

## THE BOUNDARY BETWEEN LINEARITY AND NONLINEARITY OF THE DYNAMICS OF GEAR DRIVES

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**Abstract:** *The vibration of the gear pair of the locomotive wheelset drive gearbox excited by kinematic transmission errors is presented. The mathematical model of the wheelset drive is used for determination of constant gear mesh regions by means of the operational parameters – locomotive speed and longitudinal creepage of both wheels. Using the conditions of constant gear mesh, the areas of constant and interrupted gear mesh are determined. The method is applied to a particular gear drive of the locomotive SKODA 109E.*

**Keywords:** *Gear drive, constant gear mesh conditions, constant gear mesh map.*

### 1. Introduction

Gear drives are often high speed drive systems with front helical gears. Dynamic properties of gear drives are usually significantly affected by manufacturing tolerances and elevation gear modifications (Byrtus & Zeman, 2011). Because of the presence of gear kinematic deviations, they can be assumed as the dominant high-frequency excitation source. The computational model simulates kinematic deviations as a fictitious wedge inserted between the side meshing considering ideal involute teeth (Fig. 1).

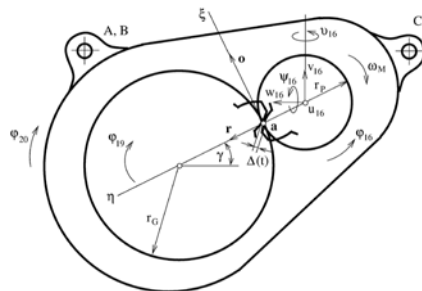


Fig. 1: Scheme of gear mesh

### 2. Mathematical model of the individual wheelset drive

The individual wheelset drive of the locomotive SKODA 109 E was modelled in (Zeman, Hlaváč & Byrtus, 2010) and provided the spatial oscillations of its components. The mathematical model of the wheelset drive can be written as

$$\mathbf{M}\ddot{\mathbf{q}}(t) + (\mathbf{B} + \mathbf{B}_{RM} + \mathbf{B}(s_0, v))\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{f}^G(t) + \mathbf{f}_0 \quad (1)$$

where  $\mathbf{M}, \mathbf{B}, \mathbf{K}$  are mass, damping and stiffness matrices,  $\mathbf{B}_{RM}, \mathbf{B}(s_0, v)$  are rotor damping matrix and matrix of the adhesion properties in contact of wheels and rails. The vector of internal kinematic excitation generated in gear meshing can be expressed in form  $\mathbf{f}^G(t) = (k_z \Delta_z(t) + b_z \dot{\Delta}_z(t))\mathbf{c}_z$ , where  $\mathbf{c}_z$  is the global vector of geometrical parameters of the gearing. The function  $\Delta_z(t)$ , defining kinematic transmission error of gearing can be expressed by Fourier series

$$\Delta_z(t) = \sum_{k=1}^K (\Delta_{z,k}^C \cos k\omega_z t + \Delta_{z,k}^S \sin k\omega_z t)$$

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where meshing frequency  $\omega_z = \frac{\pi n}{30} z$  is function of operation speed  $n$  [rpm] and  $z$  denotes the gear teeth number.

### 2.1. Constant gear mesh conditions

The practice is very important to determine conditions for stable working side shot teeth. Solution (1) can be found in the shape of the sum of static and oscillating component  $\mathbf{q}(t) = \mathbf{q}_{st} + \mathbf{q}_{dyn}(t)$ , where vector  $\mathbf{q}_{st} = \mathbf{K}^{-1}\mathbf{f}_0$ . From the equations of motion (1) for oscillating component of motion we get

$$\mathbf{M}\ddot{\mathbf{q}}_{dyn}(t) + (\mathbf{B} + \mathbf{B}_{RM} + \mathbf{B}(s_0, v))\dot{\mathbf{q}}_{dyn}(t) + \mathbf{K}\mathbf{q}_{dyn}(t) = [k_z\Delta_z(t) + b_z\dot{\Delta}_z(t)]\mathbf{c}_z \quad (3)$$

The constant gear mesh conditions by gearing deformation steady state is

$$\min_{t \in \langle 0, 10T_z \rangle} d_z(t) = \min_{t \in \langle 0, 10T_z \rangle} \{ \mathbf{c}_z^T [\mathbf{q}_{st} + \mathbf{q}_{dyn}] + \Delta_z(t) \} > 0$$

For an illustration, Fig. 2 includes map of constant gear mesh of the gear drive SKODA 109E. Fig. 2 shows splitting of the operational area into two regions. The gray colored one corresponds to area of uninterrupted gear coupling contact and the white one corresponds to the area of gear mesh contact interruption, i.e. the gray colored area corresponds to area where the linear model holds and the white area corresponds to the area where the model is nonlinear.

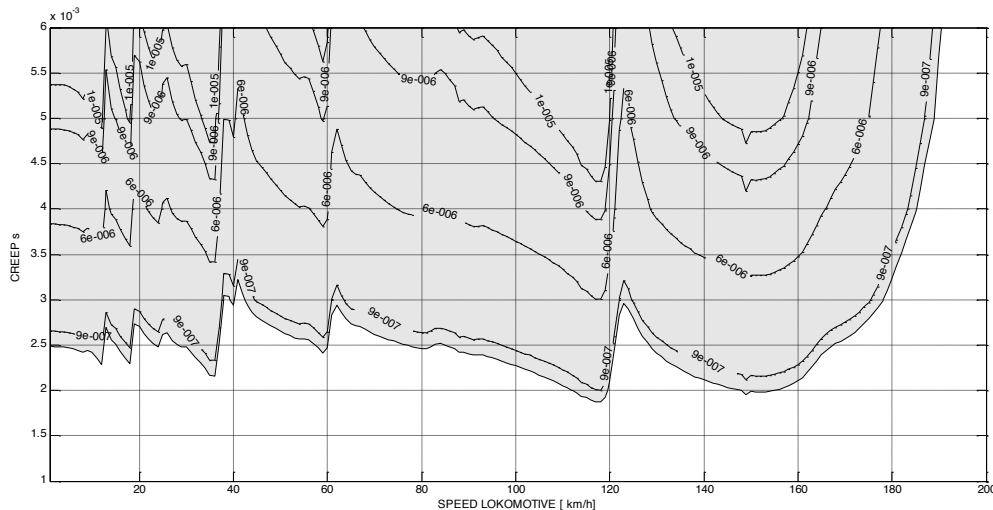


Fig. 2: Map of constant gear mesh of the gear drive SKODA 109E

### 3. Conclusions

The paper provides a method for modeling vibration gear drives and allows investigation of the region of constant gear mesh in dependence on the operating parameters – speed of locomotive and creep of between wheel and rail. Using the map of constant gear mesh, one can clearly determine when the nonlinearities in gear coupling employ.

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