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# COLLISION OF A ROTATING SPHERICAL PARTICLE WITH FLAT WALL IN LIQUID

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**Abstract.** *The collision of the rotating spherical particle with a flat wall in liquid was studied experimentally. The glass and steel beads rotating in water and silicon oil were used. A high-speed video system recorded the bead motion before and after the collision. It was shown that the restitution coefficient depends not only on the Stokes number but also on the particle angular velocity; the restitution coefficient decreases with increasing of the rotational Reynolds number and decreasing of the Stokes number. These results can be useful in modelling of the two-phase flows near solid boundaries.*

**Key words:** ***restitution coefficient, spherical particle, particle rotation, liquid viscosity.***

## 1. Introduction

The mechanisms of solid particles collision with a solid boundary or with other particles is a subject of interest for many years, especially in case of a two-phase flow, like processes of mixing in chemical reactors, slurry pipeline transport, pneumatic transport, bed load transport and erosion in channel, particle sedimentation, fluidization, filtering and suspension thickening.

The modeling of particle-particle or particle-wall collisions requires a detailed understanding of the mechanics of impact and rebound. The energy dissipation due to an inelastic contact is usually characterized by a coefficient of restitution *e*, defined as the ratio of the rebound velocity to the impact velocity (normal components)

$$e = | v_r / v_i |, \tag{1}$$

where $v_i$ and $v_r$ are the impact and rebound velocities, respectively, i.e. the velocity just before and after the collision. The coefficient of restitution characterizes the energy losses during collision; the initial kinetic energy is transformed into elastic strain energy stored in the bodies and then restored into kinetic energy of the rebounding particle (Ruiz-Angulo & Hunt, 2010).

Under negligible fluid resistance, in a fully elastic collision coefficient of restitution can be approximately unity, $e \approx 1$, whereas for a perfectly plastic collision $e = 0$. Coefficient of restitution is less than one for the most part of collisions, i.e. collisions with inelastic losses. Inelasticity of the collision is evoked by plastic deformation, viscoelasticity or vibration. Their effects lead to energy losses, which are dependent on the impact velocity, and coefficient of restitution decreases with impact velocity with power law with small exponents (Gondret et all., 2002).

Most of the known studies deal with so called dry collisions, i.e. collisions in vacuum or gas, where the fluid resistance is negligible. Only a few works take into account effect of fluid viscosity on the coefficient of restitution and collision process, but we have not found any study dealing with the effect of particle rotation. For collision of solid particles in liquid surrounding, the effect of liquid is important and the restitution coefficient is often named the effective restitution coefficient.

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Davies et all. (1986) declared, that the pertinent number for the collision is not the Reynolds number, but the particle Stokes number which compares the particle inertia to the fluid viscous forces

$$St = (2/9)\,\rho_p\,|\,U\,|\,r\,/\,\mu = (1/9)\,(\,\rho_p\,/\,\rho_f\,)\,Re, \tag{2}$$

$$Re = d\,U\,\rho_f\,/\,\mu, \tag{3}$$

where $\rho_p$ is a particle density, $\rho_f$ is a fluid density, $r$ is a particle radius, $\mu$ is fluid dynamic viscosity, and is a Reynolds number based on the particle impact velocity $U$ and particle diameter $d$.

Let us suppose that the particle shape is spherical, it moves in liquid and collides with a plane wall. The rebound of the particle after collision depends on the material of particle and the wall, on impact velocity and the restitution coefficient $e = e\,(St)$, which is a function of the particle Stokes number. For $St < St_c$, where $St_c \approx 10$ is a critical Stokes number, the effective restitution coefficient is equal zero, and no rebound occurs. The restitution coefficient increases with increase of the Stokes number, and reaches the maximum value when the Stokes number is about 2000 – 3000. In this region the restitution coefficient is close to the value of restitution coefficient determined for a collision in gas or in vacuum (Gondret et all., 2002).

## 2. Experimental equipment and procedure

The goal of the present work is to examine experimentally the effect of liquid viscosity and of particle rotation on the restitution coefficient of the spherical particle rebounding from a plane wall in liquid. Two series of experiments were conducted. In the first series it was proved, that particle impact with wall causes the particle rotation. The second series deals with effect of particle rotation on the restitution coefficient. The experiments were focused on determining trajectory of a solid sphere falling in water onto a plane glass wall and evaluation of the effect of liquid viscosity and particle rotation on the restitution coefficient.

Water and silicon oil were used as a liquid; viscosity of the oil was about 200 higher than that of water, what made possible to obtain different values of Stokes number. Smooth glass and steel spherical particles of diameter $d$ = 14, 16, and 19 mm were used. Hairlines were drawn along two perimeters of the particle with the angle of $90^{\circ}$ to make it possible to visualize the particle rotation. A sheet of glass of thickness 21 mm was used as impact surface.

The particle was rotated about a horizontal axis in a spinning device developed in the Institute of Hydrodynamics AS CR, v. v. i., situated above the water level (see Fig. 1). The particle was held between cups and rotated around with an initial angular velocity $\omega_0$, which was measured by a tachometer. When the trigger was released, the springs pulled the cups apart, allowing the particle to fall freely in water. The spinning device ensured the required particle rotation in the given plane and translational velocity of the particle was reached by free fall of the particle (Lukerchenko et all., 2012). The device allowed to spin the particle up to 6 500 rpm (revolutions per minute).

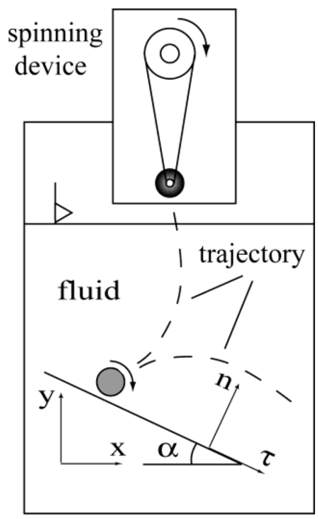


*Fig. 1: Experimental setup.*

Immediately before the collision the angular velocity becomes 1.5 – 2 times less than $\omega_0$. After the collision with the wall, the particle rebounded and the combined translational and rotational motions were recorded with a frequency of 1000 frames per second by the digital video camera MotionPro X High-Speed CMOS Digital Camera. Only experiments in which the plane of the particle trajectory was parallel to the plane of the video camera objective were chosen. The software *MotionPro X Studio* was used for data processing.

## 3. Results and discussion

For example, in the case of bed load transport or multi-phase flows in chemical reactor, the motion of particles in fluid near solid wall results in the particle-wall collisions and, when the impact angel is not equal π/2, the sequent particle rotation. The particle angular velocity $\omega$ could reach values of a few tens or even hundreds of revolutions per minute during the particle saltation pattern. Let us now consider the spherical shape of a particle. The sphere rotation in fluid generates the secondary flow, the centrifugal effect will produce a swirling radial jet directed outward of the equatorial region of the sphere, which is reason for flow of fluid towards the poles in the direction of the axis of rotation. The pole-ward flow exert equal and opposite forces on each hemisphere so that the resultant force is zero in unbounded area. When the particle rotates around an axis parallel to the wall the presence of the wall generates forces in normal and tangential directions, which are functions of the rotational Reynolds number, particle relative distance from the wall, and relative roughness of the particle and the wall. In neighbourhood of a wall, the boundary destroys the symmetry around the particle and the particle becomes a subject of forces, which can dump the impact velocity, and in consequence decrease the restitution coefficient. The forces would affect motion of the particle, attract or repulse it when the particle moves towards the boundary. Small particle-wall gap would bring into play strong viscous effect, which causes repulsive forces (Liu and Prosperetti, 2010).

The value of the repulsive force depends on the angular velocity and roughness of the sphere surface. The larger roughness of the sphere surface gives the larger induced secondary fluid flow. In the case of a smooth sphere, the effect is noticeable for the large angular velocities only, whereas in the case of the rough sphere it can be observed even for the moderate or small angular velocity range. For the intensification of the secondary flow we used a golf ball with a rough surface (diameter $d$ = 0.0428 m, mass $m$ = 0.0458 kg). The glass wall was covered by the waterproof sandpaper and situated under angle of $25^0$ to the horizontal vessel bed (see Fig. 1).

The main difficulty of data processing of the conducted experiments is that time of a collision duration is much smaller than the time period between two successive images ($\Delta t = 10^{-3}$ s) of the used camera. According to Gondret et all. (2002) the collision duration is typically $10^{-5}$ s (it was measured with a piezoelectric sensor). The values of the horizontal $x$ and vertical $y$ coordinates of two ends of a diameter ($x_1$ , $y_1$) and ($x_2$ , $y_2$) were obtained. The coordinates of the ball center ($x_0$, $y_0$) and the angle of the ball rotation $\varphi$ were calculated as functions of time:

$$x_0 (t) = 0.5[x_1 (t) + x_2 (t) ], \tag{4}$$

$$y_0 (t) = 0.5[y_1 (t) + y_2 (t) ]; \tag{5}$$

$$\varphi (t + \Delta t) = \varphi (t) + \arccos [\boldsymbol{a} (t).\boldsymbol{a}(t + \Delta t)/d^2], \tag{6}$$

where

$$\boldsymbol{a} (t) = [x_2 (t) - x_1 (t); y_2 (t)- y_1 (t) ] \tag{7}$$

is the vector coincided with the diameter. The dependences (4) – (6) were smoothed and then differentiated, and the translational and angular velocities immediately before and after the collision were calculated.

Fig. 2 shows the time dependences of the coordinates of the ends of the horizontal diameter ($x_1$ , $y_1$) and ($x_2$ , $y_2$) and of the ball center ($x_0$ , $y_0$) in the case without the ball rotation before the collision ($\omega_0$ = 0). The slope angle of the plots allows us to define the components of the velocity of the ball center before and after the collision. As indicated in the picture the ball begins to rotate after the collision. The characteristics of the spherical particle motion before and after the collision for the experiments, where three different initial angular velocity $\omega_0$ were used ($\omega_0$ = 0; 4500, and 5200 rpm), are given in Table 1. Immediately before the collision the angular velocity of the ball becomes 1.5 – 2 times less than the initial angular velocity $\omega_0$ (Lukerchenko & Kvurt, 2011).

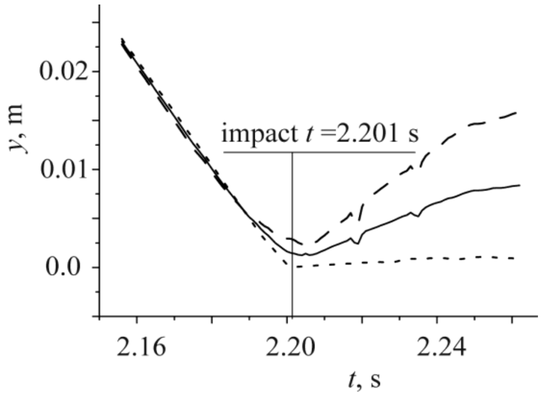


*Fig. 2: Coordinates of the ends of the horizontal diameter* $(x_1,y_1)$ [- - -] *and* $(x_2,y_2)$ [·····] *and of the ball center* $y_0$ [—] *versus time t before and after the particle impact* ($\omega_0 = 0$).

Table 1 shows that the maximum value of the restitution coefficient *e* is when the ball does not rotate before the collision ($Re_{\omega,r} = 0$). The rotational Reynolds number

$$Re_{\omega,r} = \omega \cdot r^2 \rho_f / \mu \qquad (8)$$

is the dimensionless analogue of the angular velocity of a sphere. The larger angular velocity immediately before the collision corresponds to the smaller restitution coefficient. Based on these results, it can be conclude that the larger angular velocity immediately before the collision corresponds to the smaller value of the effective restitution coefficient.

*Table 1: The characteristics of the ball motion before and after the collision.*

| No. | $\omega_0$ [rpm] | $v_{i,n}$ [m/s] | $v_{r,n}$ [m/s] | *e* | *St* | $\omega_i$ [rpm] | $Re_{\omega,r}$ |
|---|---|---|---|---|---|---|---|
| 1 | 0 | -0.39 | 0.27 | 0.70 | 1 850 | 0 | 0 |
| 2 | 4 500 | -0.51 | 0.31 | 0.60 | 2 420 | 2 005 | 95 300 |
| 3 | 5 200 | -0.75 | 0.35 | 0.46 | 3 570 | 3 065 | 145 500 |

Examples of experimental trajectories and velocity components for rotating glass spherical particle (diameter d = 16 mm) falling in water are illustrated in Fig. 3. The time between two successive images was $\Delta t = 10^{-3}$ s. Initial angular velocity $\omega_0$ = 5 800 rpm, due to the pass through water level and drag in water the angular velocity just before the 1st impact was significantly less, only $\omega_1$ = 3 288 rpm, and further $\omega_2$ = 1 846 rpm, and $\omega_3$ = 895 rpm for the 2nd and 3rd impact, respectively. The three first impact and jump after the collision of the particle with horizontal glass plane wall were recorded.

The distance of the particle centre $y_0$ to the wall is displayed as a function of time *t* and of the horizontal coordinate *x*, respectively, in upper part of Fig. 3. The particle rotated in clockwise direction and due to the Magnus force its direction was downward and in negative *x*-axis direction. Both the trajectory in *xy*-plane and the record of $y_0 = f(t)$ show steady uniform motion before impact and gradual decrease of jump height due to the effect of the drag force and decreasing of the effective restitution coefficient. Interesting is effect of the particle rotation on change in *x*-direction during the first impact. The height of the jump gradually decreased in successive jumps on the contrary to length of the jumps.

The instantaneous particle velocity components in vertical (normal) and horizontal (tangential) direction were computed as the time derivative of the co-ordinate increment between two successive images, and the corresponding velocity components $v_y$ and $v_x$ were plotted as function of time in bottom part of Fig. 3, which illustrate character of velocity changes during collision. The values of velocity components just before and after collision ($v_i$ and $v_r$, respectively) can be determined and used for the effective restitution coefficient *e* calculation, see Eq. (1). The particle vertical (normal) velocity component reached the maximum just before the first collision; during the individual jumps normal velocity component reached very quickly maximum value and then gradually decreased due to drag of the surrounding liquid. This behaviour is, due to the viscosity of water, different from that of the parti lic.

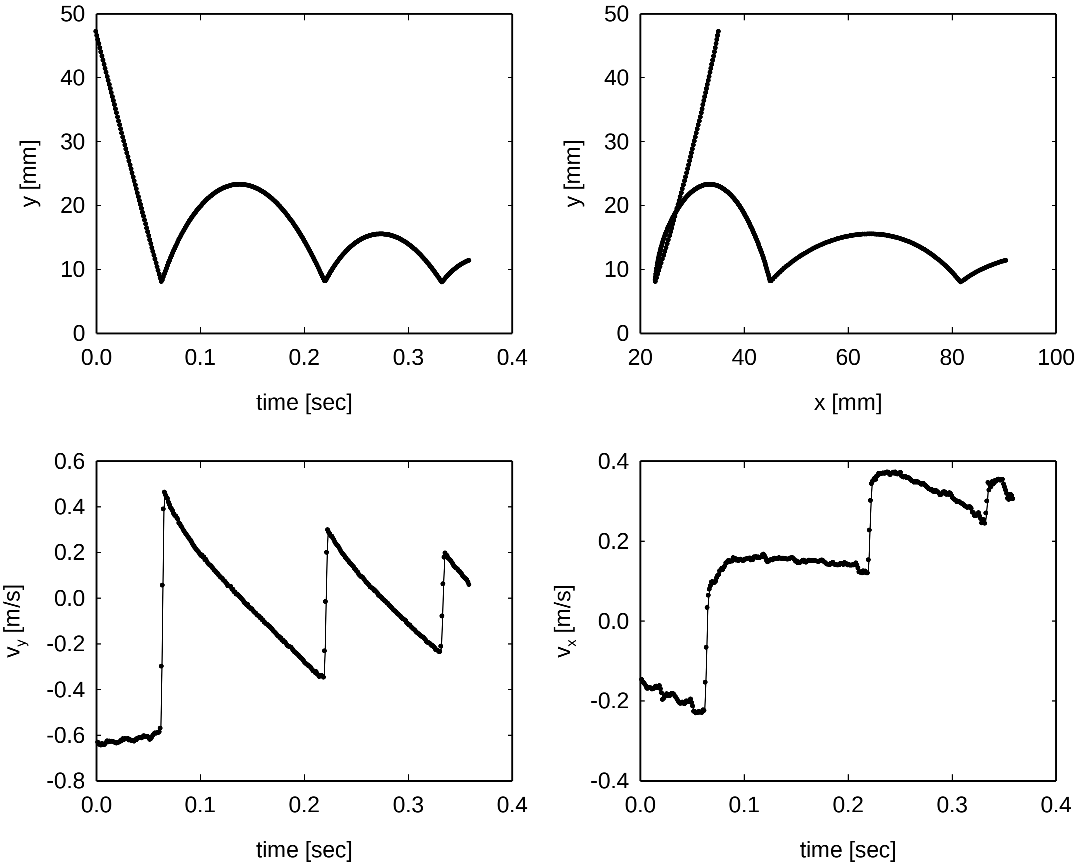


*Fig. 3: Trajectories and velocity components of rotating glass spherical particle falling in water*

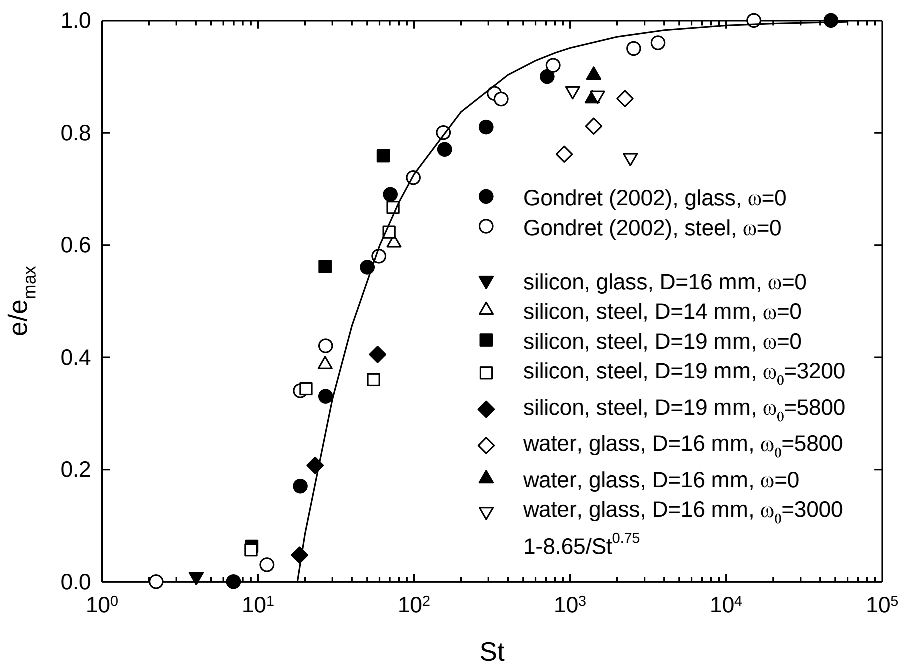


*Fig. 4: Dependence of* the dimensionless restitution coefficient *$e/e_0$ on the Stokes number St; comparison of present experiments with Gondret et all. (2002) results.*

The normal velocity decreased nonlinearly with time and its modulus was significantly smaller at the end of the jump than at the beginning. On the contrary, the tangential velocity increased after the particle collision due to the Magnus force.

As it was mentioned above, for particle impact in liquid the viscous effects are important, since the particle rebound depends on both, the plasticity and viscous losses. Fig. 4 shows the comparison of our experimental data of the restitution coefficient *e* with results of Gondret et all. (2002), Ruiz-Angulo & Hunt (2010). For current experiment water and silicon oil were used as liquid, smooth glass and steel spherical particles of diameter 14, 16, and 19 mm and resistant glass wall (of thickness 21 mm) was used as impact surface. The silicon oil of viscosity about 200 higher than that of water made possible to obtain results in region of low Stokes number with used particles of relatively large diameter. Fig. 4 shows that present experimental data of particles without rotation relatively well coincide with data from above mentioned literature. However, rotation of the particle entailed decreasing of the effective restitution coefficient. This finding is supported for instance by the data ◊ (glass particle in water, $d$ = 16 mm, $\omega_0$ = 5 800 rpm), which were also used for illustration of particle trajectories and velocity components in Fig. 3.

According to Ruiz-Angulo & Hunt (2010) the solid line

$$e/e_0 = 1 - 8.65/St^{0.75} \tag{9}$$

is the best fit of experimental data of experiments, which were conducted with no permanent deformation of either the impacted surface or of the particle (Joseph et all., 2001). The maximum coefficient of restitution $e_0$ is value measured with given particle and plane wall in air. This curve was used as a reference for elastic collisions, corresponding to the greatest possible value of the restitution coefficient at a given Stokes number (Ruiz-Angulo & Hunt, 2010). We can see that for low values of the Stokes number, below approximately $St$ = 10, the particles did not rebound due to the viscous effect. For the relatively large Stokes number, i.e. $St > 10^3$, the viscous effect became negligible and the effective restitution coefficient reached the elastic limit, $e/e_0 = 1$.

For constant parameters of particle and carrier liquid the increase of the Stokes number depends on the impact velocity. In the Stokes number interval between St = 20 and St = $10^3$, as the impact velocity increased, the energy consumed by plastic deformation increased and energy consummated to displace the fluid from area between particle and wall decreased. However, the used velocities did not cover the regimes of high impact velocities where the plastic deformation dominates the hydrodynamic effects.

## 4. Conclusions

The collision of the rotating glass and steel spherical particle with a flat wall in water and silicon oil was studied experimentally. It was conducted that the restitution coefficient depends not only on the Stokes number, but also on the particle angular velocity, the restitution coefficient decreases with increasing of the rotational Reynolds number.

It was confirmed, that the restitution coefficient increases with increase of the Stokes number, and reaches the maximum when the Stokes number $St > 10^3$, where the viscous effect became negligible and the restitution coefficient is close to the value of restitution coefficient determined for a collision in gas or in vacuum. For the Stokes number $St < St_c$, where $St_c \approx 10$ is the critical Stokes number, the effective restitution coefficient is equal zero, and no rebound occurs.

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