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EXPERIMENTAL AND COMPUTATIONAL INVESTIGATION OF A SIMPLE FIBRE-MASS SYSTEM

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Abstract: *Experimental measurement is performed on a simple fibre-mass system: a moving mass coupled with the frame by a fibre. Dynamic response of the mass is measured. The same system is numerically investigated by means of a simple multibody model. The influence of the model parameters on the coincidence of results of experimental measurements and the simulations results are evaluated. The simulations aim is to create a phenomenological model of a fibre, which will be utilizable in fibre modelling in the case of more complicated mechanical or mechatronic systems.*

Keywords: ***Fibre-mass system, vibration, experiment, simulation.***

1. Introduction

The replacement of the chosen rigid elements of manipulators or mechanisms by fibres or cables (Chan, 2005) is advantageous due to the achievement of a lower moving inertia, which can lead to a higher machine speed, and lower production costs. Drawbacks of using the flexible elements like that can be associated with the fact that cables should be only in tension (e.g. Valášek & Karásek, 2009) in the course of a motion.

Experimental measurements focused on the investigation of the fibre behaviour were performed. The simple fibre-mass system consists of moving weight coupled with a frame by a fibre (see Fig. 1a). The same system is numerically investigated using a simple multibody model created in the **alaska** simulation tool (Maißer et al., 1998). The influence of the model parameters on the coincidence of results of experimental measurements and the simulations results are evaluated. The simulation aim is to create a phenomenological model of a fibre, which will be utilizable in fibre modelling in the case of more complicated mechanical or mechatronic systems. The motivations for introducing this paper is a logical continuation of the paper Polach et al. (2013), in which first pieces of knowledge concerning the creation of the phenomenological model of the referred simple fibre-mass system are mentioned, and a finishing of the first phase of the phenomenological model of fibre creating. In contradiction to the paper Polach et al. (2013), in order to define more general conclusions a large amount of tests, which were the subject of experimental measurements, were processed using the computer simulations.

2. Experimental stand

Originally it was supposed that for the experimental measurement focused on determining properties of fibres an inverted pendulum driven by two fibres attached to a frame (see Fig. 2) would be used. Its properties were investigated very thoroughly applying calculation models (see e.g. Polach et al., 2012a). But strength calculation results drew attention to a high loading of fibres which were to be used in the experimental measurements (carbon or wattled steel wire) and to the possibility of their breaking (Polach et al., 2012b). Due to those reasons a different mechanical system was chosen for the experimental measurements (its geometrical arrangement was changed several times on the basis of

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various pieces of knowledge). In this system the fibre is driven only with one drive and is led over a pulley and on its other end there is a prism-shaped steel weight (weight 3.096 kg), which moves on an inclined plane (see Fig. 1b). The fibre length is 1.82 meters, the pulley diameter is 80 mm. Drive exciting signals can be of a rectangular, a trapezoidal and a quasi-sinusoidal shape and there is a possibility of variation of a signal rate. Displacement of the weight, displacement of the drive and the force acting in the fibre will be the measured quantities. At the experiments it is planned to use Kevlar fibre and two types of carbon fibres: a simple fibre (see e.g. Polach et al., 2012b) and a fibre composed of two simple fibres with a common silicone coating. Angle of inclination of the inclined plane can be changed.

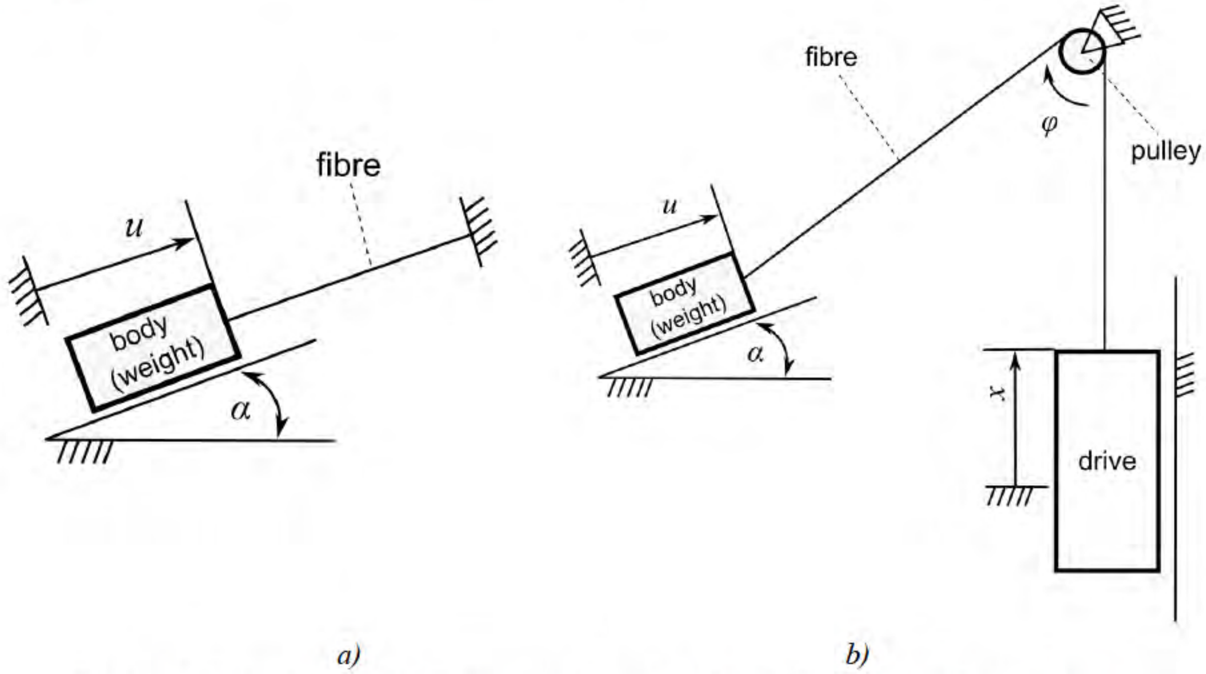


Fig. 1: a) Weight-fibre mechanical system, b) weight-fibre-pulley-drive mechanical system.

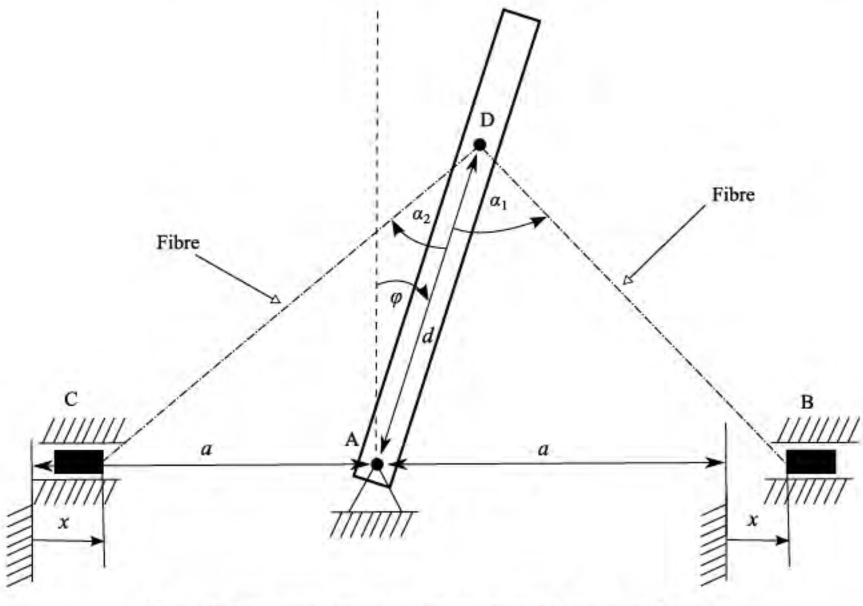
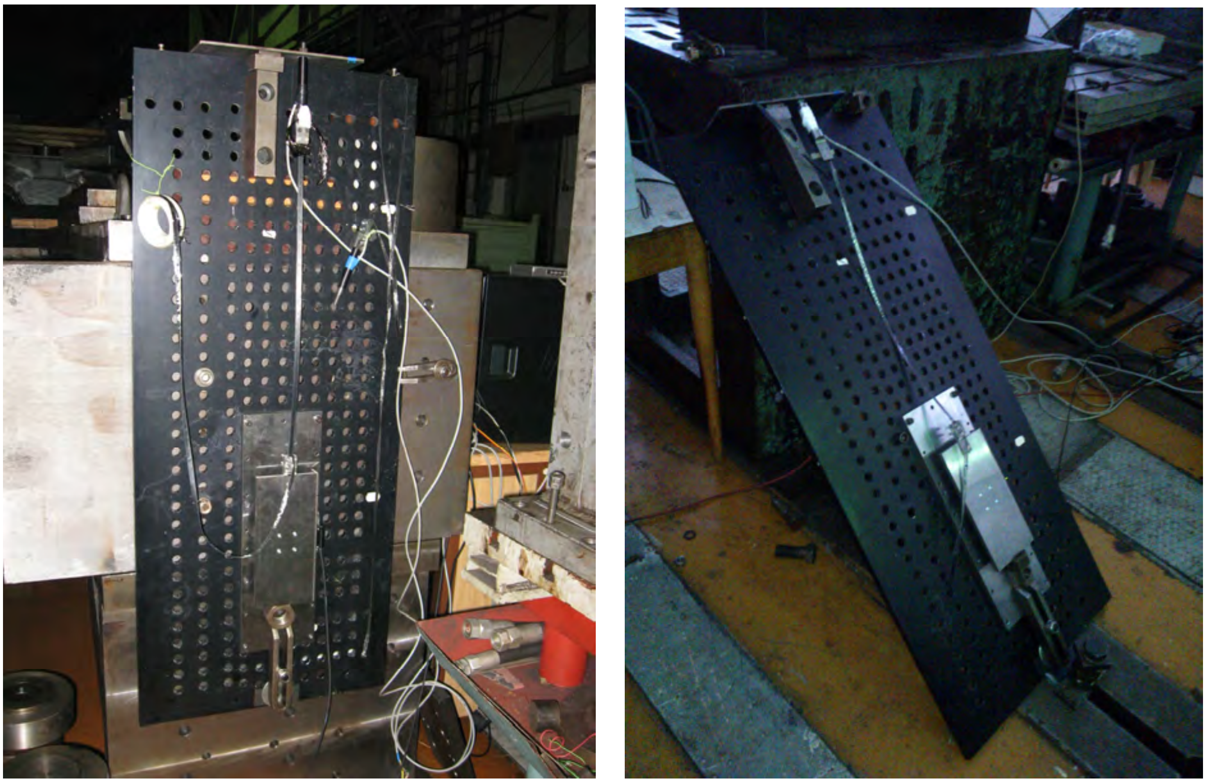


Fig. 2: Inverted pendulum actuated by the fibres.

Investigation of the single fibre (carbon or wattled steel wire) properties eliminating the influence of the drive and of the pulley (see Fig. 1a) was an intermediate stage before the measurement on this stand. Fibre was fixed on a force gauge of a rectangular thin-wall cross-section profile 15 by 15 millimeters and thickness of wall 1 millimeter. In the other end of the fibre the already mentioned prism-shaped steel weight (i.e. of weight 3.096 kg) was fastened – see Fig. 3. The weight was lifted to a certain height (from 5 to 20 millimetres) and then let to fall in the vertical direction or to slide down the inclined plane – see Fig. 3. The weight moved in prismatic linkage. The time histories of the weight position (in the direction of the inclined plane; measured by means of a dial gauge) and of the force acting in the fibre (measured on a force gauge) were recorded using sample rate of 2 kHz.



a) *b)*

Fig. 3: Weight-fibre mechanical system, a) in a vertical position, b) on an inclined plane.

3. Possibilities of the fibre modelling

The fibre (cable, wire etc.) modelling should be based on considering the fibre flexibility and suitable approaches can be based on the flexible multibody dynamics (see e.g. Shabana, 1997).

The fibre (cable, wire etc.) modelling (Hajžman & Polach, 2011) should be based on considering the fibre flexibility and the suitable approaches can be based on the flexible multibody dynamics (see e.g. Shabana, 1997; Gerstmayr et al., 2012). The branch of the flexible multibody dynamics is a rapidly growing area of computational mechanics and many industrial applications can be solved by newly proposed flexible multibody dynamics approaches. Studied problems are characterized by the general large motion of interconnected rigid and flexible bodies with possible presence of various nonlinear forces and torques. There are many approaches to the modelling of flexible bodies in the framework of multibody systems (Hajžman & Polach, 2008). Comprehensive reviews of these approaches can be found in Shabana (1997) or in Wasfy & Noor (2003). Further development together with other multibody dynamics trends was introduced in Schiehlen (2007). Details of multibody formalisms and means of the creation of equations of motion can be found e.g. in Stejskal & Valášek (1996).

The simplest way how to incorporate fibres in equations of motion of a mechanism is the force representation of a fibre. It is assumed that the mass of fibres is low to such an extent comparing to the other moving parts that the inertia of fibres is negligible with respect to the other parts. The fibre is represented by the force dependent on the fibre deformation and its stiffness and damping properties. This way of the fibre modelling is probably the most frequently used one in the cable-driven robot dynamics and control (e.g. Zi et al., 2008). The system of fibre-mass fulfils all requirements for modelling the fibre using the force representation of a fibre.

A more precise approach is based on the representation of the fibre by a point-mass model (e.g. Kamman & Huston, 2001). The fibre can be considered either flexible or rigid. It has the advantage of a lumped point-mass model. The point masses can be connected by forces or constraints. In the case of the fibre-mass system the fibre is discretized using 10 point masses, the fibre is considered to be flexible and point masses are connected using force elements. The stiffness and the damping coefficient between the masses are determined in order to keep the global properties of the massless fibre model.

The fibre model is considered to be phenomenological and it is modelled by the forces which comprise e.g. influences of fibre transversal vibration, etc. The weight is considered to be a rigid body. When assuming the massless fibre model the system is considered two-dimensional, when assuming the point-mass model of the fibre the system is (partly) considered three-dimensional (Polach et al., 2013). Behaviour of these nonlinear systems is investigated using the **alaska** simulation tool (Maißer et al., 1998).

4. Simulation and experimental results

As it was already mentioned the simulations aim is to create a phenomenological model of a fibre. When looking for compliance of the results of experimental measurement with the simulation results influences of the following system parameters are considered: the fibre damping coefficient, the fibre stiffness and the friction force between the weight and the prismatic linkage.

Results of experimental measurements and simulations of six tested situations are presented. Carbon fibre (fibre length is 599 millimetres; fibre mass is 1.63 grams) was used at all of the tests. In the first case the weight was let to fall by free fall (see Fig. 3a), in the other cases the weight was let to slide down the inclined plane of the alpha angle (see Fig. 1a and Fig. 3b). The input parameters of experimentally and computationally investigated tested situations are given in Tab. 1. When simulating with multibody models simulation times were selected identical with those at experimental measurement (see Tab. 1).

Tab. 1: Parameters of the tested situations.

Tested situation	Alpha angle [deg] (see Fig. 1a)	Shift of the weight (in the inclined plane direction) [mm] (see Fig. 3)	Time of simulation [s]
1	90	4.912	0.63
2	45	4.94	0.516
3	45	10.2216	0.895
4	45	15.0224	0.843
5	30	5.032	0.6495
6	30	15.1296	0.7745

As it has been stated the value of the fibre damping coefficient, the value of the fibre stiffness and the friction force between the weight and the prismatic linkage were variable parameters at finding the suitable phenomenological model.

Influence of those parameters on time histories of the weight position and also on time histories of the force acting in a fibre was evaluated partly visually and partly on the basis of the value of correlation coefficient between the records of the experimental measurement and that of the simulation results. Application of the approach based on the calculation of the statistical quantities that enables to express directly the relation between two time series has appeared to be suitable for comparing two time series in various cases – e.g. Polach & Hajžman (2008), Polach et al. (2012c).

Correlation coefficient $R(\boldsymbol{p})$ (Rektorys et al., 1994) defined for two discrete time series $x^{(1)}$ (the time history recorded at experimental measurement) and $x^{(2)}(\boldsymbol{p})$ (the time history determined at simulation with the multibody model; function of the investigated parameters $\boldsymbol{p}$) was calculated

$$R(\boldsymbol{p}) = \frac{\sum_{i=1}^{n}\left(x_i^{(1)} - \mu_1\right)\cdot\left[x_i^{(2)}(\boldsymbol{p}) - \mu_2(\boldsymbol{p})\right]}{\sqrt{\sum_{i=1}^{n}\left(x_i^{(1)} - \mu_1\right)^2 \cdot \sum_{i=1}^{n}\left[x_i^{(2)}(\boldsymbol{p}) - \mu_2(\boldsymbol{p})\right]^2}}, \tag{1}$$

where μ_1 and $\mu_2(\boldsymbol{p})$ are mean values of the appropriate time series. The correlation coefficient values range between zero and one. The more the compared time series are similar to each other the more the correlation coefficient tends to one. The advantage of the correlation coefficient is that it quantifies very well the similarity of two time series by scalar value, which is obtained using a simple calculation.

Starting values at the phenomenological model creating were fibre stiffness measured on a tensile testing machine at Department of Mechanics, Faculty of Applied Sciences, University of West Bohemia (Zemčík, 2011) and on the basis of experience derivated fibre damping coefficient. The values were used at calculations e.g. in Polach et al. (2012b). The starting friction force between the weight and the prismatic linkage was considered to be zero.

When looking for the fibre model, which would ensure the similarity of time histories of the weight position and time histories of the force acting in a fibre as great as possible, fibre stiffness and fibre damping coefficient were considered to be constant (in a current phase of the fibre behaviour research). Friction force course (in dependence on the weight velocity) was considered nonlinear. Basis for the determination of friction force course was especially the article Půst et al. (2011).

The monitored quantities at the experimental measurements and the computer simulations are presented in Figs 4 to 9. In Tab. 2 and in Fig. 10 there are parameters of the phenomenological models of the fibre-mass mechanical system (it is necessary to note that various parameters for various system configurations). In Tab. 3 and Tab. 4 there are values of correlation coefficient $R(\boldsymbol{p})$ before and after "tuning" the parameters of the mechanical fibre-mass system model. The parameters values were "tuned" for the fibre-mass system model with the force representation of a fibre and not for the system models with the representation of the fibre by the point-mass model (this is evident from comparing the correlation coefficient $R(\boldsymbol{p})$ values for both types of the model – see Tab. 3 and Tab. 4). Further, as it is evident from Tab. 2, identical values of fibre stiffness and fibre damping coefficient for all the simulations of the weight sliding down the inclined plane, independently of the alpha angle, were chosen to the exclusion of the achieved coincidence of the simulations results and the experimental measurements results. This approach seemed to be logical. At simulating weight falling by free fall fibre stiffness and fibre damping coefficient were tuned independently.

Course of the friction force was considered different for each plane incline (and, of course, different for the simulation of weight falling by free fall, i.e. alpha = 90°). The smaller the plane incline angle the larger the friction force – see Fig. 10. For the weight velocities lower than -0.1 m/s and higher than 0.1 m/s the friction force courses are constant. In the course of friction forces the frictional drag of contact in the stick phase of loading before the beginning of the weight motion is taken into account.(Půst et al., 2011; see Fig. 10).

In contradiction to Polach et al. (2013) at simulating the weight sliding down the inclined plane the model to define friction force as a quadratic function in dependence of the weight distance from the equilibrium position of the fibre-mass system was left.

Tab. 2: Values of stiffness and damping coefficient of the fibre model.

Tested situation (see Tab. 1)	Stiffness [N/m] of the fibre model		Damping coefficient [N·s/m] of the fibre model	
	Starting value	Final value	Starting value	Final value
1	$305\cdot10^3$	$163\cdot10^3$	152.58	113.94
2 to 6	$305\cdot10^3$	$104\cdot10^3$	152.58	83.47

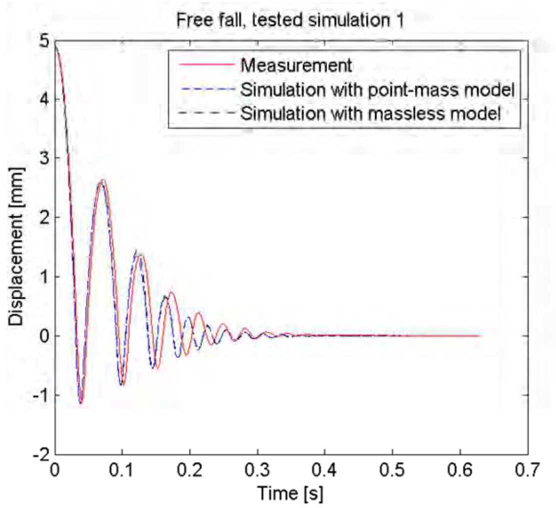
*Tab. 3: Values of correlation coefficient R(**p**) [-] – comparison of time histories of weight displacement.*

Tested situation (see Tab. 1)	**Force model of fibre**		**Point-mass model of fibre**	
	Starting value	**Final value**	**Starting value**	**Final value**
1	0.8941	0.9621	0.9047	0. 9582
2	0.9504	0.9885	0.9575	0.9816
3	0.993	0.988	0.9929	0.991
4	0.9762	0.9839	0.9799	0.9884
5	0.8774	0.91	0.9091	0.9422
6	0.9497	0.9851	0.9609	0.988

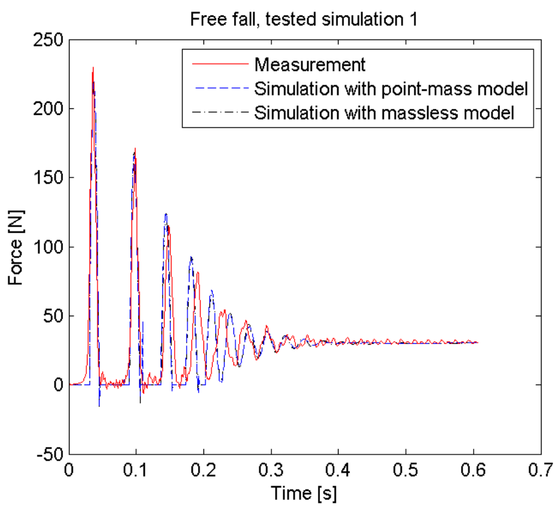
*Tab. 4: Values of correlation coefficient R(**p**) [-] – comparison of time histories of force acting in fibre.*

Tested situation (see Tab. 1)	**Force model of fibre**		**Point-mass model of fibre**	
	Starting value	**Final value**	**Starting value**	**Final value**
1	0.5038	0.8166	0.5387	0. 7947
2	0.7465	0.8557	0.7664	0.5799
3	0.3734	0.8155	0.7031	0.9694
4	0.2976	0.696	0.4918	0.71
5	0.3819	0.696	0.3546	0.2945
6	0.2606	0.5878	0.3598	0.3796

From the time histories of the monitored quantities given in Figs 4 to 9 it is evident that in contradiction to the simulation of free fall when simulation results are independent of the used fibre model the results of the simulation of the weight sliding down the inclined plane partly differ in connection with the utilization of various fibre models. When the weight slides down the inclined plane gravitation force acts on the fibre in direction different from direction of the weight moving. Despite the fact that the fibre mass is very low in comparison with the weight mass it becomes evident in the longer time of the system transition to the equilibrium position when using the point-mass fibre model.

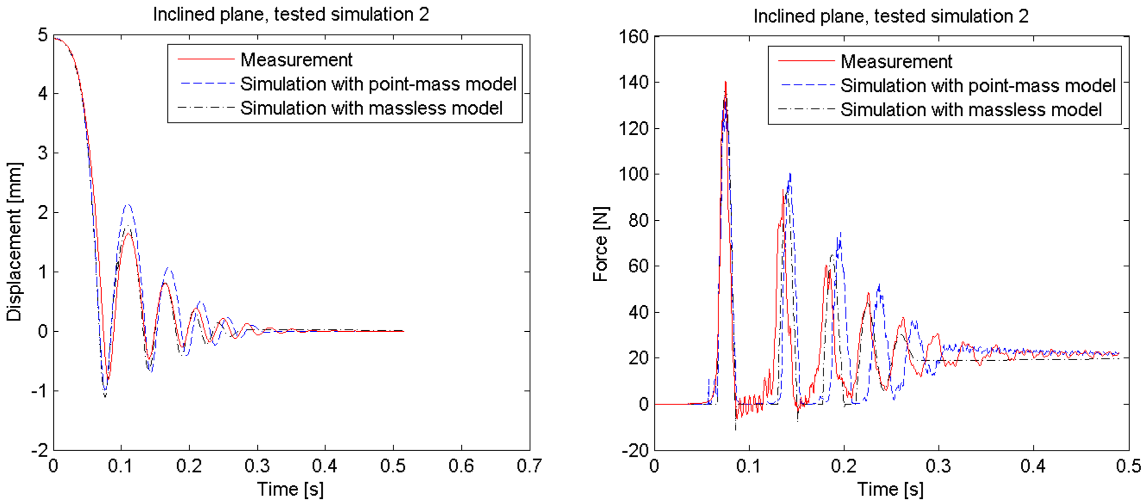


a)



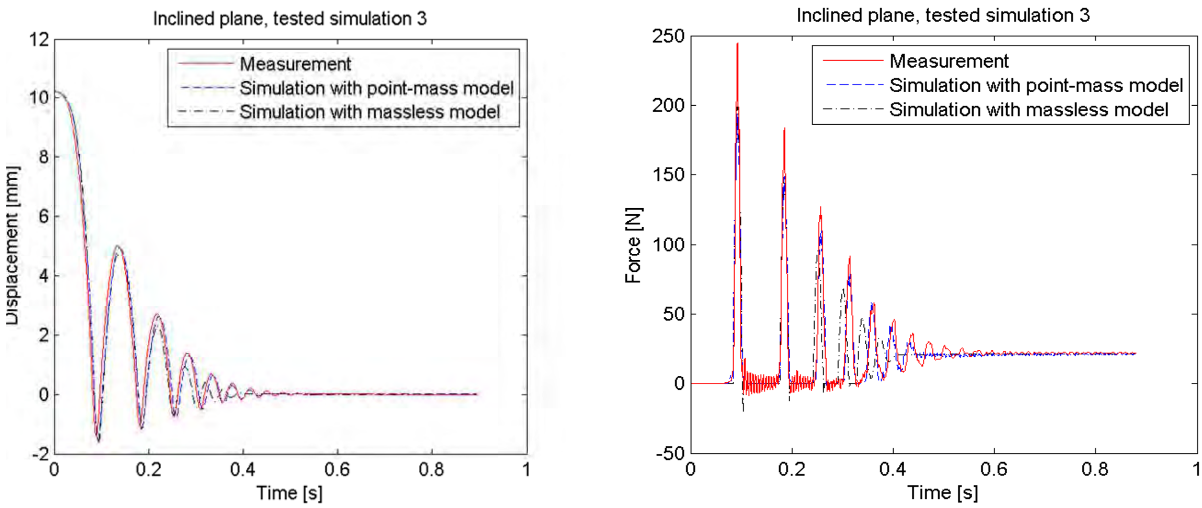
b)

Fig. 4: Time histories at free fall, a) the weight position, b) the force acting in fibre.



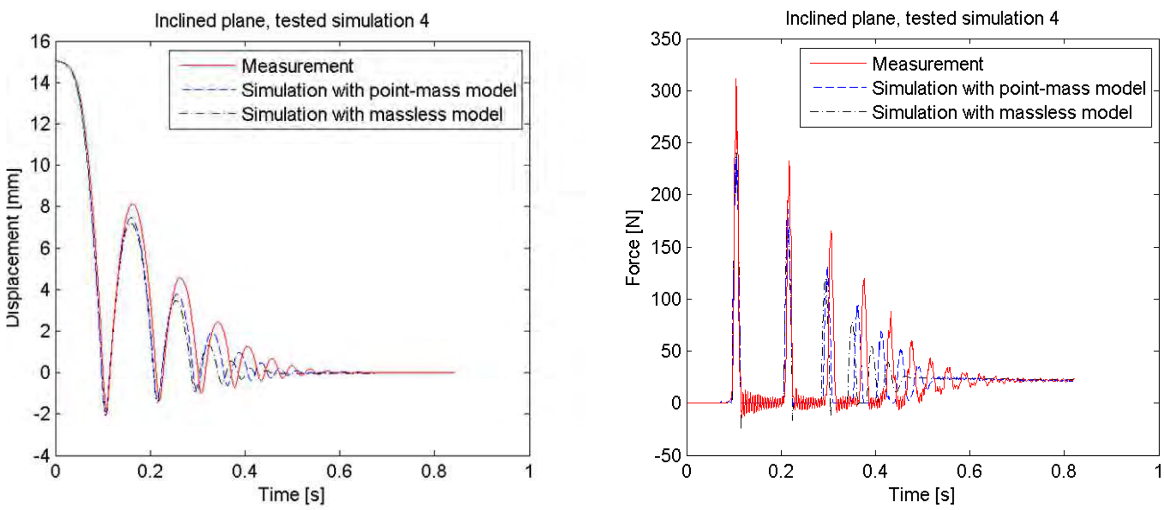
a) *b)*

Fig. 5: Time histories at sliding down the inclined plane, tested situation 2, a) the weight position, b) the force acting in fibre.



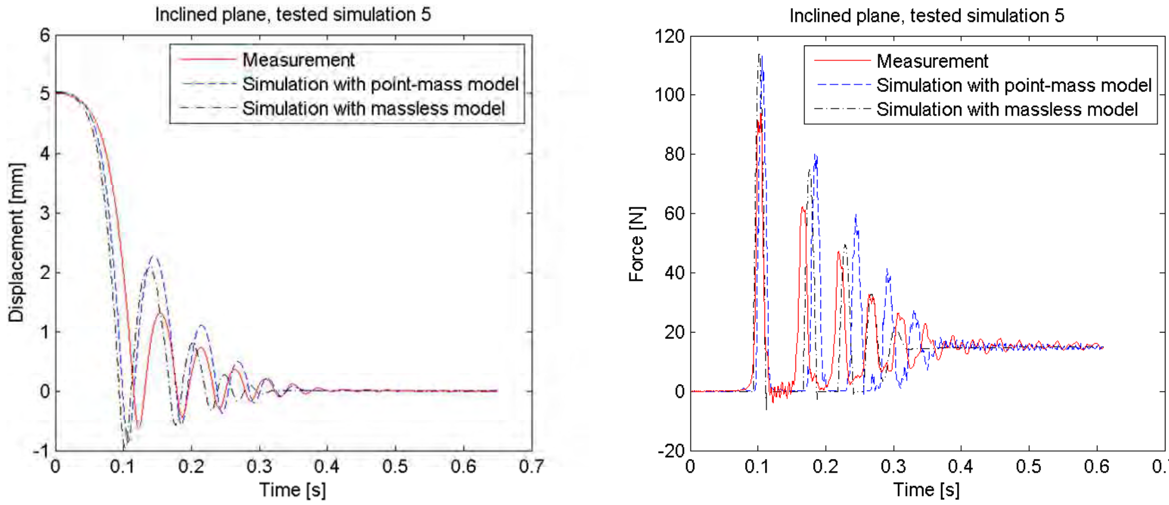
a) *b)*

Fig. 6: Time histories at sliding down the inclined plane, tested situation 3, a) the weight position, b) the force acting in fibre.



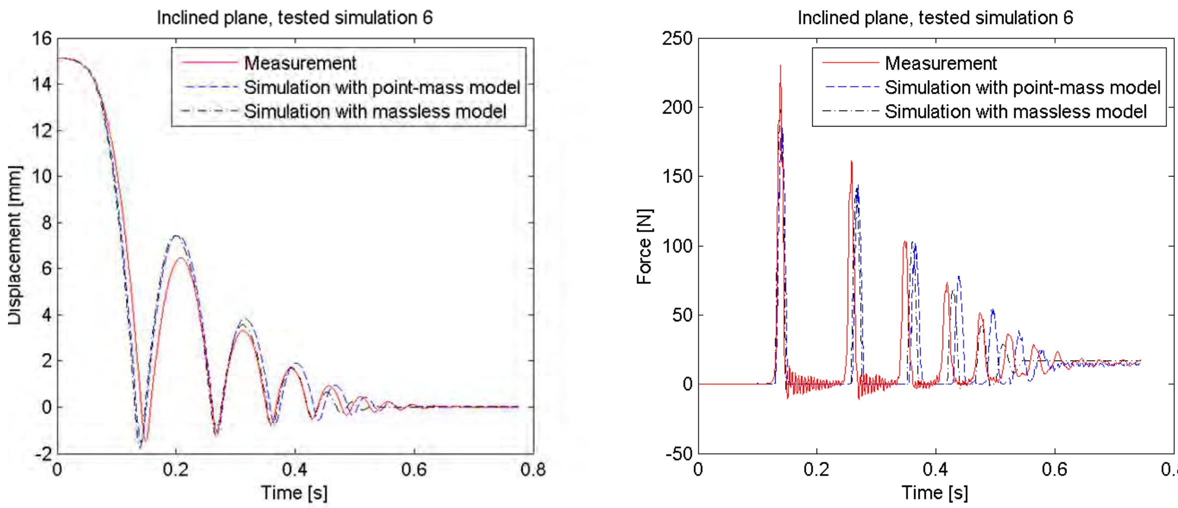
a) *b)*

Fig. 7: Time histories at sliding down the inclined plane, tested situation 4, a) the weight position, b) the force acting in fibre.



a) *b)*

Fig. 8: Time histories at sliding down the inclined plane, tested situation 5, a) the weight position, b) the force acting in fibre.



a) *b)*

Fig. 9: Time histories at sliding down the inclined plane, tested situation 6, a) the weight position, b) the force acting in fibre.

The delay in the time histories of the measured forces acting in the fibre caused by the time delay in force gauge sensors was “manually” eliminated in the presented results (see Figs 4b to 9b) compared with the paper Polach et al. (2013).

The following pieces of knowledge (some of them expected) resulted from creating the fibre phenomenological model:

1. When the stiffness of fibre is lower, the local extremes of the time histories of weight displacement are higher and local extremes of the time histories of force acting in fibre are lower. When the stiffness of fibre is lower the weight vibrates with a lower frequency. The only parameter which can “tune” the global minimum of the time history of the weight displacement (e.i. the first minimum from the beginning of the tested situation), is the fibre stiffness.

2. When the damping coefficient of fibre is lower, both the local extremes of the both time histories of weight displacement and of force acting in fibre are higher. When damping coefficient of fibre is lower the weight vibrates with a lower frequency.

3. The lower the friction force between the weight and the prismatic linkage (in its whole domain – see Fig. 10), the higher the first local maximum of the both time histories of the weight displacement and of the force acting in fibre and at the same time the lower second local maximum of both time histories.

4. The lower the static friction force between the weight and the prismatic linkage (i.e. force acting in stick phase of loading before the beginning of the weight motion), the lower the local extremes of both time histories of the weight displacement and of the force acting in fibre during vibration fading. At the same time at vibration fading the weight vibrates with a lower frequency. The weight vibration fades longer.

5. The lower the friction force (acting in the slip phase of the weight motion; according to Coulomb law – e.g. Půst et al., 2011) between the weight and the prismatic linkage the higher, the local extremes of both time histories of the weight displacement and of the force acting in fibre are higher. The weight vibrates with the lower frequency. The weight vibration fades longer.

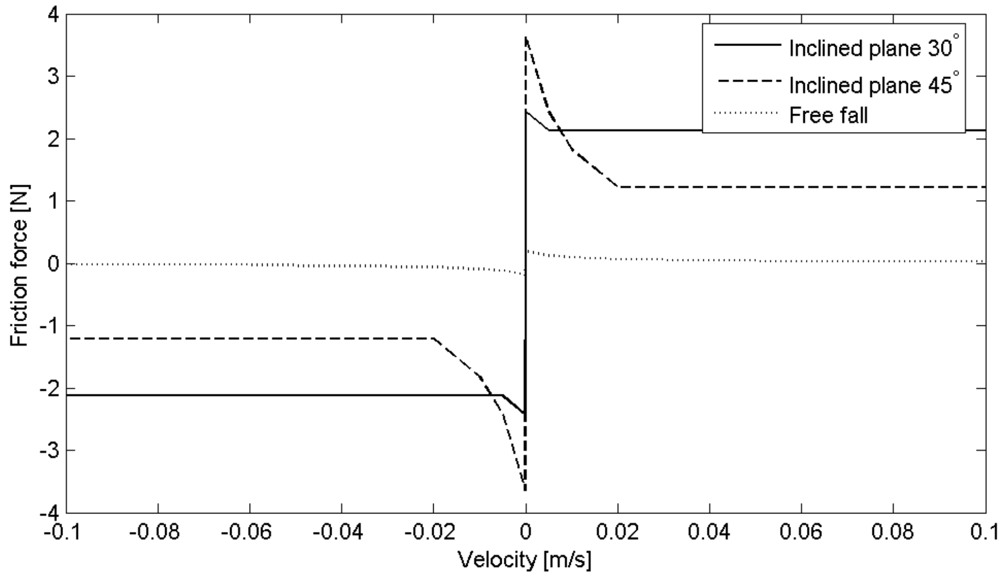


Fig. 10: Friction forces between the weight and the prismatic linkage.

5. Conclusions

The approach to the fibre modelling based on the force and on the lumped point-mass representations was utilised for the investigation of the motion of the weight fastened to a fibre. A case when the weight was lifted to a certain height and then let to fall freely or to slide down an inclined plane was investigated. The simulation aim is to create a phenomenological model of a fibre, which will be utilizable in fibre modelling in the case of more complicated mechanical or mechatronic systems. The created phenomenological model is assumed to be dependent on the fibre stiffness, on the fibre damping coefficient and on the friction force acting between the weight and the prismatic linkage in which the weight moves.

From the achieved results it is evident that the general phenomenological model of the fibre was not determined. General influences of the individual parameters on the system behaviour, which are usable for all systems containing fibre-mass subsystem(s), were assessed. Suitable fibre models, but only in dependence on the definite simulated test situation, were determined. In the case of the investigated simple fibre-mass mechanical system the created models are dependent on the angle of inclination of the (inclined) plane on which the weight moves. It is obvious that it would not be possible to generalize the created models either for the weight-fibre-pulley-drive mechanical system, which will be experimentally investigated, or for the other similar systems. Naturally, the cause of problems can also be in the experimental measurements (e.g. “suspicious” time delay in the measured forces, which was different at each measurement).

Development of the fibre phenomenological model will continue. It can be supposed that in more sophisticated phenomenological model of a fibre more complicated dependencies of the fibre stiffness and of the fibre damping coefficient than currently used constant values will be considered and the usability of the functions of higher orders for the model of the friction force will be verified. The question is if it is possible to create the phenomenological model like this.

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References

Chan, E.H.M. (2005) *Design and Implementation of a High-Speed Cable-Based Parallel Manipulato*r. PhD Thesis, University of Waterloo, Waterloo.

Gerstmayr, J., Sugiyama, H. & Mikkola, A. (2012) Developments and Future Outlook of the Absolute Nodal Coordinate Formulation, in: *Proc. The 2nd Joint International Conference on Multibody System Dynamics* (P. Eberhard & P. Ziegler eds), University of Stuttgart, Institute of Engineering and Computational Mechanics, Stuttgart, USB flash drive.

Hajžman, M. & Polach, P. (2008) Modelling of flexible bodies in the framework of multibody systems, in: *Proc. 6th International Conference Dynamics of Rigid and Deformable Bodies 2008* (B. Skočilasová ed.), Faculty of Production Technology and Management, UJEP in Ústí nad Labem, Ústí nad Labem, pp. 33-42.

Hajžman, M. & Polach, P. (2011) Modelling of Cables for Application in Cable-Based Manipulators Design, in: *Proc. ECCOMAS Thematic Conference Multibody Dynamics 2011* (J.-C. Samin & P. Fisette eds), Université catholique de Louvain, Brussels, USB flash drive.

Kamman, J.W. & Huston, R.L. (2001) Multibody Dynamics Modeling of Variable Length Cable Systems. *Multibody System Dynamics*, 5, 3, pp. 211-221.

Maißer, P., et al. (1998) ***alaska***, *User Manual, Version 2.3*. Institute of Mechatronics, Chemnitz.

Polach, P. & Hajžman, M. (2008) Design of Characteristics of Air-Pressure-Controlled Hydraulic Shock Absorbers in an Intercity Bus. *Multibody System Dynamics*, 19, 1-2, pp. 73-90.

Polach, P., Hajžman, M. & Šika, Z. (2012a) Influence of crucial parameters of the system of an inverted pendulum driven by fibres on its dynamic behaviour. *Applied and Computational Mechanics*, 6, 2, pp. 173-184.

Polach, P., Václavík, J. & Hajžman, M. (2012b) Load of fibres driving an inverted pendulum system, in: *Proc. 50th annual international conference on Experimental Stress Analysis* (M. Růžička, K. Doubrava & Z. Horák eds), Czech Technical University in Prague, Faculty of Mechanical Engineering, Tábor, pp. 337-344.

Polach, P., Václavík, J. & Hajžman, M. (2012c) Verification of the Multibody Models of the TriHyBus on the Basis of Experimental Measurements, in: *Proc. The 6th Asian Conference on Multibody Dynamics ACMD2012*, Shanghai Jiao Tong University, Shanghai, CD-ROM.

Polach, P., Hajžman, M. & Václavík, J. (2013) Simple fibre-mass model and experimental investigation, in: *Proc. National Colloquium with International Participation Dynamics of Machines 2013* (L. Pešek ed.), Institute of Thermomechanics Academy of Sciences of the Czech Republic, Prague, pp. 79-84.

Půst, L., Pešek, L. & Radolfová, A. (2011) Various Types of Dry Friction Characteristics for Vibration Damping. *Engineering Mechanics*, 18, 3/4, pp. 203-224.

Rektorys, K., et al. (1994) *Survey of applicable mathematics, Vol. II*. Kluwer Academic Publishers, Dordrecht.

Shabana, A.A. (1997) Flexible Multibody Dynamics: Review of Past and Recent Developments. *Multibody System Dynamics*, 1, 2, pp. 189-222.

Schiehlen, W. (2007) Research trends in multibody system dynamics. *Multibody System Dynamics*, 18, 1, pp. 3-13.

Stejskal, V. & Valášek, M. (1996) *Kinematics and Dynamics of Machinery.* Marcel Dekker, Inc., New York.

Valášek, M. & Karásek, M. (2009) HexaSphere with Cable Actuation. *Recent Advances in Mechatronics: 2008-2009*, Springer-Verlag, Berlin, pp. 239-244.

Wasfy, T.M. & Noor, A.K. (2003) Computational strategies for flexible multibody systems. *Applied Mechanics Review*, 56, 6, pp. 553-613.

Zemčík, R. (2011) *Determination of stiffness and strength of carbon fibres in axial direction*. Research report to the order no. 11-02200707, University of West Bohemia, Plzeň. (in Czech)

Zi, B., Duan, B.Y., Du, J.L. & Bao, H. (2008) Dynamic modeling and active control of a cable-suspended parallel robot. *Mechatronics*, 18, 1, pp. 1-12.