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FEM SIMULATION OF HIGH VELOCITY SHOCK WAVES IN FIBER REINFORCED COMPOSITES

M. Žmindák[*], Z.Pelagič[*]

Abstract: *Fiber reinforced composites are very efficient in damping shock waves. Computational simulations enable to evaluate the damping properties and to design the structure of the fiber reinforced materials (FRM) and response of the whole structure to explosion and impact load. Especially important is the case when fibers are much stiffer than the matrix. The shock waves reflect, refract and interact in such material and the shock wave is damped and attenuated in this way. Material properties of fibers, matrix and volume content of both components are material design parameters, which define the structural response to explosion and impact load in computational simulations by multi-level modeling. We will present computational models for elastic material. The modulus of elasticity of fibres is 100 times larger than that of the matrix. .*

Keywords: ***Computational simulation, high velocity impact, fiber reinforced composite.***

1. Introduction

Composite materials have been used more extensively in the recent years in many fields of industry. Composite materials reinforced by stiff particles or fibers are important materials possessing excellent mechanical and also thermal and electro-magnetic properties. The main advantages of such materials are high strength and stiffness together with low weight, and possibility of designing desired mechanical properties of the structure in different locations and directions. This fact involves higher demands on the computational analysis, especially when compared to corresponding analysis of conventional materials. One of the important tasks is to assess the critical stress state for static or dynamic loadings. There are many criteria which are used to predict the failure of composite materials. The accuracy of prediction of failure strongly depends on the criterion (Kormaníková, et al., 2011).

Properties of fiber composites reinforced by long fibers significantly depend on the selection of fiber and matrix and on the way of how they are combined, fiber volume fraction, fiber length, fiber orientation, laminate thickness and presence of bond medium for improvement of fiber-matrix bond. Glass fibers have high strength at low costs; carbon fibers have very high strength, stiffness and low density. Kevlar fibers have high strength and low density, they prevent the spread of fire and are penetrable by transparent to radio waves. Polyesters are most often used matrices, because they offer good properties at relatively low costs. The best properties of epoxies and application of polyamides at temperatures predestinate them to special use, but they are expensive. Strength of composites increases by fiber volume fraction and fiber orientation parallel to load direction. By increasing length of fibers, the reinforcement is more effective at load carrying. Shorter fibers are better for manufacturing and less expensive.

In present time creating new scientific discipline „ Simulation-Based Engineering Science (SBES)“, which on basis of mathematical methods and computer simulated engineering system behavior. Computer simulation represents an extension of theoretical science in that it is based on mathematical models. Simulation also provides a powerful alternative to the techniques of experimental science and the observation when phenomena are not observable or when measurements are impractical or too expensive.

[*] Prof. Ing. Milan Žmindák, CSc.: University of Žilina, Faculty of Mechanical Engineering, Department of Applied Mechanics, Univerzitná 8215/1; 010 26, Žilina; SK, e-mail: milan.zmindak@fstroj.uniza.sk

[*] Zoran Pelagič, Bc. University of Žilina, Faculty of Mechanical Engineering, Department of Applied Mechanics, Univerzitná 8215/1; 010 26, Žilina; SK, e-mail: ropfly@gmail.com

Wave propagation in heterogeneous material is a very old and complex problem (Brepta, 1997; Okrouhlik, 2001). The phenomenon of material and geometric dispersion are so far very little studied. It is a complex problem with regard to interaction of pressure and tension phase waves generated on the boundary of an inhomogeneous material. The smaller the particles, the greater the number of material interfaces, which interact with each other and wave progresses, the greater the reduction and dispersion. Interaction of the bow shock wave with the secondary pressure waves resulting dissipation, shock-wave attenuation.

Currently, for wave propagation modeling in composite structures at high speeds are mainly used the Finite Element Method (FEM) (Bathe, 2011), Boundary Element Method (BEM) (Sládek, 2002), Fast Multipole BEM (Liu, 2009), respectively, Finite Volume Method (FVM), mesh free formulations and recently connection of FEM and element free based formulations, used in commercial program systems LS-DYNA (LS-DYNA ,2006), AUTODYN and PAM CRASH etc.. However very effective methods appear to be spectral FEM (SFEM) and wavelet methods. The SFEM is a numerical method evolved from the Fourier Transform based method. More details about the solution is given in (Tsai, 2006; Gopalakrishnan, 2008).

Shock wave propagation in heterogeneous materials is a complex matter (Datta, 2009). Phenomenon of material and geometric dispersion is yet poorly understood as complex pattern is generated by a continuous interaction of compression and rarefaction waves generated by inter-face in non-homogeneous material. The smaller is the particle size, the greater is the number of interfaces that interact with propagating stress waves and the higher is attenuation and dispersion. Interaction of leading shock front with secondary compression waves results in dissipation of the shock wave. The behaviour of materials by shock wave loading can be simulated and studied by commercial softwares like LS DYNA, AUTODYN, DYTRAN, ABAQUS, PAM-SHOCK, etc., but the models require large number of equations that have to be solved and resolved during propagation of the shock waves and so, very efficient computers/supercomputers are necessary for modelling of complicated problems.

The aim of this paper is to contribute better understanding and modelling of scattering and dispersion of shock waves using commercial software. It is supposed that readers are familiar with basics of continuum mechanics and basic methods of simulation classical problems of statics and dynamics using FEM. Some special methods and corresponding governing equations used especially in simulations of shock propagation in solids are introduced.

2. Wave propagation in elastic solids

In this section we first present the basic considerations about physical problems and then we mentioned about most important wave forms in solids. We note that we consider only an elastic, isotropic homogeneous isotropic medium.

2.1 Governing equations

We will not present here all governing equations for shock wave propagation as it would contain basic relations of continuum mechanics, which can the reader find in textbook of continuum mechanics (Malvern, 2007, Meyres, 1999 & Wu, 2005). These equations are used to describe:

- the kinematics of solid continuum, the equations which present relation between displacements and corresponding displacement gradient for finite displacements in material and spatial description, strain tensors for finite strain formulation, strain measures and strain rate tensors,
- material and spatial time derivatives of deformations, velocity and velocity gradients,
- corresponding stress measures,
- formulation of equilibrium,
- conservation equations (conservation of mass, momentum and energy),

Thermodynamic laws give:

- the first law – the conservation of total energy,
- the second law – change in entropy,
- thermodynamic potentials – internal energy, enthalpy, Helmholtz and Gibbs free energy.

Further, the constitutive equations, which have to be thermodynamically consistent, give the relation between stress and strain measures. Dynamic deformation processes, especially when shock wave formation is involved, are usually modelled by decomposed stress tensor. The decomposition splits the stress tensor into a deviatoric tensor S_{ij}

$$\sigma_{ij} = S_{ij} - p\,\delta_{ij} \tag{1}$$

The usefulness of the decomposition results from the needed nonlinear character of equations of state (EOS) to describe shock waves. In general, a pure material can be solid, fluid and gas.

We will deal further with solids and only with special problems concerning shock waves propagation. More general problems can be found in textbooks, e.g. (Hiermaier, 2008; Malvern 1969; Meyers at al., 1999; Wu, 2005). Dynamic compressive behaviour of materials at strain rates in the regime of 10^6 [s^{-1}] is typical for shock loading resulting elastic-plastic characteristic.

For most engineering applications involving equations of states, empirical relations with experimentally derived data are used. It is most simple representation is the so called *linear equation of state* which assumes isothermal processes and a linear pressure-volume or pressure-density relation. Via the bulk modulus K the linear equation of state is formulated as:

$$p = K_{\varepsilon kk} = K(\rho / \rho_0 - 1) = K\mu \tag{2}$$

with the compression term μ describing the ratio of change in volume and density ρ from its initial state, ρ_0.

For isotropic materials, the bulk modulus K is linked to the Young's modulus E and the shear modulus G via the Poisson ratio v by:

$$K = E/(3-6v) = 2G(1+v)/(3-6v) \tag{3}$$

meaning that the knowledge of any two other elastic constants provides the needed material dependent input for the linear equation of state.

Whenever the linear elastic region described in equation (2) is left, which is for example the case when a wide spectrum of pressure and energy shall be covered by the EOS, nonlinear relations are needed. A polynomial description of an equation of state can for instance be written as:

$$p = K_1\mu_1 + K_2\mu_2 + K_3\mu_3 + (B_0 + B_1\mu)\rho_0 \tag{4}$$

where K_i and B_i are material constants usually defined separately for compression and expansion, respectively. An important difference to the linear equation (2) is marked by the energy dependency, *e,* of the last term in (4). Whereas the linear equation is only a compression curve along an isotherm, the latter can really be called equation of state in the sense of (4).

The observation of shock wave propagation can provide information to identify the material parameters in (4). The underlying theory is composed of the thermomechanics of shock waves, i.e. essentially the Rankine-Hugoniot equations providing a line of reference configurations on the state surface, used to identify the parameters K_i and an assumption on the pressure change of the Hugoniot-line along isochores, defining the constants B_i.

In the case of quasi-static loads, wave effects are not investigated since the loading duration is long compared to the duration of multiple reflections throughout the structure. In addition, the resulting structural deformation and material state is not influenced in a comparable way by single wave transition. However, if the induced waves take the shape and amplitude of shock waves or the load speed is in the order of magnitude of the local sound speed, then wave effects and their propagation through the structure needs to be resolved in time and space.

2.2 Wave forms

Waves in solids are basically perturbations in the velocity field propagating through the continuum in different forms and at related different velocities. The propagating perturbation leads to wave form specific motion of the particles. The most important wave forms in solids are:

- *Longitudinal waves* of compressive or tensile type which cause particle deflections along propagating direction. They are the fastest wave forms in solids and are also called *primary waves* and the velocity is given by

$$c_L = \sqrt{\frac{E}{\rho}} \tag{5}$$

  The next fastest waves are the *shear* or *secondary waves* causing particle motion perpendicular to the wave propagation. The speed is given by

$$c_S = \sqrt{\frac{G}{\rho}} \tag{6}$$

  where G is shear modulus of material.
- Along the surface of solids propagate so called *Rayleigh waves* setting surface particles into elliptic motion and decaying in direction perpendicular to the surface.
- In structures of finite bending stiffness *flexural wave* propagate upon dynamic loading. In structures of complicated form a complicated combination of all basic wave forms can be observed.

Characteristic properties of all shock waves are extremely short rise times as well as high pressure, density and temperature amplitudes. Basically, shock waves can arise as a sequence of both wave superposition and dispersion effects:

- If the source of a pressure disturbance is moving at a speed of sound of the surrounding medium or faster, superposition of the propagated disturbance and thus pressure waves leads to increased amplitudes and pressure gradients.
- In case of nonlinear pressure-density relations the corresponding dispersion effects lead to the formation of shock waves if faster wave components overtake earlier induced waves of lower propagation speed.

External dynamic compressive loads, initiated e.g. by impact or detonation, can possibly cause very strong waves with extremely short rise times inside structure. Superposition of different wave components is responsible for the steepening of the wave front. Superposition takes place as a consequence of dispersion, an effect that arises with nonlinear compressive behaviour.

In the initial elastic regime (p_0, V_0) compressive waves are propagated at the elastic wave speed (see Fig.1)

$$c_{elastic} = c_0 = -\frac{1}{V^2}\sqrt{\left.\frac{\partial p}{\partial V}\right|_0} \tag{7}$$

As the load rises to higher pressures beyond the plastic threshold, the gradient and thus the propagation speed decreases drastically. Enhancement of pressure beyond the state of (p_1 , V_1) leads to a gradual increase of the modulus. From that turn around point onwards, pressure waves are initiated that propagate faster than others before. Consequently, a superposition of slower wave packages by faster ones with higher amplitude occurs.

In the light of these observations and with a mathematical description for the slopes in the $p-V$ diagram of **Chyba! Nenašiel sa žiaden zdroj odkazov.**, conditions for the formation of shock waves can be formulated as:

$$\frac{\partial p}{\partial V} < 0 \tag{8}$$

$$\frac{\partial^2 p}{\partial V^2} > 0 \tag{9}$$

In materials with an elastic-plastic compressive behaviour according **Chyba! Nenašiel sa žiaden zdroj odkazov.**, only loading conditions achieving pressures of $p1$ or more can lead to shock waves. In gases and fluids, however, compression shocks can arise from ambient pressures onwards since no regions with $\left(\partial^2 p / \partial V^2\right) \leq 0$ exist.

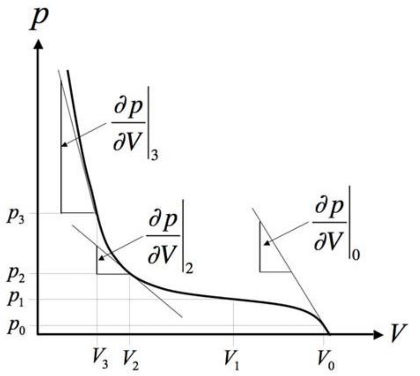


Fig. 1 Nonlinear compression curve of a solid elastic-plastic material allowing for dispersion driven shock waves.

Another necessary precondition for the shock formation is the rapid loading. Imagine a quasi-static load application to a pressure level indicated by p_1 in **Chyba! Nenašiel sa žiaden zdroj odkazov.**. Still, information about the applied load would be transported by waves at the sound speed defined by dispersion effects, i.e. depending on $\partial p/\partial V$. But time delay for each pressure increment along a certain equilibrium path would avoid the formation of a shock wave.

Only if the load application is fast enough, the wave fronts of the faster packages keep up with the earlier wave fronts. The result is a steepened wave front and shorter rise times to higher pressures. Often, the wave components from the elastic regime are fast enough that a so called *elastic precursor* is formed. It is, however, also possible that even the elastic precursor is overtaken by very fast plastic waves. Whether or not this happens is only a matter of the load application speed and the achieved maximum pressure level.

3. Computational simulations

As it was shown in the previous section the shock wave velocity is influenced by material properties, modulus of elasticity, material density, temperature, etc. Modern composite materials are reinforced by particles, fibres or layers from materials of much higher stiffness than that of the matrix. Such materials are important for many applications, as its stiffness and strength is often much higher than that of the homogeneous material (Harper, 1971; Kompiš et al., 2010; Kompiš et al., 2011). Also the dynamical properties of such materials differ from homogeneous materials by much higher damping which is important for impact by low and high velocities, but also by other loading conditions. Computational simulations described below document great importance of the damping in materials reinforced by fibres.

In following example the composite material with modulus of elasticity and density equal to 210 GPa and 7830 kg m^{-3}, respectively, is reinforced by straight fibres regularly distributed parallel to the upper surface (see Fig. 2). The modulus of elasticity of fibres is 100 times larger than that of the matrix. The loading of the material is perpendicular to the surface and it is increasing from zero to 0.0315 GPa in 0.05 μs and decreasing back to zero in same time.

In order to describe the stress behaviour of a FRM, the material is modelled as a Representative Volume Element (RVE). This element describes the homogenous behaviour of the two phases, matrix and fiber, inside the RVE. Simulations are carried out with fibers with different diameters and volume fraction of fibers. We assume perfect adhesion between the fibers and matrix.

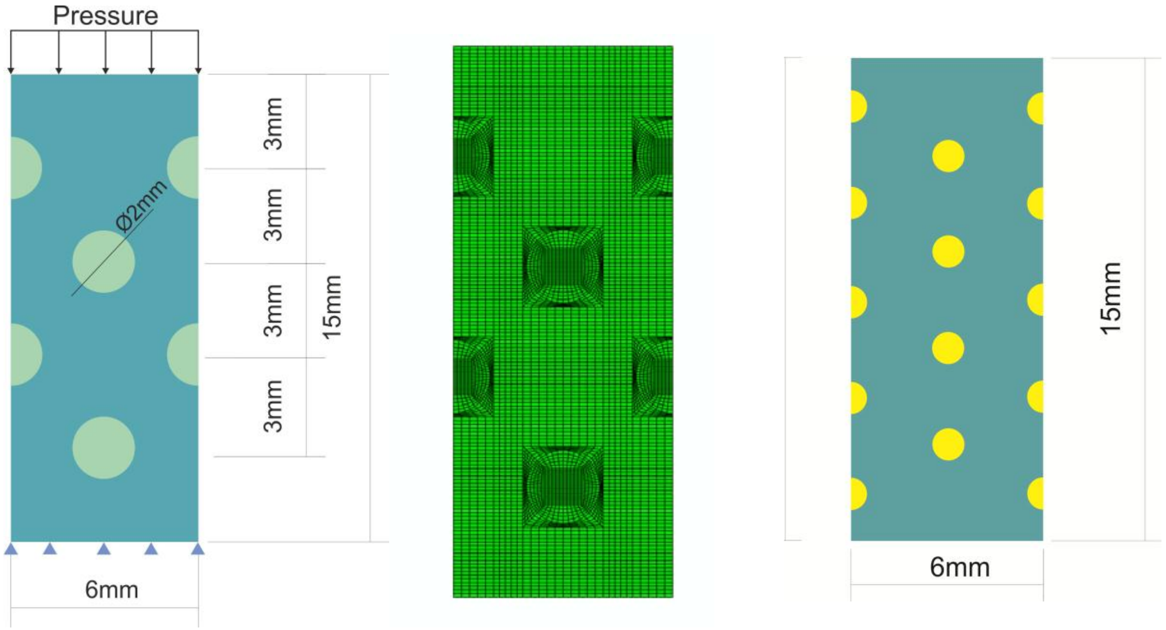
It is a 2D problem (plain stress, t =1) and computational simulations were performed in FE software ABAQUS. The geometric parameters of the RVE are given in fig. 2. The regular FE mesh of the model consist CPS4R, 4-noded, bilinear plane stress quadrilateral element with linear base functions with hourglass control. Most number of elements (27 348) and nodes (27856) has variant 3. The problem was solved in 2000 cycles with time step 2.5 x 10^{-10} s in the model.

Boundary conditions, FE mesh are described in fig 2a and in fig. 2b is described the mesh of 14 fibers. Dimensions are the same as for RVE with 6 fibers. The pressure load is applied on the upper side of RVE. The bottom side of RVE is fixed in the Y-direction and axis symmetry was applied on the both vertical sides. Calculations were made for following 4 variants:

Variant 0 - model without fibers.

Variant 1 - model with fiber radius r_f= 1 mm and volume fraction of fiber v_f= 35%.
Variant 2 - model with r_f= 0.5 mm and v_f = 17,5%.
Variant 3 - model with r_f= 0.5mm and v_f= 35%.

In fig. 3a are described coordinates of investigated point for all four variants and in tab. 1 are their numerical values.

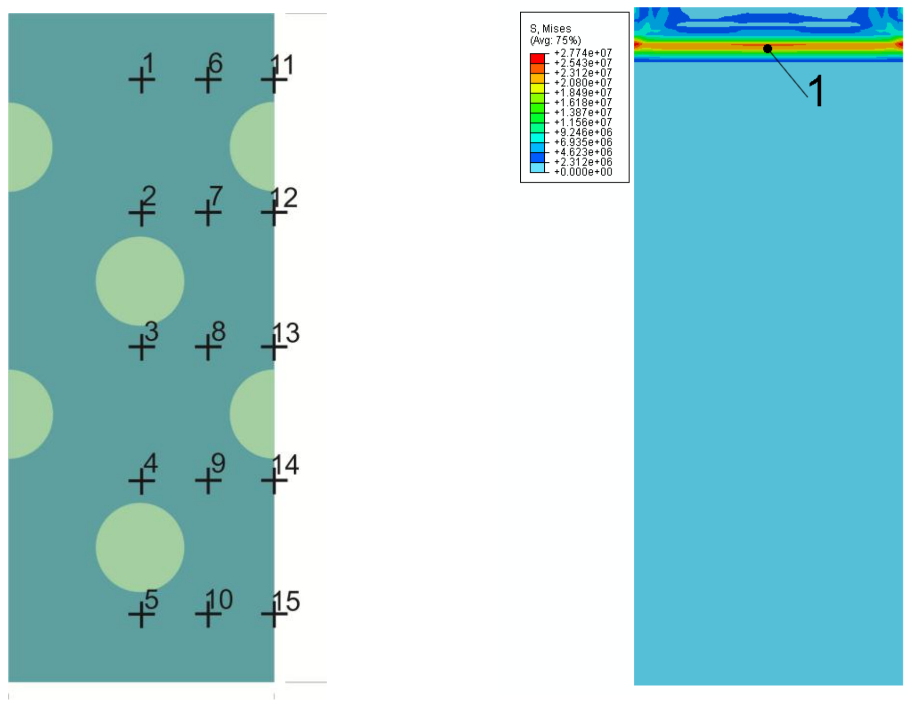


a) Boundary conditions and FEM mesh *b) Mesh of 14 fibers*

Fig. 2 Problem definition and boundary conditions

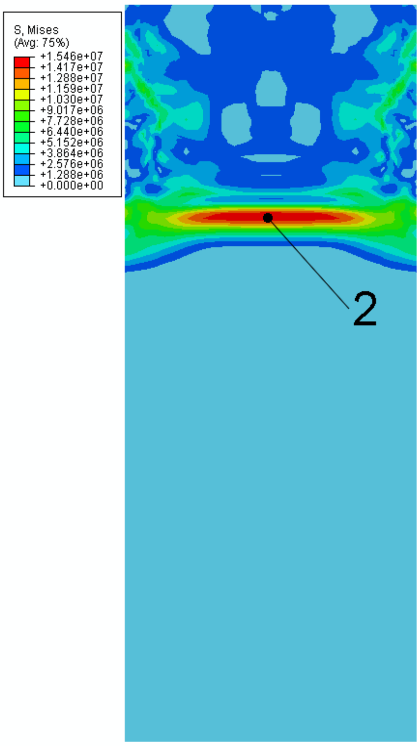
Tab. 1: Coordinates of investigated points

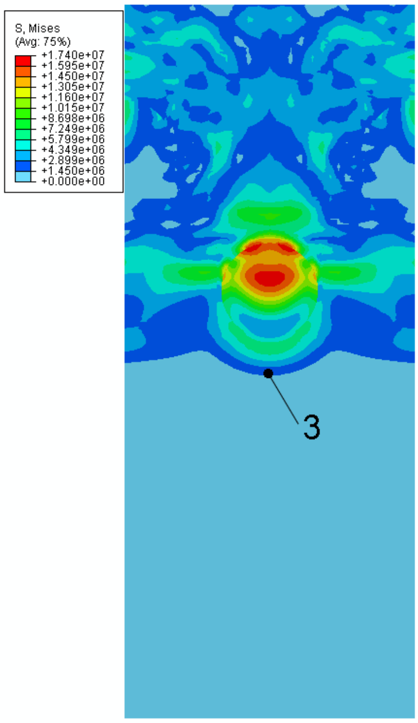
Point:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
X[mm]	0	0	0	0	0	1.5	1.5	1.5	1.5	1.5	3	3	3	3	3
Y[mm]	6	3	0	-3	-6	6	3	0	-3	-6	6	3	0	-3	-6



a) evaluated points

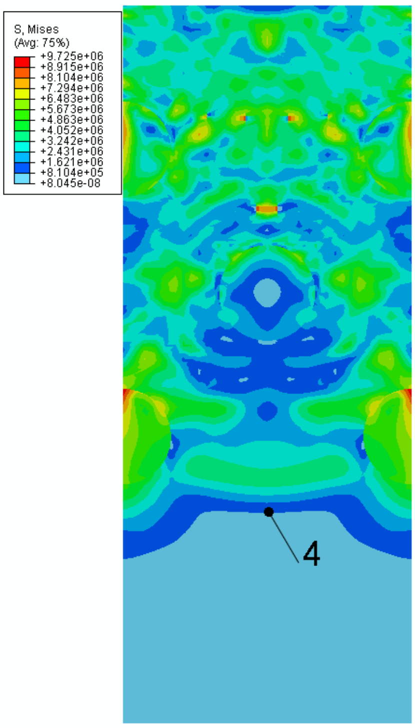
b) Point 1, t = 2.3275 E-07 s

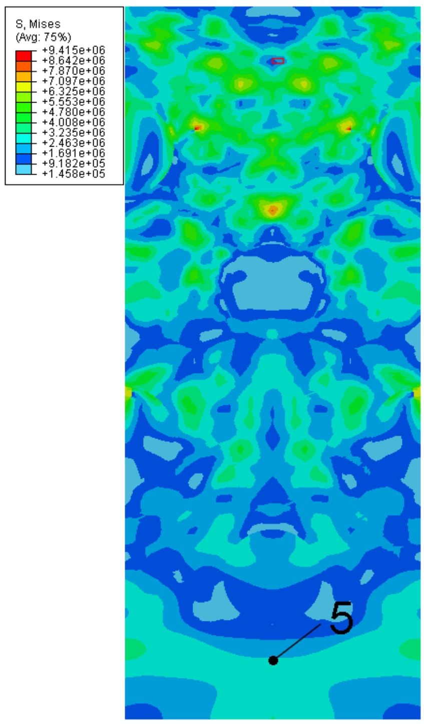




c) Point 2, t =8.4774 E-07 s

d) Point 3, t = 1.1477 E-06 s

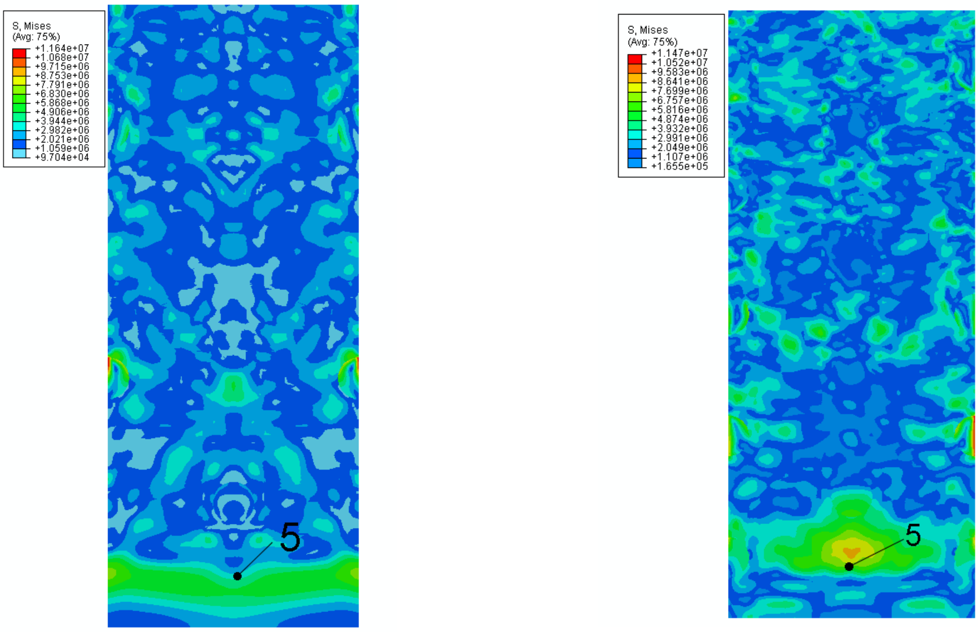




e) Point 4, t = 2.6301 E-06 s

f) Point 5, t = 2.81165 E-06

Fig. 3 Von Mises stress in investigate points 1 to 5 for variant 1



a) Variant 2, *t* = 3.0227 E-06 b) Variant 3, *t* =3.005 E-06

Fig. 4 Von Mises stress in point 5

At beginning the wave propagates parallel to the surface without any interaction (Fig.3b). After reaching first fibres the wave reflects from the fibre and interact with reflected part, however, the front of the wave is still expressive (Fig.3c). After the front of wave continues to propagate to lower part under the surface, the maximum in corresponding point is not as high as in many other points closer to the surface. Fig. 3e and fig.3f corresponds to the moment when the effective stress is maximal in point 5, but the stresses in points closer to the upper surface are larger because of complicated interactions of the waves.

Fig. 4a and fig. 4b shows the same situation for variant 2 and variant 3. Fig.5 to fig.6 shows time course of the von Misses stresses after the shock achieving the surface of the material all in points 1 to 5. The red colour corresponds to the point 1 close to the surface and other colours to the other points below the first one. From the figures we can find the movement of the front as well the maximum of the stress in time. In fig.7 and fig.8 are described time courses of the von Mises stress for variant 2 and variant 3.

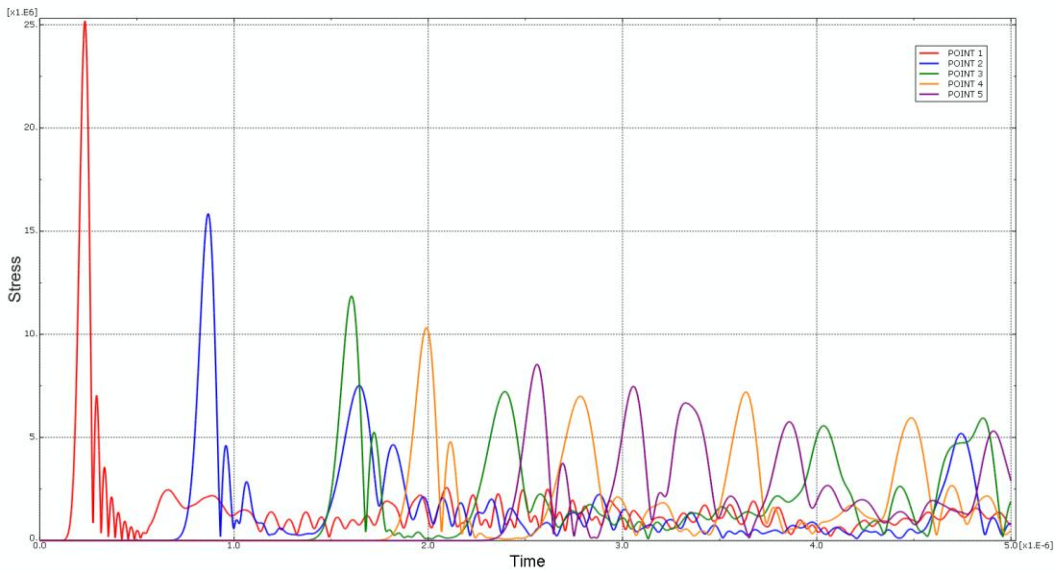


Fig. 5 Time course of the von Mises stress for variant 0

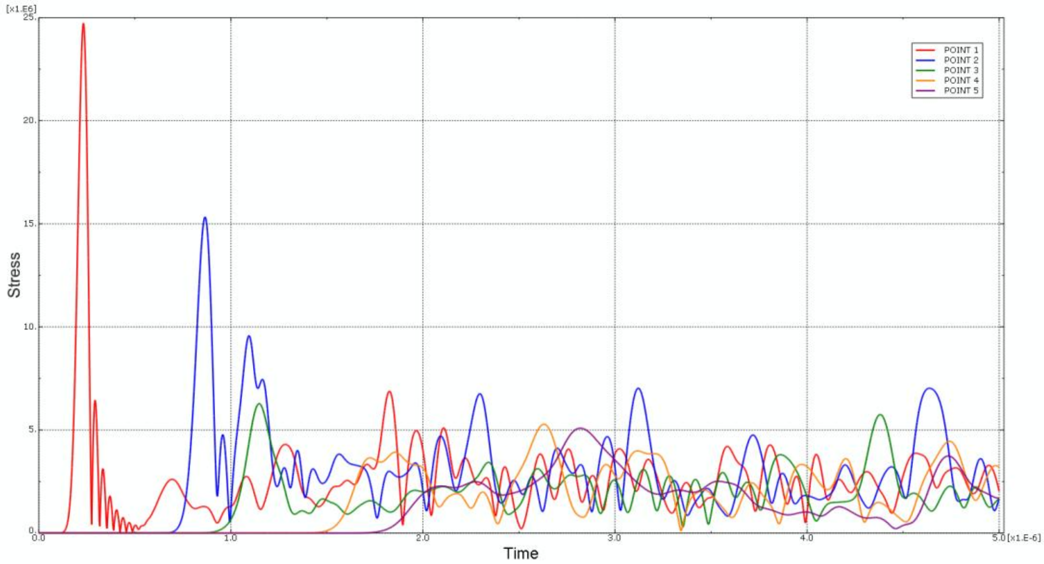


Fig. 6 Time course of the von Mises stress for variant 1

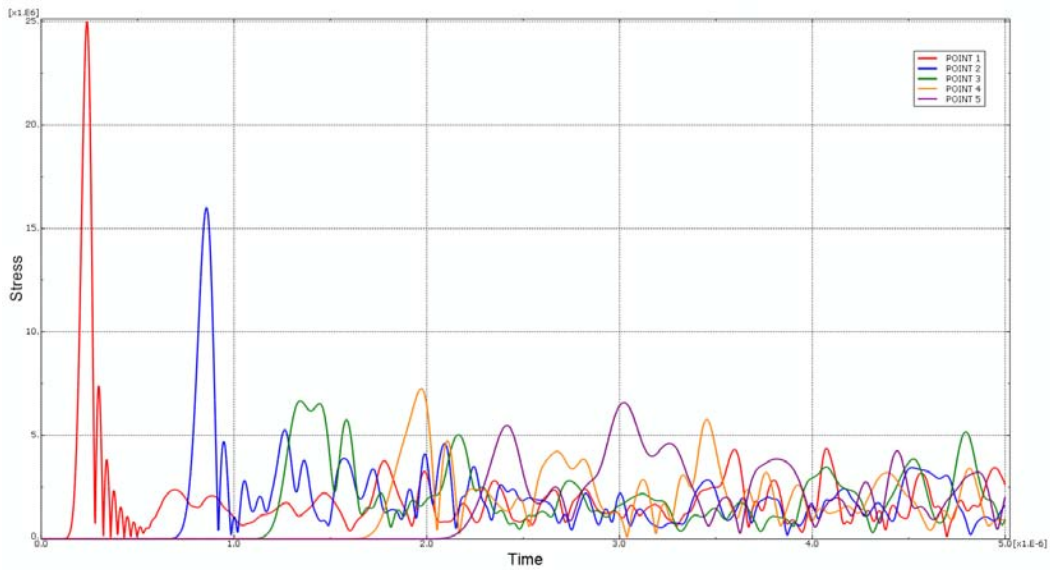


Fig. 7 Time course of the von Mises stress for variant 2

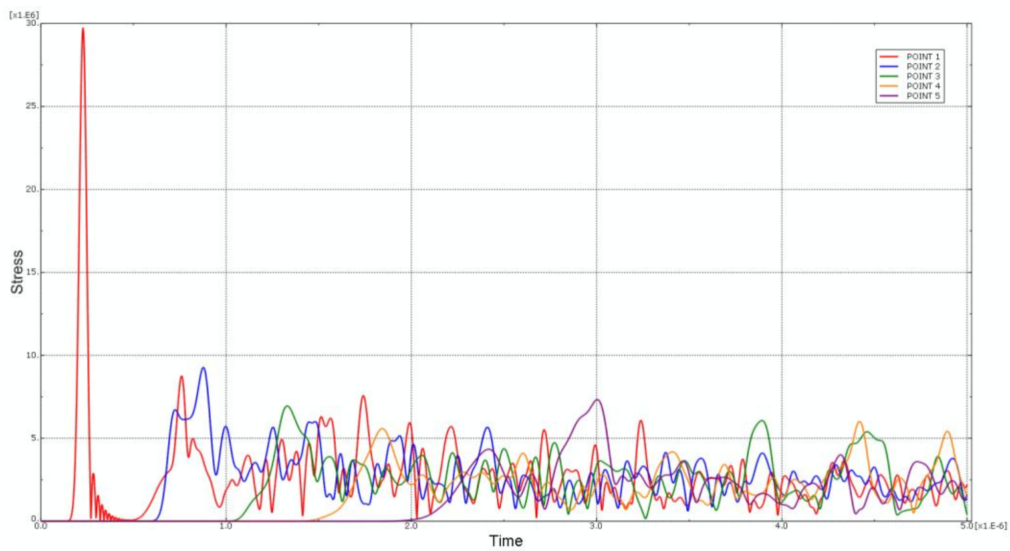


Fig. 8 Time course of the von Mises stress for variant 3

Finally, all results are summarised in tab.1, which gives maximum von Mises stress for all 4 variants in 15 investigated points throughout the period of time. In fig.9 is course of maximum von Mises stress for all variants and all investigated points. The sharp drop in maximum von Mises stress from point 1 to point 2 and 11 to 12 for variant 3 is due to passing stress wave through the fiber. The von Mises stress drop is modest from point 6 to 11 because the plane defined by points 6-10 is between fibers.

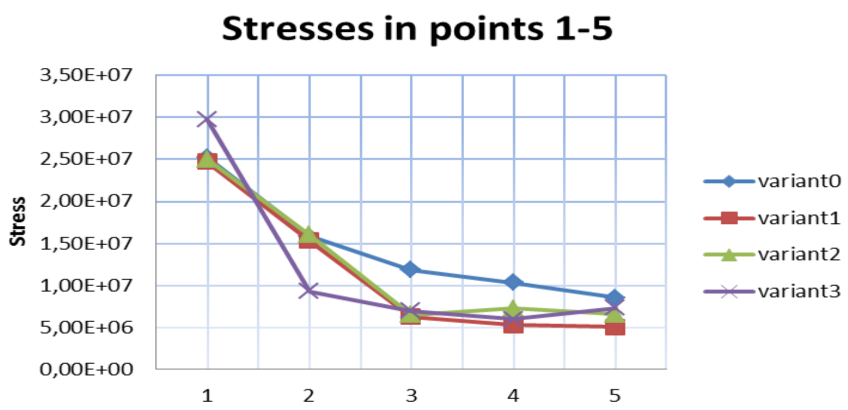
Tab. 1: maximum von Mises stresses (Pa) in points for different variants

Point	Variant 0	Variant 1	Variant 2	Variant 3
1	2.51873E+007	2.46752E+007	2.50021E+007	2.97331E+007
2	1.58461E+007	1.53314E+007	1.60142E+007	9.27268E+006
3	1.18519E+007	6.27792E+006	6.53354E+006	6.94687E+006
4	1.03242E+007	5.28174E+006	7.27257E+006	6.0145E+006
5	8.54604E+006	5.08785E+006	6.58414E+006	7.32889E+006
6	2.51873E+007	2.47847E+007	2.43585E+007	2.96361E+007
7	1.52088E+007	1.25627E+007	1.47267E+007	1.94574E+007
8	1.08391E+007	6.52019E+006	1.05614E+007	1.1574E+007
9	9.49899E+006	3.99877E+006	7.04418E+006	7.59478E+006
10	9.75734E+006	5.03414E+006	7.21751E+006	5.78076E+006
11	2.80318E+007	2.44781E+007	2.80414E+007	7.74334E+007
12	1.7466E+007	6.97456E+006	1.01383E+007	1.98204E+007
13	1.3209E+007	6.71266E+006	8.95498E+006	1.49838E+007
14	1.18789E+007	5.92797E+006	7.30872E+006	1.07567E+007
15	1.04222E+007	3.62426E+006	1.02094E+007	1.09097E+007

If we define damping ratio as von Mises stress drop in 3 planes vertical to the upper surface defined by investigated points 1-5, 6-10 and 11-15 as

$$Dampingratio = \frac{(\sigma_{von})_i - (\sigma_{von})_{i-4}}{(\sigma_{von})_i}, \; i = 5,\, 10,\, 15, \qquad (10)$$

then damping ratios of stress waves for going through indicated planes are given in tab. 2. As expected, the largest value of damping ratio is for variant 3 in plane 11-15 and is 0.8591. This means that the drop of maximum von Mises stress is almost 86 percentage.



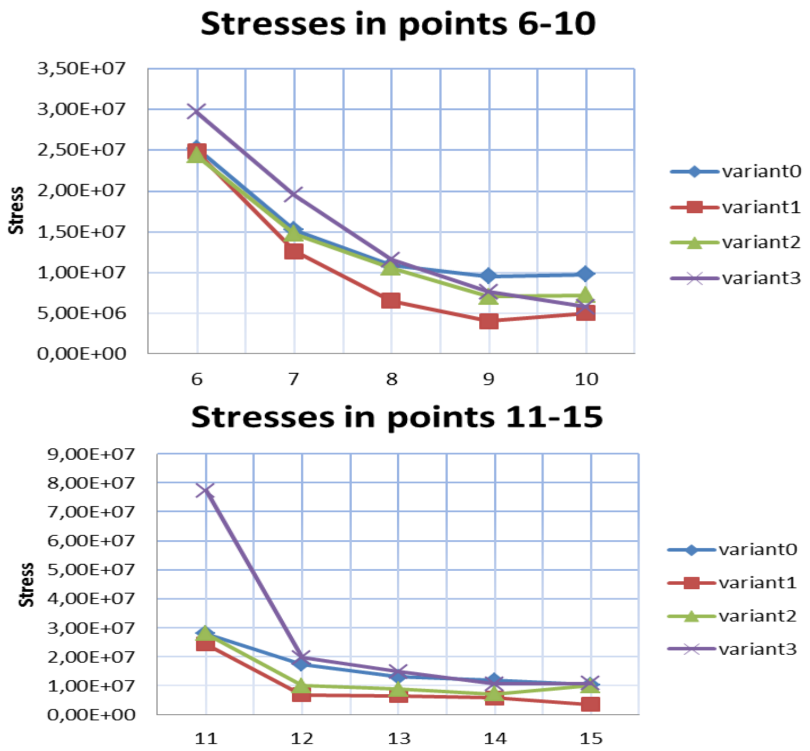


Fig. 9 Course of von Mises stress in investigated points

Tab: 2 Damping ratios of stress waves

Plane	Variant 0	Variant 1	Variant 2	Variant3
1 - 5	0.6607	0.7938	0.7367	0.7535
6-10	0.6126	0.7969	0.7037	0.8049
11-15	0.6282	0.8519	0.6359	0.8591

We note that computational models do not contain any other damping except of the interaction of the shock wave with fibres (each material contains some imperfections in the structure and in material properties and so, there is some material damping also in homogeneous material) and shows as the reinforcing fibres because of very different material properties of both material components result in very efficient damping of shock waves and thus such composite can be very efficient in defence against explosion.

The shock wave in homogeneous material is not influenced by propagation through material and only when it is reflected on the boundaries there is an interaction with propagated wave [Hermaier, (2008); Kompiš, at al. (2010)]. On the other side there is very complicated interaction of the wave by reflection, and refraction on the interface between softer matrix and stiffer fibres leading to strong damping of the shock wave.

4. Conclusions

The problem of shock wave propagation was studied in this work. Computational simulations were performed in FE software ABAQUS. The FEM simulation was based on the RVE model. Simulations are carried out with fibers with different diameters and volume fraction of fiber. The proposed procedure allow very effectively without expensive experiments to study the behaviour of composite materials from all points of view, the material structure topology, material properties of components, percentage of reinforcement, etc. That the longest calculation time was for variant 3 and used CPU time was only 1:47:17 on Pentium desktop computer with Intel core i5 with 2,6 GHz frequency and 8 GB RAM.

Acknowledgement

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References

ABAQUS 6.11 (2011), *Theory-manual*, DS Simulia, Providence, RI.

Barker, L. M. & Barker, Z. B. (2006) *The VALYN multi-beam VISAR*, Valyn Press, Albuquerque, New Mexico.

Bathe, K.J., (2011) A finite element method enriched for wave propagation problems, Int. J. of Computers and Structures (to be published).

Brepta, R (1997) *Shocks and stress waves in solid elastic bodies*. CVUT publisher, (in Czech)

Datta, K.S., Shah, A.H. (2009) *Elastic Waves in Composite Media and Structures wit Applications to ultrasonic Nondestructive Evaluation*, CRC Press, Taylor & Francis Group , LLC.

Gopalakrishnan, S., Chakraborty, A., Mahapatra D.R. (2008) *Spectral Finite element Method, Wave Propagation, diagnostics and Control in Anisotropic and Inhomogeneous Structures*. Springer –Verlag London Limited.

Harper, E. Y. (1971) *Method and means for attenuating shock waves propagating within a solid*, US Patent 3592147.

Hiermaier, S. J. (2008) *Structures under crash and impact, Continuum mechanics, discretization and experimental characterization*, Springer, New York.

Kompiš, V., Vančo, M. & Ferencey, V. (2010) Shock waves in composite materials, *Mechanical Engineering Journal*, pp 73-87.

Kompiš, V., Qin, Q.H., Fu, Z., Chen, C.S., Droppa, P., Kelemen & M., Chen, W. (2011) Parallel computational models for composites reinforced by short fibres, *Engineering Analysis with Boundary Elements*, 36, pp. 47-52.

Kormaníková, E., Riecky, D., Žmindák, M. (2011): Strength of Composites with Short Fibers. In: Eds, J. Murín: *Computational Modelling and Advanced Simulations*, Springer Series: Computational Methods in Applied Sciences, Vol. 24.

Liu, Y. (2009) Fast Multipole Boundary Element Method, Theory and Applications in Engineering, Cambridge University Press.

LS DYNA theory-manual, Livermore Software Technology Corporation, Livermore, California, 2006.

Malvern, L.E. (1969) *Introduction to mechanics of a continuous medium*, Prentice Hall, Englewood Cliffs, New Jersey.

Mader, C.R. (2008) *Numerical modeling of explosives and propellants*, CRC Press Taylor & Francis, Boca Raton.

Meyers, M.A., Armstrong, R.W. & Kirchner, H.O.K. (1999) *Mechanics and materials. Fundamentals and linkages*, John Wiley & Sons, New York.

Meyers, M. A. (1994) *Dynamic behaviour of materials*, John Wiley, New York.

Okrouhlik M. (2001) Computational Aspects of Stress Waves Problems in Solids . *2nd European Conference on Computational Mechanics*, Abstracts Vol. 1, Cracow, Poland, pp. 1- 30.

Sládek, J, Sládek, V, Jakubovičová, J., (2002) *Application of Boundary Element Methods in Fracture Mechanics*. University of Žilina, Faculty of Mechanical Engineering, Žilina.

Tsai, L. (2006) *Shock wave structure and spall strength of layered heterogeneous glass polymer composite*, PhD Thesis, Case Western Reserve University.

Wu, H.C. (2005) *Continuum mechanics and plasticity*, Chapman and Hall/CRC London.