if the fatigue crack is not revealed it is likely that it exists but the fatigue crack size is so small that it cannot be detected under existing conditions.

In order to fix the time for the next inspection, it is necessary to determine the conditioned probabilities, $P(F_{(T)} | U_{(t_I)})$ and/or $P(F_{(T)} | D_{(t_I)})$, which can be expressed using the Bayes' formula:

$$P(F_{(T)} | U_{(t_I)}) = \frac{P(F_{(T)}) - P(F_{(t_I)}) - P(D_{(t_I)}) \cdot P(F_{(T)} | D_{(t_I)})}{P(U_{(t_I)})}, \quad (21)$$

$$P(F_{(T)} | D_{(t_I)}) = \frac{P(F_{(T)}) - P(F_{(t_I)}) - P(U_{(t_I)}) \cdot P(F_{(T)} | U_{(t_I)})}{P(D_{(t_I)})}. \quad (22)$$

If redistribution of stress from a point that is weakened by the crack is not taken into account, the crack propagation crack is usually rather high in the practical range of measurable values. If a fatigue crack is found during the inspection of the construction, it is necessary to monitor the safe growth of the crack or to take actions that will slow down or stop further propagation of the fatigue crack. In order to time the inspections well, the equation which defines the failure probability in $T > t_I$ is most important - provided that no fatigue cracks have been revealed during the last inspection. It is clear from the equation that the results of the failure probability are influenced by mutual relations between the three crack sizes - the initiation crack size, measurable crack size and acceptable crack size.

When the failure probability reaches the designed failure probability p_d , an inspection should be carried out in order to reveal fatigue cracks, if any, in the construction component. The inspection may result in one of the mentioned phenomena with corresponding probabilities. The entire calculation can be repeated in order to ensure well-timed inspections in the future.

4. FCProbCalc software

FCProbCalc was developed using the aforementioned techniques. By means of FCProbCalc, it is possible to carry out the probabilistic calculation of propagation of fatigue cracks in a user friendly environment. The attention is paid to propagation of fatigue cracks from the surface and edge. On the basis of these data, times for the first and next inspections are determined, the goal being to identify fatigue damage to structural details which most tend to be damaged by fatigue.

The reference probabilistic calculation used input quantities from the publication which had included the probabilistic assessment of a steel/reinforced concrete bridge on the highway in a point where a longitudinal beam connects to a transversal beam. Tables 1 and 2 show the input quantities which were entered deterministically and stochastically. The required reliability was expressed by the reliability index $\beta = 2$ which corresponds to the failure rate of $p_d = 0.02277$.

Using FCProbCalc it is possible to specify propagation of fatigue crack from the edge for a certain time interval - the resistance of the structure $R_{(a_d)}$ and $R_{(a_{ac})}$ (numerical integration using Simpson's rule with 1000 differences was used), Fig. 3), load effect S as well as probability of elementary phenomena U , D and F which are the source information for determination of the time of the first inspection. If no fatigue damage is found during the inspection, times for next inspections have been determined on the basis of the conditioned probability.

The calculated probabilities of the random phenomena, U , D and F (Fig. 4), are the source information for next inspections of the bridge based on the conditioned probability. If the edge crack cannot be detected during the first inspection in the 50th year of operation, the next inspection will take place in the 58th year of operation. And if the crack is not identified there again, the next year of inspection is the 63th year. After that year, the inspection intervals will become shorter considerably (operation years: 66,

Tab. 1: Overview of variable input quantities expressed in a histogram with parametric distribution of probabilities

Quantity	Type	Mean value	Standard deviation
Oscillation of stress peaks $\Delta\sigma$ [MPa]	Normal	30	3
Total number of oscillation of stress peaks per year N [-]	Normal	10^6	10^5
Yield stress f_y [MPa]	Lognormal	280	28
Nominal stress in the flange plate σ [MPa]	Normal	200	20
Initial size of the crack a_0 [mm]	Lognormal	0.2	0.05
Smallest detectable size of the crack a_d [mm]	Normal	10	0.6

Tab. 2: Overview of input quantities expressed in a deterministic way

Quantity	Value
Material constant m	3
Material constant C	$2.2 \cdot 10^{13}$
Width of the flange plate b_f [mm]	400
Thickness of the flange plate t_f [mm]	25
Nominal probability of failure p_d	0.02277

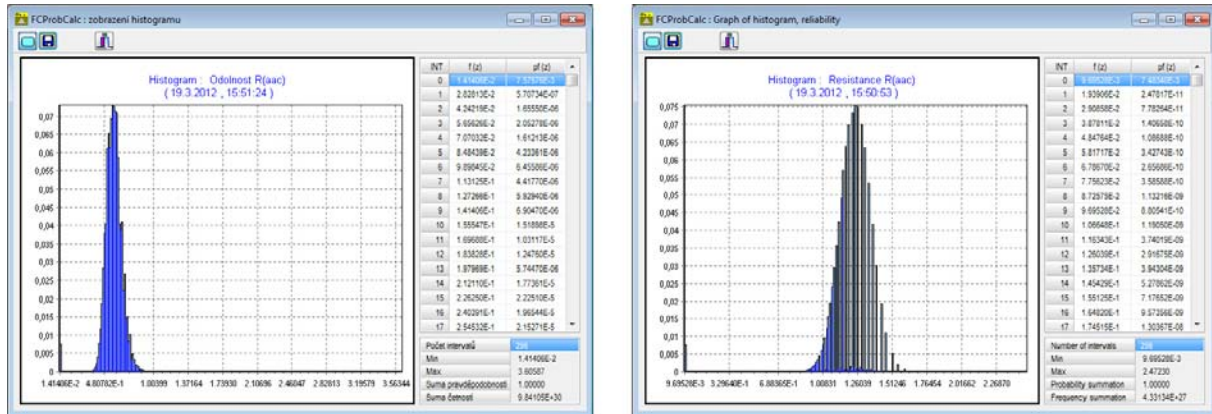


Fig. 3: FCProbCalc program output: Resulting histogram of the structural resistance $R(a_{ac})$ for propagation of fatigue crack from the edge (left) and from the surface (right).

69, 71, 73 and 74). If the crack is not identified during the 75th year, it can be assumed that - if the input values have not changed (in particular, the intensity and efficiency of the operation load) - the medium value of the initial crack will be less than 0.2 mm or there is not any fatigue crack at all.

The similar approach is used in the probabilistic calculation for propagation of a fatigue crack from surface (calculated time of inspections in years: 110, 123, 131, 137, 142 and 147).

The Fig. 5 shows the graphic results of the inspections calculated using DOProC method.

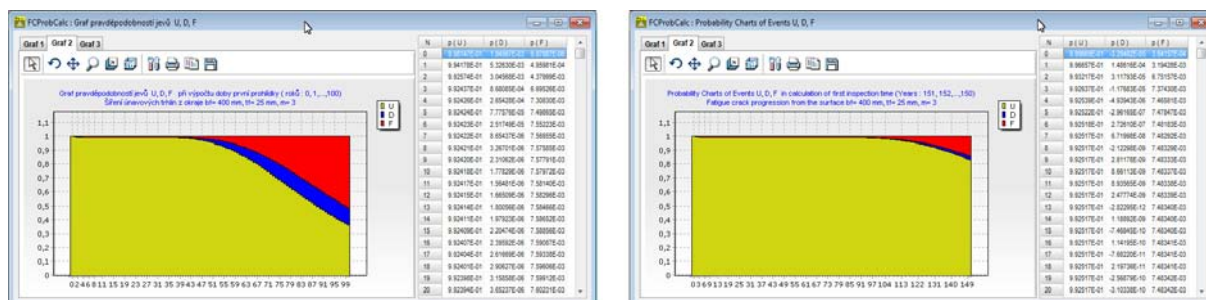


Fig. 4: FCProbCalc program output: Probabilities of the phenomena U , D and F for the propagation of fatigue crack from the edge (0 to 100 years of operation, left) and for the propagation of fatigue crack from the surface (0 to 150 years of operation, right).

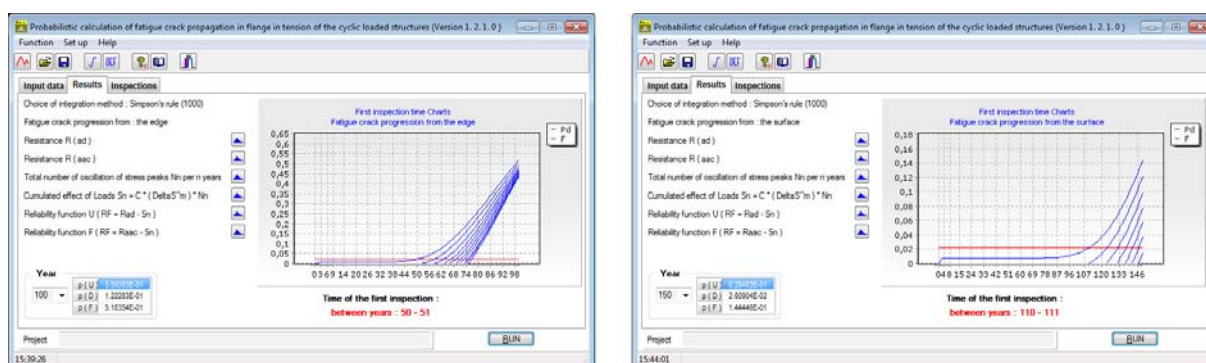


Fig. 5: FCProbCalc program output: failure probability p_f , depending on the years of operation for the propagation of fatigue crack from the edge (0 to 100 years of operation, left) and for the propagation of fatigue crack from the surface (0 to 150 years of operation, right).

5. Conclusions

This paper addresses application of the new probabilistic method, DOProC, for probabilistic assessment of steel structures which are subject to load cycles and which tend to suffer from fatigue cracks. The method was included into FCProbCalc which is the software which makes it possible to solve very efficiently the probabilistic task of fatigue crack propagation in a user-friendly environment.

FCProbCalc was used for the probabilistic assessment of fatigue damage to a bridge structure where cracks were propagating from both the surface and edge. Times were specified for inspections of the bridge structure, where the purpose was to monitor occurrence of certain fatigue cracks. The comparison proved that velocity of propagation of the fatigue crack from the surface is considerably slower than that from the edge.

It should be pointed out that FCProbCalc still provides many other options to be used. Future development of this software will focus, in particular, on numerical integration in calculation of structural resistance and on impacts of the chosen numerical methods and input parameters on the computational time and final accuracy of the calculations in the context of the time of the proposed inspections.

Appendix

For a lite version of FCProbCalc and for other software products based on DOProC method please visit web pages <http://www.fast.vsb.cz/popv>, Janas et al. (2012).

Acknowledgments

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References

- Anderson, T. L. (2005) *Fracture mechanics: fundamentals and applications*. Third edition, CRC Press, Taylor & Francis Group, Boca Raton, Florida. ISBN 0-8493-1656-1.
- Carpinteri, A. & Paggi M. (2007) Self-similarity and crack growth instability in the correlation between the Paris' constants. *Engineering Fracture Mechanics*, vol. 74, issue 7, pp 1041–1053 (13 p), DOI 10.1016/j.engfracmech.2006.12.007.
- Chen, N. Z., Wang, G. & Soares, C. G. (2011) Palmgren-Miner's rule and fracture mechanics-based inspection planning. *Engineering Fracture Mechanics*, vol. 78, issue 18, pp 3166–3182 (17 p), DOI 10.1016/j.engfracmech.2011.08.002.
- Janas, P., Krejsa, M. & Kolos, I. (2003) Reliability assessment of structure expressed to impact. *Proceedings of national conference with international participation Engineering Mechanics 2003*, Svratka. Editors Naprstek, J. and C. Fischer. (10 s). ISBN 80-86246-18-3.
- Janas, P., Krejsa, M. & Krejsa, V. (2012) *ProbCalc Software and DOProC Method*. Web pages. [on-line]. <http://www.fast.vsb.cz/popv>. VSB – Technical University Ostrava.
- Janas, P., Krejsa, M. & Krejsa, V. (2009) Structural Reliability Assessment using a Direct Determined Probabilistic Calculation. *Proceedings of the 12th International Conference on Civil, Structural and Environmental Engineering Computing*, Civil-Comp Press, Stirlingshire, Scotland. Paper 79, ISBN 978-1-905088-31-7.
- Janas, P., Krejsa, M. & Krejsa, V. (2009) Using the Direct Determined Fully Probabilistic Method for determination of failure. *Proceedings of the European Safety and Reliability Conference, Esrel 2009*. Taylor & Francis Group, London, ISBN 978-0-415-55509-8 (set of 3 volumes + CD-ROM), ISBN 978-0-203-85975-9 (e-book).
- Janssen, M., Zuidema, J. & Wanhill, R. J. H. (2002) *Fracture Mechanics*. Second edition, Delft University Press. ISBN 90-407-2221-8.
- Kala, Z. (2005) Sensitivity Analysis of the Stability Problems of Thin-Walled Structures. *Journal of Constructional Steel Research*, vol. 61, issue 3, pp 415–422 (8 p), DOI 10.1016/j.jcsr.2004.08.005.
- Konecny, P., Tikalsky, P. J. & Tepke, D. G. (2007) Performance Evaluation of Concrete Bridge Deck Affected by Chloride Ingress. *Transportation Research Record*, issue 2028, pp 3–8 (6 p), DOI 10.3141/2028-01.
- Kralik, J. (2009) *Reliability Analysis of Structures Using Stochastic Finite Element Method*. Slovak university of technology in Bratislava. (143 s) ISBN 978-80-227-3130-0.
- Krejsa, M. & Tomica V. (2010) Calculation of Fatigue Crack Propagation Using DOProC Method. *Transactions of the VSB – Technical University of Ostrava*, no.1, vol.X, Civil Engineering Series, paper #11 (9 p), DOI 10.2478/v10160-010-0011-6. Publisher Versita, Warsaw, ISSN 1213-1962 (Print) ISSN 1804-4824 (Online).
- Krejsa, M. & Tomica V. (2011) Determination of Inspections of Structures Subject to Fatigue. *Transactions of the VSB – Technical University of Ostrava*, no.1, vol.XI, Civil Engineering Series, paper #7 (11 p), DOI 10.2478/v10160-011-0007-x. Publisher Versita, Warsaw, ISSN 1213-1962 (Print) ISSN 1804-4824 (Online).
- Krejsa, M. (2011) *Using DOProC Method for Structural Reliability Assessment*. Habilitation thesis, VSB – TU Ostrava. ISBN 978-80-248-2385-0.
- Krivy, V. & Marek P. (2007) Probabilistic Design of Steel Frame Structures. *Stahlbau*, vol. 76, issue 1, pp 12–20 (13 p), DOI 10.1002/stab.200710003.
- Moan, T. (2005) Reliability-based management of inspection, maintenance and repair of offshore structures. *Structure and Infrastructure Engineering*, vol. 1, issue 1, pp 33–62 (30 p), DOI 10.1080/15732470412331289314, Taylor & Francis Ltd.
- Sanford, R. J. (2003) *Principles of Fracture Mechanics*. Pearson Education, Inc., U.S.A. ISBN 0-13-092992-1.
- Straub, D. (2009) Stochastic Modeling of Deterioration Processes through Dynamic Bayesian Networks. *Journal of Engineering Mechanics*. DOI 10.1061/(ASCE)EM.1943-7889.0000024.
- Tomica, V. & Krejsa, M. (2007) Optimal Safety Level of Acceptable Fatigue Crack. *Proceedings of 5th International Probabilistic Workshop*, Ghent, Belgium. Edited by L.Taerwe and D.Proske. (12 p). ISBN 978-3-00-022030-2.