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INPUT SHAPING CONTROL OF ELECTRONIC CAMS

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Abstract: *The paper deals with the input shaping control of electronic cams that eliminates their residual vibrations. The models of several different kinds of electronic cams are described, i.e. the simple traditional one, the serial one, the parallel one, the multi-input one. Then the input shaping control and its generalization is described. The generalization means the shaper of arbitrary time length and/or of arbitrary rate combined with the set of shaping functions of reentry kind. It is demonstrated and explained that some more complex shaping functions in comparison with the simple Heaviside pulse shapers are more robust against model misalignment. This generalized input shaping control is applied to different kinds of electronic cams.*

Keywords: ***Input shaping, electronic cams, residual vibration, reentry commands***

1. Introduction

Conventional cam drives in modern machines can be replaced by properly controlled servomotors (Jirásko, 2010). This concept is generally called an electronic cam and can be further divided into several groups according to system structure – e.g. serial, parallel or multi-input electronic cams. The demand for fast and precise positioning is crucial in all mentioned cases but it could be easily corrupted by the residual vibration. To suppress the unwanted dynamics of flexible system the standard control input can be reshaped in such a way that it doesn't excite flexible modes or, more generally, that all the energy put into flexible modes is completely relieved at the end of the travel (Miu, 1989). The difference between original unshaped and shaped signal as well as the response of the two-mass model is shown in Fig. 1.

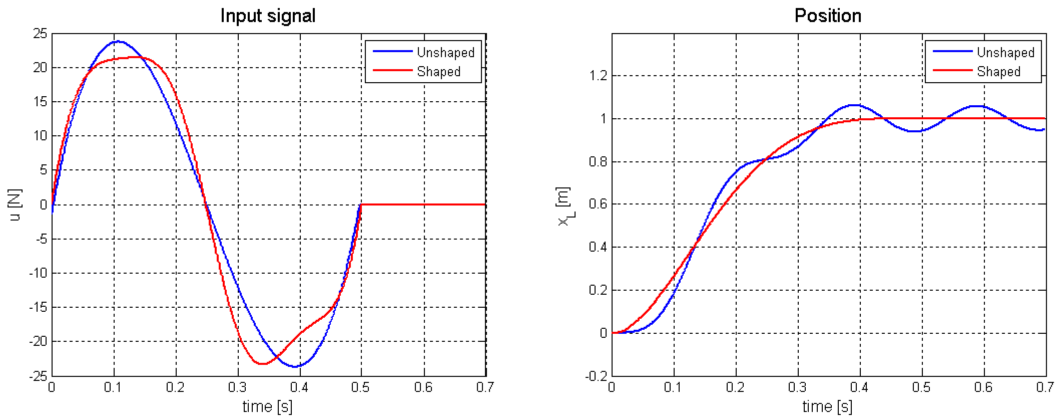

Fig. 1:Comparison of shaped and unshaped control input.

2. Necessary conditions

The control input that ensures no-vibration positioning has to fulfill some necessary conditions. For the system described using state space formulation as

$$\dot{\boldsymbol{y}} = \boldsymbol{A}\boldsymbol{y}(t) + \boldsymbol{B}\boldsymbol{u}(t), \tag{1}$$

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these conditions can be derived in the form

$$\sum_{l=1}^{n} U_l(s)|_{s=\boldsymbol{A}} \cdot \boldsymbol{b}_l = e^{-\boldsymbol{A}T}\boldsymbol{y}(t_2) - e^{-\boldsymbol{A}T}\boldsymbol{y}(t_1), \tag{2}$$

where $U_l(s)$ is the finite time Laplace transform (Miu, 1989) of the l-th input, $\boldsymbol{b}_l$ is the corresponding column of $\boldsymbol{B}$ matrix, t_1 and t_2 is the start and the finish time, n is the number of inputs. The solution $u_l(t)$ in the time domain is the inverse Laplace transform of $U_l(s)$.

Now this approach would be applied to the simple electronic cam that can be modeled as a two mass spring-dumper system in Fig. 2.

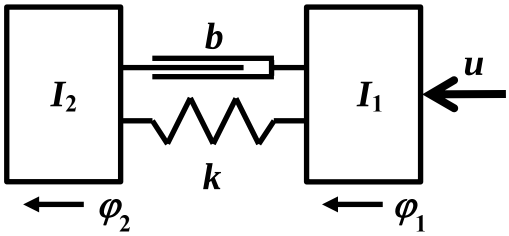

Fig. 2: Two-mass model of the electronic cam.

This system is described by the equation

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \boldsymbol{B}\dot{\boldsymbol{x}}(t) + \boldsymbol{K}\boldsymbol{x}(t) = \boldsymbol{F}(t), \tag{3}$$

where

$$\boldsymbol{x} = \begin{bmatrix}\varphi_1\\ \varphi_2\end{bmatrix}, \boldsymbol{M} = \begin{bmatrix}I_1 & 0\\ 0 & I_2\end{bmatrix}, \boldsymbol{B} = \begin{bmatrix}b & -b\\ -b & b\end{bmatrix}, \boldsymbol{K} = \begin{bmatrix}k & -k\\ -k & k\end{bmatrix}, \boldsymbol{F} = \begin{bmatrix}u\\ 0\end{bmatrix}. \tag{4}$$

No-vibration conditions in the final position Φ_f of point-to-point translation are

$$\boldsymbol{x}_\mathrm{f} = \begin{bmatrix}\Phi_\mathrm{f}\\ \Phi_\mathrm{f}\end{bmatrix}, \dot{\boldsymbol{x}}_\mathrm{f} = \begin{bmatrix}0\\ 0\end{bmatrix}. \tag{5}$$

The differential equation of the second order (3) can be rewritten as a set of first order equations and transform to the Jordan canonical form

$$\underbrace{\begin{bmatrix}\dot{y}_1\\ \dot{y}_2\\ \dot{y}_3\\ \dot{y}_4\end{bmatrix}}_{\dot{\boldsymbol{y}}} = \underbrace{\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & p & 0\\ 0 & 0 & 0 & p^*\end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix}y_1\\ y_2\\ y_3\\ y_4\end{bmatrix}}_{\boldsymbol{y}} + \underbrace{\begin{bmatrix}0\\ 1\\ 1\\ 1\end{bmatrix}}_{\boldsymbol{B}} u, \tag{6}$$

where p and p^* are complex conjugated poles of flexible modes.

The boundary conditions (5) are transformed to the equation

$$\boldsymbol{y}(t_2) = [\Phi_\mathrm{f}; 0; 0; 0]^T. \tag{7}$$

Assuming $t_1 = 0$ and zero initial conditions equation (2) can be rewritten in the component form

$$\begin{aligned} \frac{dU(s)}{ds}|_{s=0} &= \Phi_\mathrm{f},\\ U(s)|_{s=0} &= 0,\\ U(s)|_{s=p} &= 0,\\ U(s)|_{s=p^*} &= 0. \end{aligned} \tag{8}$$

This simple analytical formulation of necessary conditions for no-vibration translations used by Bhat & Miu (1991) is the result of the system description in the canonical form. Other state space representations usually need a numerical solution of (2).

Described approach leads to the control input in the form of pre-computed curve. However if it is rewritten to the form of a dynamical block it acts like a filter that transform any arbitrary signal to no-vibration one (Beneš & Valášek, 2008). And in contrast with patented input shaping technique by Singhose & Seering (1990) the length of this shaper is not dependent on the system natural frequency and can be set arbitrary.

3. Additional conditions and the control input synthesis

There are an infinite number of input functions $u(t)$ that fulfill equations (8). But these only ensure zero residual vibration. Therefore additional restrictions have to be applied e.g. for the to time domain continuity of the input signal

$$u(0) = 0, u(t_2) = 0. \tag{9}$$

Other restrictions are defined by maximal torque and rate of the actuator available etc.

The straight forward method of the control input synthesis is to assume analytical form of the control input with some variable parameters, e.g. the polynomial function with unknown coefficients

$$u(t) = \sum_{i=0}^{n} a_i \cdot t^i. \tag{10}$$

The exact value of parameters a_i is then calculated with respect to (8) and all other defined restrictions. It is possible to use some optimization methods as well.

Generally the number of parameters should be at least the same as the number of restricted conditions, however a smart choice of an analytical form of the input could automatically filled some of them. For example this form of a control input

$$u(t) = \sum_{i=1}^{n} a_i \cdot \sin\left(\frac{2\pi \cdot i}{t_2} t\right) \tag{11}$$

automatically filled conditions (9).

4. Serial electronic cam

The two-mass model in Fig. 2 is probably the most common demonstrator of different input shaping methods. Speaking about electronic cams the two masses represent the actuator (index 1) and the cam (index 2). The solution of the two-mass problem ensures precise positioning of the cam only. But in real systems the cam is connected to the rest of the system that usually has its own flexibility. The modified serial structure consisted of three bodies is in Fig. 3.

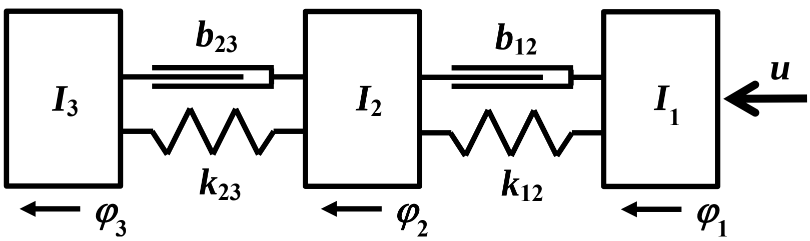

Fig. 3: Serial structure of electronic cam.

This system has two pairs of complex conjugated flexible modes and the rigid body mode. For simulation experiments it was described according to (1) and the control input was considered in the form of a polynomial function. Coefficient were calculated using (2) for $t_1 = 0\,s$, $t_2 = 1\,s$ and $\Phi_f = 1\,rad$. The value of all masses was set to 1 *Nm²*, stiffness 100 *Nm/rad* and the dumping was neglected. The simulated system response is shown in Fig. 4.

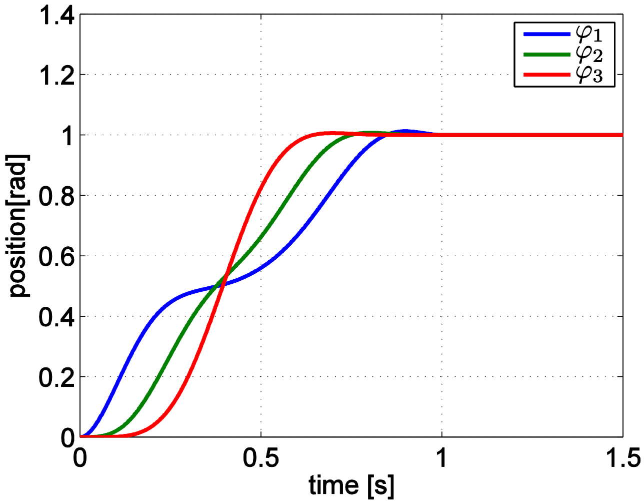

Fig. 4: Serial electronic cam – system response.

5. Parallel/two-input electronic cam

Input shaping techniques are usually applied only to systems with a single input. However many systems has two or more actuators. The schema of simple electronic cam with two parallel inputs is shown in Fig. 5. The position of the cam I_2 is controlled by actuators I_1 and I_3. Both actuators act on the same axis. The real application of this structure is that I_3 is primary force element, e.g. asynchronous motor, but with low accuracy of positioning. The I_1 is a fast servo motor that ensures precise positioning and/or vibration suppression.

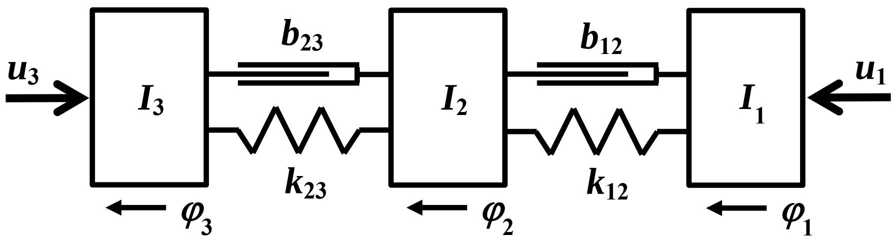

Fig. 5: Parallel/two-input structure of electronic cam.

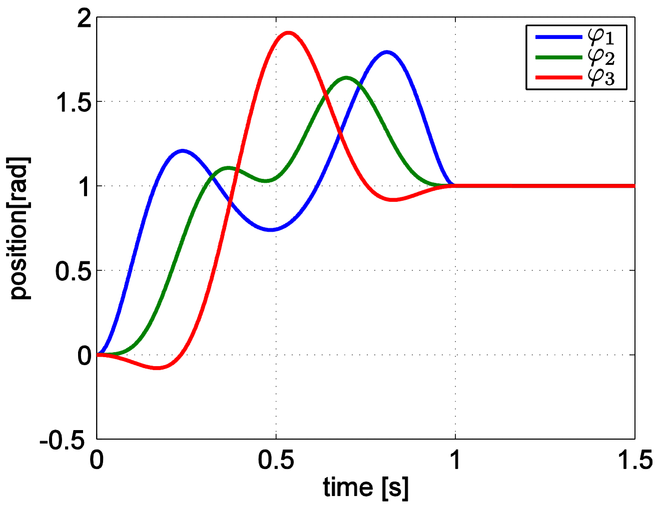

Fig. 6: Two-input electronic cam – system response.

Parameters of the model were the same as in the previous chapter and the control input u_1 was considered in the form of a polynomial function (10). The input u_3 was set as a constant. The values of coefficients were calculated using (2). Simulated system response is shown in Fig. 6. Note that there is no residual vibration but all the masses slightly “overshoot” the final position during the travel. The reason is that no restrictions were defined to deal with this problem, but it is possible to add them to existing calculating procedure.

6. Robustness

Being a feed-forward method all control shaping techniques need precise system models. The vibration suppression is in fact caused by placing zeros of the control input into the poles of the system. Therefore incorrect system model causes that the control input is not design properly and vibrations are not canceled. To increase robustness to modeling errors it is possible to formulate additional constrains that either introduced more zeros to the control input or increase the order of existing ones. The price for that is the increase of necessary acting force or longer settling time. Comparison of spectral analysis of a standard shaper and a robust one is in Fig. 7.

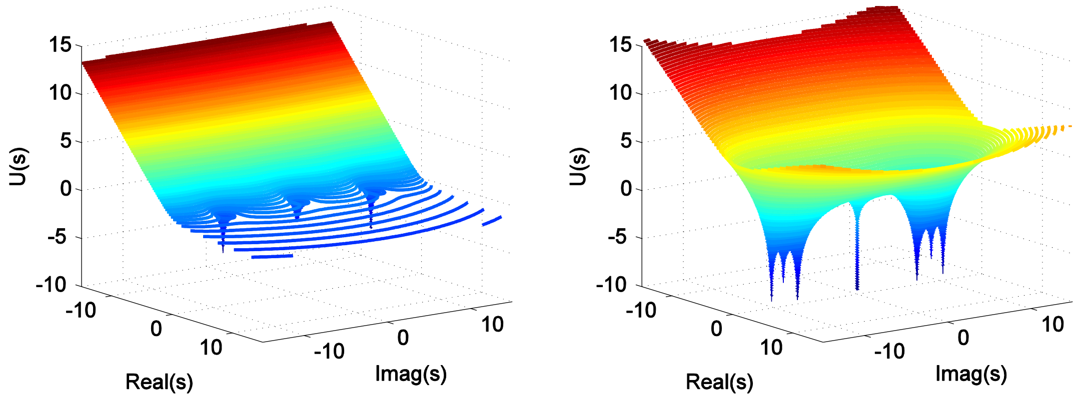

Fig. 7: Spectral analysis of a) standard shaper b) robust shaper.

7. Experiment

The test stand that was used for an experimental evaluation of simulations is in Fig. 8. Its structure is similar to Fig. 2, but the gearbox with ratio 1:5 was added. So for desired position $\varphi_2(t_2) = 2\pi\, rad$ the motor position has to be $\varphi_1(t_2) = 10\pi\, rad$. The settling time was $t_2 = 0{,}5\, s$. The control input was in the form (11) and two zeros were added to control input nearby modeling system poles.

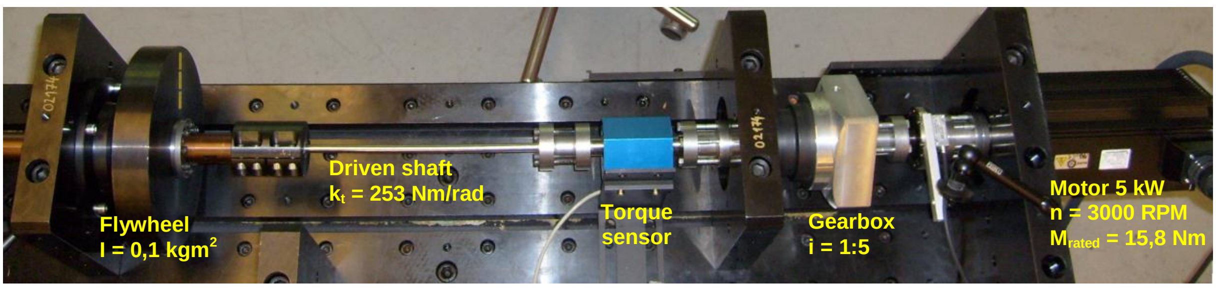

Fig. 8: Test stand.

The computed control input and the system response are in Fig. 9 and Fig. 10. The experiment proved simulation results and no vibration appeared.

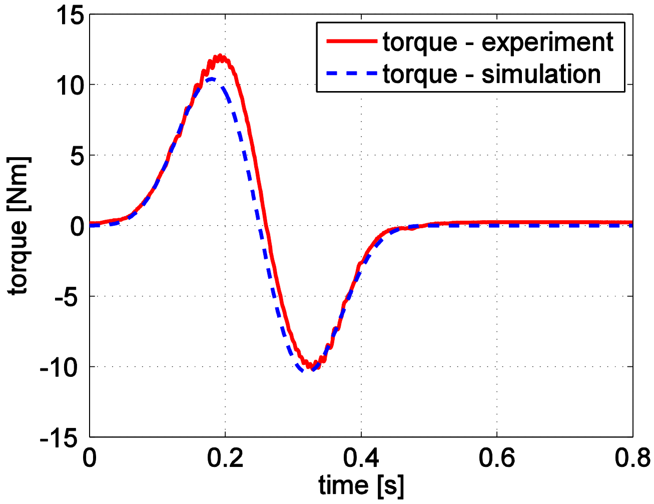

Fig. 9: Experiment – control input.

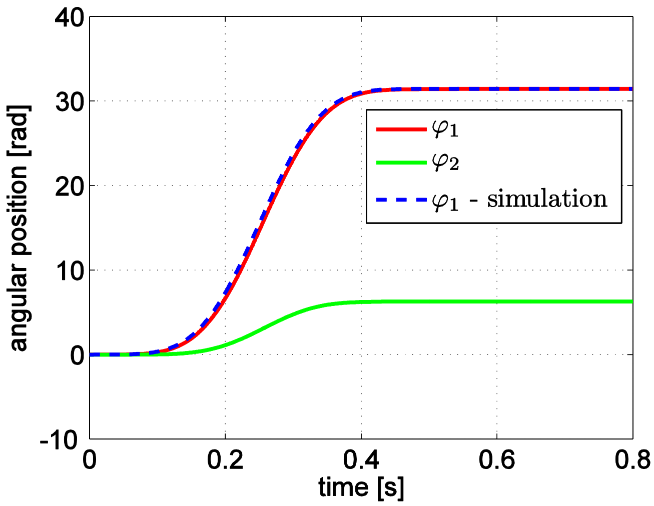

Fig. 10: Experiment – system response.

8. Conclusions

The presented approach to control of electronic cams and other flexible systems combines advantages of two different control shaping techniques. It produces command shapers of arbitrary length with reentry property as well. It is opened to formulation of additional constrains that ensure robustness to modeling errors.

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