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VIBRATION CONTROL OF CARBON BRIDGES

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Abstract: *Tuned vibration control in aeroelasticity of slender carbon fiber and laminated wood bridges is treated in present paper. The approach suggested takes into account multiple functions in aeroelastic analysis and flutter of such slender bridges subjected to laminar and turbulent wind flow. Tuned vibration control approach is presented with application on actual bridge. Some results obtained are discussed.*

Keywords: ***Bernoulli equation, damping, flutter response, turbulent wind motion, wood bridge.***

1. Introduction

A multilevel approach for the local micromechanics analysis of carbon nanotube based composite materials is suggested. Carbon nanotubes are seen as graphene sheets rolled into hollow cylinders composed of hexagonal carbon cells. The hexagonal cell is repeated periodically and binds each carbon atom to three neighboring atoms with covalent bonds, creating one of the strongest chemical bonds today, with impressive mechanical properties.

A long-standing difficulty in designing of carbon fiber composites is the formulation of a consistent theory that describes their failure behaviour under nonuniform stress fields. As problem appears there the discrepancy between the four-point bend and simple tensile test data. The bend specimens fail at higher strain compared with the tensile specimens. When the bend and tensile data are analysed using classical linear elastic theory the bend stress at any strain prior to failure of a tensile test specimen is 20 - 35 % higher than corresponding uniaxial tensile stress. Such strength discrepancy remains unresolved even when corrections are made for the nonlinearity of the stress-strain curves. By attempts to explain such discrepancy only very limited success has been achieved with failure theories, including the Weibull`s statistical model and the fracture mechanics approach. Similar experiences also appeared by the application of linear fracture mechanics or couple-stress theory.

The carbon fiber composites adopted in present structural engineering are made of typical components listed as:

1. carbon fibers, with strength and elasticity moduli in scope 2.2 – 5.7 GPa and 300 – 700 GPa, respectively,
3. aramide fibers, with strength 3.5 GPa and elasticity moduli in scope 80 – 185 GPa.

The composites consist of micromechanical fibers and surface resin skin. The calculation on the micromechanical level takes into account the behaviour of single fiber in interaction with another fibers and with surface skin. In present time is made the development of new types of fiber composites equipped with surface skin on the basis of advanced ceramics or metals having high strength and load-bearing capacity as well as increased temperature resistance and fatigue reliability.

The material instability appearing in the failure process of carbon fiber composites is treated below adopting the fiber kinking theory and using the analysis on the micromechanical level. In this paper following is submitted: 1. fiber kinking approach for the failure analysis of carbon fiber composites, 2. mathematical formulation of governing equations for modeling and numerical treatment of the problem, 3. numerical and experimental assessment with actual structural application.

Advanced slender wood bridges (Fig.1) are to be designed in such a way that no load can disturb

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their reliability. However, the experiments sampled up indicate that ultimate behaviour of such structures occurring due to wind can initiate unpredictable ultimate response influencing their safety. Ultimate flutter behaviour of such bridges occurs by laminar and turbulent air-flow along the surface of the main girder. Linear theory specifies a critical pressure at which the bridge motion becomes unstable. The linear theory specifies the flutter boundary but cannot give information about ultimate flutter response. For large amplitude oscillations the nonlinear effects restrain the motion to a bounded value with growing amplitudes as dynamic pressure increases.

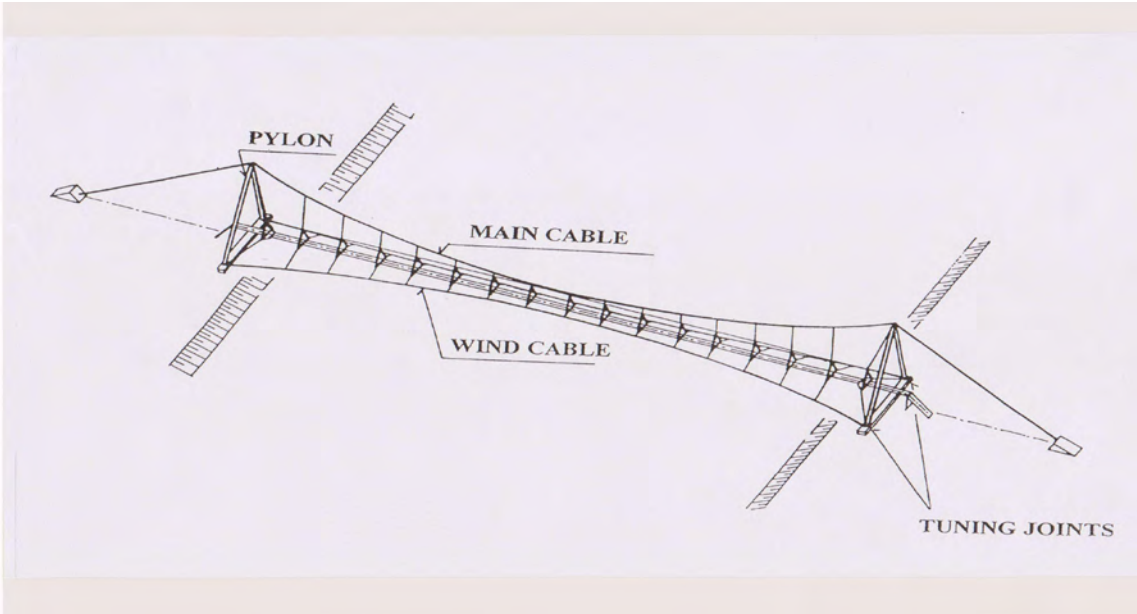

Fig. 1: Schema of the bridge with TVC-equipment

One measure to control such response is the application of tuned vibration control (TVC) in special joints adopted on the bridge. The monitoring and identification of actual parameters, the selection of target reliability and optimal tuning by evaluation of amplification curves are made in tuning joints either automatically for each forcing situation occurring or are set up stationary for the assumed range of forcing. The TVC controls the length of time interval in which the flutter response remains stable with limited amplitudes.

Slender main girders of bridges studied are made of laminated wood. Carbon fiber composites are adopted for cables. Such bridges, when subjected to laminar or turbulent air flow, can be forced into ultimate flutter response with large amplitudes and unstable aeroelastic behaviour. The monitoring submits all data for the TVC. The forces in wind cables are automatically varied in order to control the response. The TVC-software in tuning joints allows updated identification of all structural and forcing data, their evaluation, monitoring, optimization and consequently the control of structural response.

The treatment and modeling of turbulent air flow in artificial boundary layer around the bridge is a research domain based on advanced scientific technologies. They are imposed by necessity of studying the turbulent air movement in the proximity of slender structures. The models of turbulent air flow are used in the assessments being validated by tunnel testing of parameters integrated in calculation.

Aeroelastic response depends on wind speed, wind direction, wind flow (laminar or turbulent), wind temperature and humidity, snow and ice loads, geometry and configuration of wood bridges studied, dynamic properties of all structural elements.

There occur the turbulent air flows on edges of the main girder which increase the wind speeds and pressures. Regarding the variability of configurations of bridges with artificial boundary layer there appear combined laminar and turbulent wind flows. The measurements in aerodynamic tunnel submit the data required for the analysis of the problem.

2. Analysis

Slender wood bridges are prone to wind-induced vibrations for various reasons. Some issues considered in their wind resistant design are mentioned by:

1. Wind turbulences force the bridge with a considerable power and the movements owing to turbulences and associated mechanisms are stochastic in nature.
2. There is produced a strong vortex wake associated with aerodynamic drag force experienced by the bridge. Depending on wind speed and cross-section's shape, the shedding of vortices is more or less regular with shedding periods inversely proportional to the wind speed. In resonance conditions the structure's oscillation can control the rhythm of the vortex shedding.
3. Aside the known vortex trail type excitation the more general types of forcing appear in the bridge. The vortices generated by the local geometry and movement of the bridge contribute to such forcing.
4. Aeroelastic forces proportional to the movement of the bridge produce self-induced divergent vibrations at some wind speeds.
5. In the design of bridge is to be avoided that absolute value of negative aerodynamic damping exceeds the positive mechanical damping producing across-wind flexural mode instability. Associated critical wind speed is the flutter velocity while corresponding circular frequency is the flutter frequency.
6. At the onset of divergence the aerodynamic instability of the bridge is initiated.

In this paper the wind induced structural phenomena are treated by transient dynamics. Laminar and turbulent wind forcing is studied adopting the wave propagation approach. The goal is to develop the approach based on transient dynamics combined with wave propagation forcing and adopted for the analysis of aeroelastic response of slender bridges.

3. Basic principles

The wind flow field is described by the velocity field $\hat{w}$ which is the function of location vector r and of time t and is given by

$$\hat{w} = f(r,t) \tag{1}$$

The location vector in Cartesian system is

$$r = i.x + j.y + k.z \tag{2}$$

and the velocity vector is

$$\hat{w} = i.v + j.u + k.w \tag{3}$$

with parameters

$$v = f_1(x,y,z,t) \tag{4}$$

$$u = f_2(x,y,z,t) \tag{5}$$

$$w = f_3(x,y,z,t) \tag{6}$$

As velocity potential is introduced, the scalar function Φ (x,y,z) is given by

$$\hat{w} = \operatorname{grad} \Phi \tag{7}$$

The scalar terms of equation (3) are specified by velocity components

$$v = \partial\Phi/\partial x \tag{8}$$

$$u = \partial\Phi/\partial y \tag{9}$$

$$w = \partial\Phi/\partial z \tag{10}$$

The Bernoulli equation for non-stationary wind flow is given by

$$\partial\Phi/\partial t + w.w/2 + \int 1/\rho \, dp = F(t) \tag{11}$$

In order to obtain energetic relations for volume unit of the wind flow, each term of Eq. (11) is to be multiplied by the density of the wind flow ρ. The term $\rho.(\partial\Phi/\partial t+w.w/2)$ represents kinetic energy available, dp is energy of the dynamic flow occurring and $\rho.F(t)$ is energetic balance of the wind flow studied. The lift force on the bridge is given by

$$F = \int (p_d - p_u) \, dp \tag{12}$$

with p_d and p_u as values of pressures on lower and upper surface of the main girder studied. The pressure is modeled by translation flow with constant speed and by flow with circulation $\Gamma\neq0$ for each closed profile of the main girder of the bridge. There pays

$$F = \int \rho.b.\hat{w}.\Gamma \tag{13}$$

with b as width of the bridge girder studied. The circulation Γ depends on air velocity, air flow angle, geometry and environment of the bridge.

4. Wind model

Turbulent air flow and wind gusts are given by intensity, spectral distribution and coherence. In order to generate the wind effects as occuring actually there appear the wind models giving data and parameters for 10-minute constant wind velocity steps as well as frequency spectrum and coherence properties of turbulences appearing. The basic parameter of such wind model (König and Zilch, 1970) is a 10-minute average step v_G of a standard 50-year wind velocity in atmospheric boundary layer studied. The profile of 10-minute wind velocity v_w is established in accordance with exponential law by

$$v_w = v_G \, (z/z_G)^{\alpha} \tag{14}$$

with height z, with z_G as corresponding gradient height and with α as exponent for the wind profile studied. The variability of wind velocity σ is defined as a standard aberation of the gusts in the wind direction. In accordance with the wind model σ is constant along z_G and is given by

$$\sigma = z_o. \, v_G.\sqrt{(6.\beta)}/z_G \tag{15}$$

where z_o=10 is the comparative height and β is the roughness parameter (König and Zilch, 1970).

As further parameter of the wind model appears the atmospheric coherence. It describes the similarity of speed variability in various points n along the span and height of the bridge and is given by

$$C_H(A,B,n) = \sqrt{\{[G_{AB}(n).G_{AB}(n)+Q_{AB}(n).Q_{AB}(n)]/[S_{AA}(n)+S_{BB}(n)]\}} \tag{16}$$

with A and B as two nodes of the bridge model used, $S_{AA}(n)$ and $S_{BB}(n)$ as turbulence spectra measured in A and B, $G_{AB}(n)$ and $Q_{AB}(n)$ as covariance and quadrature spectra of $v_w(A,t)$ and $v_w(B,t)$, respectively. Equation (16) specifies time vs propagation of the wind gusts appearing.

5. Aerodynamic forces

Studied is the plane panel of the main girder of the wood bridge subjected to wind flow initiating aerodynamic forces as shown in Fig. 2.

In case of simultaneous action of critical velocity of the air flow and of the resonance frequency of bridge vibration there appears the flutter combination of flexural and torsional oscillations. For linear analysis of the problem the cross-section studied is an ideal smooth panel and the bridge is forced by laminar air flow along the whole length studied. The aerodynamic forces in accordance with the theory of Theodorsen (1935) are given by

$$L_1 = -2.\pi.\rho.b.v_w.C(k).[v_w.\upsilon_T + i.\omega.u_i + i.b/(2.\omega.\upsilon_T)] \tag{17}$$

$$L_2 = 2.\pi.\rho.\omega.b^2.\omega.u_i \tag{18}$$

$$L_3 = -\pi.\rho.b^2.v_w \tag{19}$$

$$L_4 = \pi.\omega^2.\rho.\upsilon_T.b^4/8 \tag{20}$$

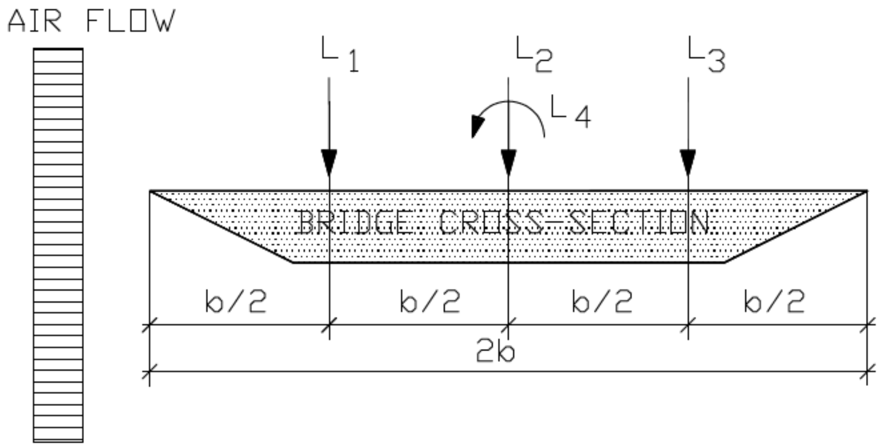

Fig. 2: Aerodynamic forces on the bridge

where u_i and υ_T are flexural and torsional deformations, i is the complex unit and C(k) is the complex Theodorsen function

$$C(k) = H_1^{(2)}(k)/[H_1^{(2)} + i.H_o^{(2)}(k)] \qquad (21)$$

with $H_i^{(2)}$ as Hankel cylindrical functions of second order and with frequency

$$k = 2.\pi.b/\lambda \qquad (22)$$

where λ is the period of natural vibration of the bridge.

6. Mechanism of damping

The wind response of slender wood bridges is influenced by material and structural damping taking into account viscoelastic properties of wood, dissipative capacity of environment as well as aerodynamic damping due to interactions bridge vs environment. The equivalent damping represents the total energy dissipation and is given by several mechanisms which specify relations between damping forces and strain velocities. The energy transformations due to material damping are given by nonstationary thermic variations caused by friction and motion of atomic groups in the material used. The dissipative capacity depends on frequency, stress and thermal effects. The interactions of these mechanisms are significant in the assessment of the problem.

For the analysis of material and structural damping the theory of hysteretic damping is adopted (Sorokin, 1957 and Tesar, 1988). The stress-strain relation in complex form is given by

$$\sigma = E_o\,(\eta_1 + i\,\eta_2).\varepsilon \qquad (23)$$

with stress and strain σ and ε, respectively, and with E_o as complex modulus of elasticity when the real part of strain converges to zero. The parameters η_1 and η_2 are real functions of damping factor η and are given by

$$\eta_1 = (1 - \eta^2/4)/(1 + \eta^2/4) \qquad (24)$$

$$\eta_2 = \eta/(1 + \eta^2/4) \qquad (25)$$

Damping factor η and logarithmic decrement of damping δ_f are given by

$$\eta = \delta_f/\pi \qquad (26)$$

If the stress σ varies sinusoidally, the strain changes with frequency ω and with phase shift α. If the stress is

$$\sigma = \sigma_o.e^{i\omega t} \qquad (27)$$

with stress amplitude σ_o, then corresponding strain is given by

$$\varepsilon = \varepsilon_o.e^{i(\omega t-\alpha)} \tag{28}$$

For complex modulus of elasticity there holds

$$E = E_1 + i.E_2 = \sigma/\varepsilon \tag{29}$$

with real and imaginary components given by

$$E_1 = \sigma_o.\cos\alpha/\varepsilon_o \tag{30}$$

$$E_2 = \sigma_o.\sin\alpha/\varepsilon_o \tag{31}$$

Damping factor η is given by

$$\eta = \mathrm{tg}\,\alpha = E_2/E_1 \tag{32}$$

and is included into complex moduli of elasticity

$$E = E_o\,(1 + i.\eta) \tag{33}$$

$$G = G_o\,(1 + i.\eta_e) \tag{34}$$

where η_e is the shear damping factor. The structural damping is approximated by complex spring characteristics

$$k_B = k_o\,(1 + i.\eta_B) \tag{35}$$

specified by elastic supports or joints. The parameters k_o and η_B are defined as the spring constant and the factor of structural damping. The structural damping appears in interaction with the material damping.

In the hysteretic model the damping forces generated by material friction are proportional to deformations. The loss factor is equivalent to energy dissipated. There can appear theoretical noncausalities because hysteretic damping approach holds only for steady state harmonic oscillations. However, the numerical and laboratory experiments have experienced the small influence of such theoretical malfunctioning in the analysis of nonperiodic dynamic problems, such as are flutter control and ultimate response of the wood bridges studied.

The TVC of wood bridges is made by computer operated variation of hysteretic, viscous and viscoelastic parameters in damping facilities of tuning joints as well as in energy absorbing members adopted. In hysteretic members the damping forces generated by material friction are proportional to the deformations occurring. The tuning thus depends on stress vs displacement dependence of the stiffness appearing. The specific work of the total damping is given by

$$D = J\,\sigma^n \tag{36}$$

where J and n are experimentally found parameters (Lazan, 1968). The total work of damping is obtained by integrating specific works of material and structural members adopted. The variability of stress causes that each material particle has its own hysteresis curve contributing to total damping. Within the element volume V_g the maximum stress σ_{max} corresponds to maximum work od damping D_{max} and the work of tuning is given by

$$D_g = D_{max}\,V_g\,\beta_1 \tag{37}$$

with nondimensional parameter β_1 (Lazan, 1968). The energy cumulated in analyzed volume is

$$U_g = 0.5\,V_g\,\sigma_{max}^2\,\beta_2/E \tag{38}$$

again with nondimensional parameter β_2 (Lazan, 1968). The factor of damping for analyzed volume is given by

$$\eta_s = D_g/(2\pi\,U_g) \tag{39}$$

and is implemented into complex modulus of elasticity in accordance with the stress available. For linear damping there holds $\beta_1/\beta_2=1$. The ultimate analysis with nonlinear damping is based on the assessment of β_1 and β_2 for given geometry and stress. An iterative scheme is used for specification of damping factors in each element of the bridge. In the first iteration the parameters $\beta_1^{(1)}$, $\beta_2^{(1)}$ and $\eta_s^{(1)}$ are specified for the stress level available. Such parameters are the basis for following iteration steps specifying actual incremental stress and corresponding parameters $\beta_1^{(i)}$, $\beta_2^{(i)}$ and $\eta_s^{(i)}$. The analysis is continued until satisfaction of convergence criterion

$$|\eta_s^{(i+1)}/\eta_s^{(i)}| - 1 \leq 0.01 \tag{40}$$

When analyzing the viscous damping in the TVC-joints adopted, the substitution of complex stiffness by an equivalent viscous damping is to be made. Such a shift from one model of damping into another one has to be made in order to ensure the same amount of energy dissipation per cycle of vibration in the hysteretic model

$$\Delta W_{hysteretic} = 2\,\pi\,\eta\,K\,u_o^2 \tag{41}$$

as well as in the viscous damping model

$$\Delta W_{viscous} = 2\,\pi\,\omega_o\,C\,u_o^2 \tag{42}$$

The terms K and C are stiffness and damping matrices, respectively, and u_o is the vector of deformations of the bridge oscillating with frequency ω_o. The damping capacity of the viscous member is defined as ratio of the energy ΔW dissipated per cycle of vibration vs maximal stored energy per cycle

$$\psi = \Delta W/W = 2\,\pi\,\xi \tag{43}$$

The damping capacity is in such a way related to hysteretic loss factor η based on complex modulus of elasticity and on viscous damping ratio ξ. The damping capacity of viscoelastic damping member

$$\psi^o = \psi_{vol} + \psi_{dev} \tag{44}$$

is splitted up into volumetric (ψ_{vol}) and deviatoric (ψ_{dev}) parts. The volumetric part is considered to be simply elastic and viscoelastic behaviour is principally related to the deviatoric part. The stress is splitted up into elastic and dissipative parts. In constitutive equation there holds

$$\sigma = \sigma_{elastic} + \sigma_{dissipative} = E\,\varepsilon + \lambda\,\acute{\varepsilon} \tag{45}$$

with Young modulus E, with strain ε, strain rate $\acute{\varepsilon}$ and with λ as viscosity constant of damping member studied. There holds

$$\lambda = \upsilon\,E \tag{46}$$

where υ is the relaxation time. In strain $\varepsilon = B.u$ and strain rate $\acute{\varepsilon} = B.\acute{u}$ the parameters u and ú are nodal vectors of deformations and velocities, respectively. The matrix B consists of derivatives of shape functions applied.

Environmental air causes additional interactive damping in aeroelastic response of wood bridges. For assessment of total damping is to be dealt with residual value of dissipative energy given by

$$W = \Delta W + \Delta L \tag{47}$$

with dissipative components of structural damping ΔW as mentioned above and dissipative part of environmental damping ΔL. When taking into account the air incompressibility which holds for wind velocities $v < 50$ m/sec (the density variability of air for such velocities is less than 1%), for constant air pressure q there holds

$$q = \rho\,v^2/2 \tag{48}$$

where ρ is the air density. Such pressure appears on the motionless bridge forced by constant air flow with velocity v. Additional damping force is given by

$$C_L = c\,A\,\rho\,v^2/2 \tag{49}$$

with area A of the bridge and with coefficient c as explained below. The damping force C_L is valid for stationary process, e.g., for constant velocity of the air flow. However, when dealing with ultimate response, the non-stationary circumstances are to be considered. The resulting force acting on the bridge is then given by

$$C_R = c_d\,A\,\rho\,v^2 + c_m\,A_D\,\rho\,a \tag{50}$$

with wind acceleration a. In Eq.(50) besides constant pressure term there appears one additional term corresponding to the gyration mass. The coefficients c, c_d and c_m are given in Davenport (1961. The gyration forces depend on additional virtual mass of the air pressed in front of the bridge. For small amplitudes the air damping is proportional to velocity of the bridge. For large amplitudes the air damping depends on the resistance force given by constant pressure of wind flow on the bridge. Aeroelastic interaction bridge vs air flow causes further additional damping of dynamic motion. There appear aerodynamic dissipative forces and damping is given by phase shifts of motions and forces appearing. The amplitudes of aerodynamic forces increase linearly with the wind velocity.

7. Tuned vibration control

The system identification for each forcing situation appearing together with structural response, optimization and monitoring are principal operations made in the TVC-joints. The system identification is a part of modeling with data basis available from updated structural and forcing situations measured. The analysis of structural response considers all linear and nonlinear interaction effects appearing. The optimization and monitoring take into account the target functions adopted in order to control the bridge response. The tuning joints contain the facilities for variability of forces in wind cables in the TVC, taking account of : - updated frequency spectrum of the bridge studied; is initiated by variability of forces in wind cables, - updated damping parameters of the bridge studied; are influenced by damping facilities and energy absorbers in structural system and in the TVC-joints adopted, - updated monitoring of time response of the bridge.

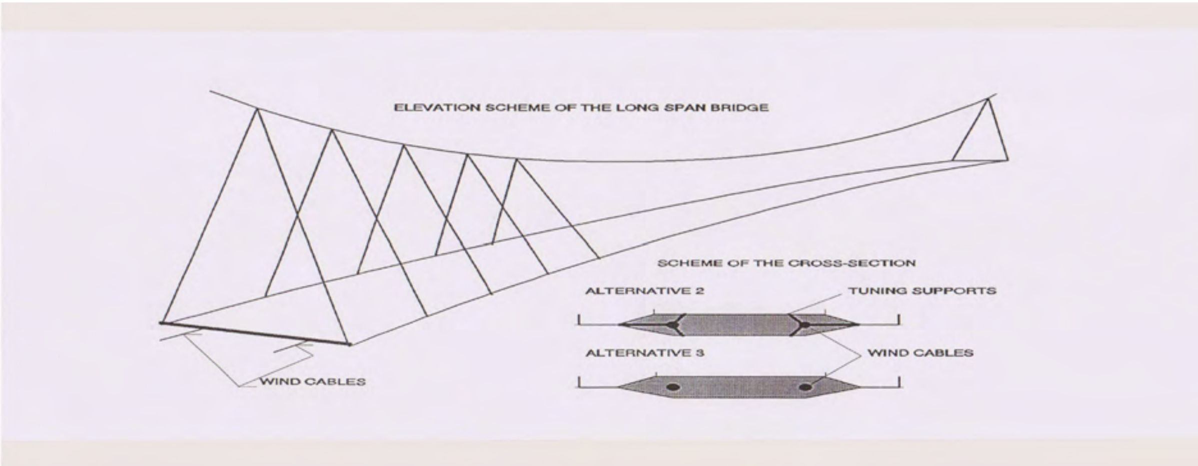

Fig. 3: TVC-alternatives

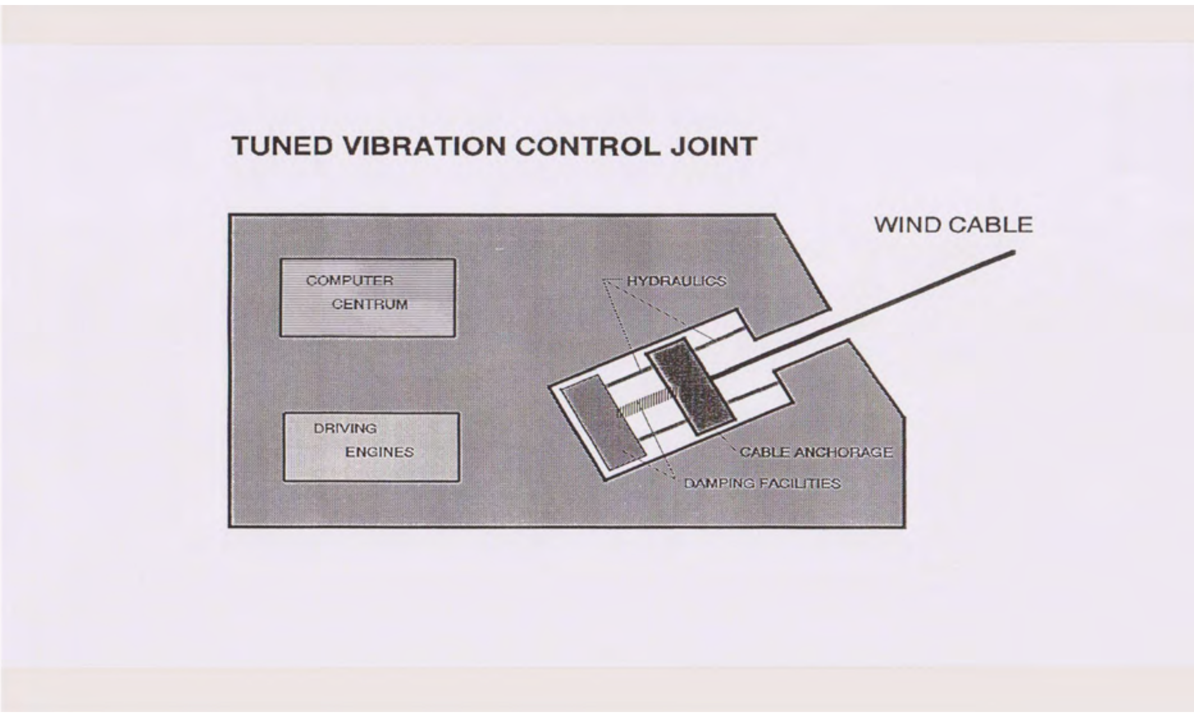

Fig. 4: TVC-joint

Other examples of the TVC-systems are based on utilization of cables located interior of the bridge girder studied (see Fig. 3 – alternatives 2 and 3). The principal scheme of such TVC facility is in Fig. 4. The TVC is performed either by variability of forces in the wind cables or by variability of distances of structural elastic supports in contact points girder vs wind cables interior the girder. The TVC is activated by adoption of simple locking mechanisms in the contact points. The TVC in scope of the alternative 3 consists of the wind cables placed and continuously supported interior of the bridge girder. Axial forces in the wind cables are varied in order to control the torsional/flexural response of the bridge.

8. Application

Ultimate flutter response of slender wood bridge as shown in Fig. 5 is studied. The span of the bridge is 100 m. The main girder of the bridge is made of laminated wood. The carbon fiber composites are adopted for the cables. The structural parameters of the bridge are: girder width 7.9 m, girder height 4.1 m and its mass per m^2 is 1830 kg.

The bridge was forced by standard laminar and turbulent air flows being measured in southern territory of Slovakia. Ultimate flutter time response during simultaneous action of flutter eigenvalues given by resonance frequency of the bridge 0.66 Hz and by critical wind velocity 23.6 m/sec was studied. Energy approach for the analysis of nonlinear time response was adopted for calculation of resulting ultimate aeroelastic response of the bridge.

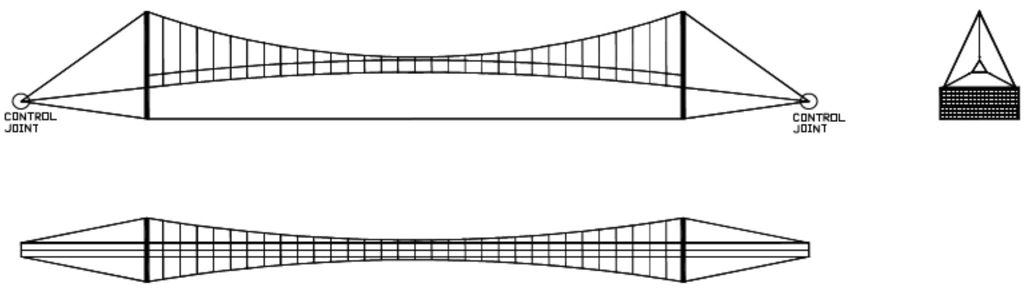

Fig. 5: Bridge studied

The assessment has shown the dominant influence of flutter rotation modes on resulting ultimate response of the bridge. Starting with simultaneous occurrence of both eigenvalues and assuming the discretization of the bridge span into n-nodes, the structural time response until the bridge collapse was studied.

In scope of the TVC the wind cables of the bridge were submitted to variable axial tensile forces. The bridge response in time points 300 sec (for axial tension force 0.1 MN), 660 sec (for axial tension force 0.25 MN) and 720 sec (for axial tension 0.5 MN) after initiation of simultaneous action of both eigenvalues, is plotted in Fig. 6.

9. Conclusion

The efficiency of the present system for tuned vibration control of slender wood bridges, adopting the variability of forces in the wind cables, is illustrated. The TVC appears there as efficient tool for the aided reliability of slender wood bridges subjected to laminar and turbulent wind forcing.

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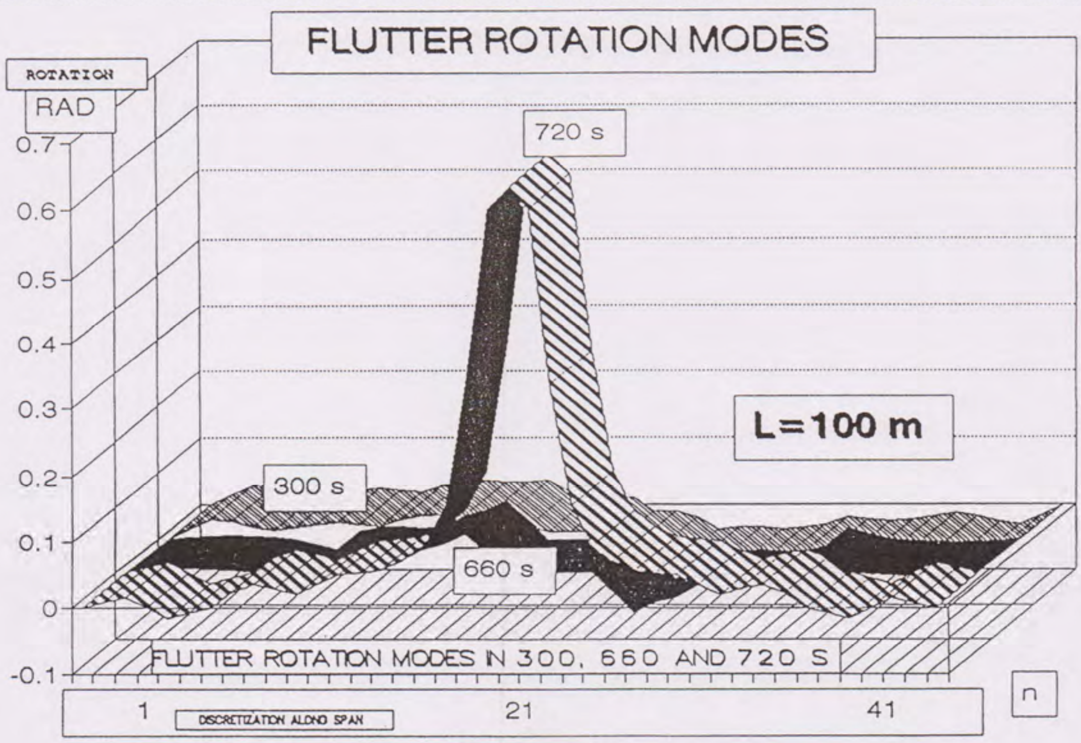

Fig. 6: Ultimate bridge response

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