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LOAD-BEARING CAPACITY OF MASONRY ARCH BRIDGES

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Abstract: *The paper is focused on development of new engineering method for determination of load-bearing capacity of buried masonry-arch bridges. This new method is based on assumptions of European standards.*

Keywords: ***Masonry arch bridges, load-bearing capacity.***

1. Introduction

Masonry arch bridges are one of the oldest kinds of bridges. There are many buried masonry arch bridges on the roads in the Czech Republic (estimate is 10.000 pcs.) and many of them are in a rather bad state. As the funds for bridge rehabilitation are limited nowadays, the correct evaluation of maximum service loading of such bridges gains in importance.

Only the European standards (or other consistent standards) should be used for structural check in present days. However, the European standards (EN 1996) do not define any method for assessment of arch structures like masonry arch bridges. Therefore, a new version of the corresponding Czech national standard ČSN 73 6213 was published recently. In this national standard the basic requirements for structural analysis and verification of masonry arch bridges are defined.

This work is focused on the development and verification of a "simple" and credible method for evaluation of the maximum service loading of buried masonry arch bridges. The method is based on the theory of materially non-linear beam.. For calculations of internal forces and determination of load-bearing capacity, a common spreadsheet program is used.

2. Structural model

2.1 Masonry arch

A beam model of unit width which assumes the non-linear material is used for structural analysis. The material (masonry) non-linearity resulted from elimination of tensional stresses in the cross-section (see Fig. 1). In the compression zone, the linear distribution of stresses is assumed. The modulus of elasticity E of masonry is determined by tests or (in common cases) is based on experience.

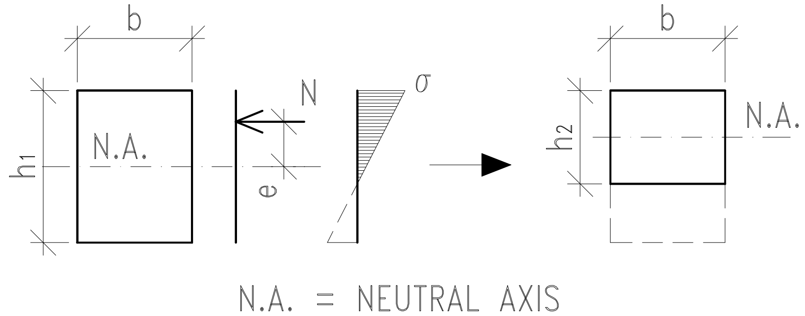

Fig. 1: Reduction of the cross-section area due to normal force eccentricity

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The compression and flexural stiffness (*EA* and *EI*) of the structure (masonry arch) is dependent on the eccentricity $e = M/N$ of the normal force (see Fig.1 and Fig.2). Functions A(e) and I(e) are defined by three different formulas in each of the basic intervals of eccentricity *e*.

$e \in <0\,;\,h/6>$ $\qquad\qquad$ $e \in (h/6\,;\,h/2)$

$$A = bh \qquad\qquad A = 3b \cdot \left(\frac{h}{2} - e\right) \tag{1}$$

$$I = \frac{1}{12} bh^3 \qquad\qquad I = \frac{9}{4} b \cdot \left(\frac{h}{2} - e\right)^3 + 3b \cdot \left(\frac{h}{2} - e\right)\left(\frac{3e}{2} - \frac{h}{4}\right)^2 \tag{2}$$

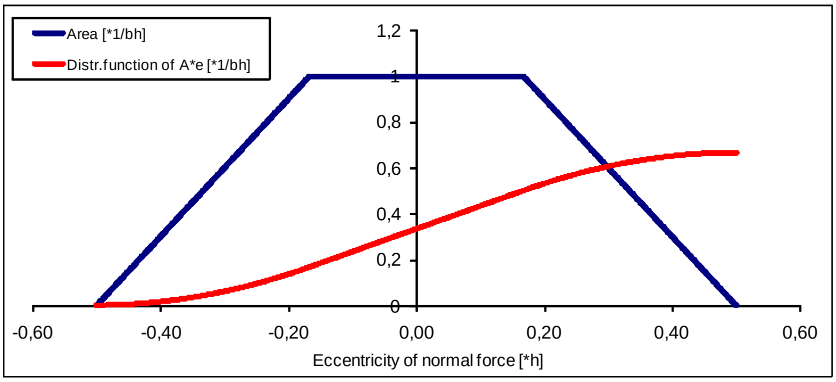

Fig. 2: The dependence of the cross-section area on the eccentricity of the normal force and the distribution function ∫A de

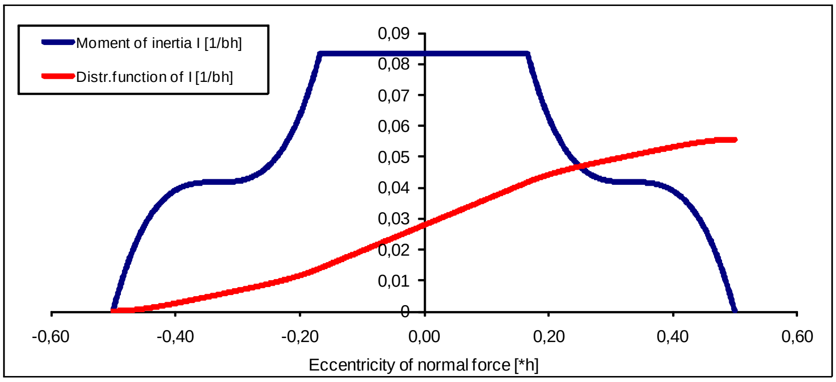

Fig. 3: The dependence of the moment of inertia on the eccentricity of the normal force and the distribution function ∫I de

In the structural model, the average values of stiffness *EA* and *EI* are used. The stiffness of the beam elements is calculated from the values of eccentricity in each step of the structural analysis. The average value of the cross-sectional area A_{av} of the beam of length of *L* and variable eccentricity $e(x)$ is derived from formula (3).

$$A_{av} = \frac{\int_0^L A(L) \cdot dL}{L} = \frac{L \cdot \int_{e(0)}^{e(L)} A(e) \cdot de}{L \cdot |e(L) - e(0)|} = \frac{\int_{e(0)}^{e(L)} A(e) \cdot de}{|e(L) - e(0)|} \tag{3}$$

For average value of the moment of inertia I_{av}, similar formula can be derived. The distribution functions shown in Fig.2 and Fig.3 are used for calculation of the average values A_{av} and I_{av} in case of general course of eccentricity *e* on the structural member (beam). Distribution functions ∫*A* and ∫*I* are composed of parts according to basic functions for *A* and *I*.

$e \in$ (-h/2 ; -h/6) $$\int A \cdot de = \frac{3b}{2} \cdot e \cdot (h + e)$$

$e \in$ <-h/6 ; h/6> $$\int A \cdot de = b \cdot h \cdot e \qquad (4)$$

$e \in$ (h/6 ; h/2) $$\int A \cdot de = \frac{3b}{2} \cdot e \cdot (h - e)$$

$e \in$ (-h/2 ; -h/6) $$\int I \cdot de = \frac{3be}{8}\left(6e^3 + 8e^2h + 4eh^2 + h^3\right)$$

$e \in$ <-h/6 ; h/6> $$\int I \cdot de = \frac{1}{12} b \cdot h^3 \cdot e \qquad (5)$$

$e \in$ (h/6 ; h/2) $$\int I \cdot de = -\frac{3be}{8}\left(6e^3 - 8e^2h + 4eh^2 - h^3\right)$$

2.2 Backfill of the bridge

The backfill is not just a passive part of the structure and therefore its behavior can be divided into two basic parts. The effects of both the parts are related to unit width of the masonry arch.

The first part represents the effect of dead load in vertical direction and traffic loading (in vertical and horizontal directions) and is independent on the deformation of arch. The dead load is composed of self-weight of the masonry arch (dependent on cross-section dimensions) and self-weight of the embankment (dependent on embankment thickness and density). The distribution of traffic loading is determined by arrangement of the loading vehicle and embankment behavior, which is assumed to be elastic. The distribution of traffic loading is usually determined by a separate embankment analysis.

The second part represents the effect of earth pressure in the horizontal direction. This part is strongly dependent on pushing of the arch into the embankment and pre-consolidation (compaction) of the embankment material during construction. The embankment resistance may be expressed by known terms for earth pressure loading and its dependence on displacements of structure (see ČSN 73 0037). An example of a multi-linear relationship of embankment resistance (resp. corresponding horizontal pressure) on the horizontal movement of the structure is shown in Fig. 4. Horizontal pressure due to pushing into embankment is applied (in each steps of non-linear calculation) on vertical projection of beam length *L*.

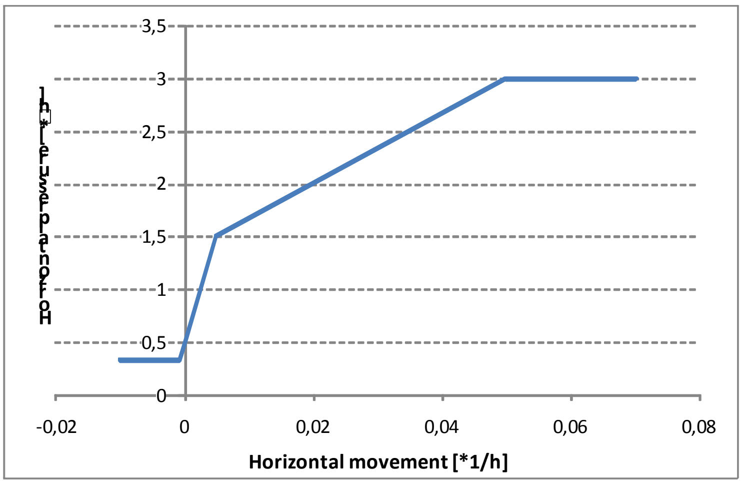

Fig. 4: The relationship of the embankment resistance on horizontal movement of structure

3. The requirements of the standards

3.1 Material

The design characteristics of masonry, i.e. compressive strength (f_k) and shear strength (f_{vk}), are given by EN 1996 and ČSN 73 6213. For existing structures, accurate method of strength determination should be used (e.g. Oniščik formula - see Drahorád, M. & Hrdoušek, V.,2012). The value of partial safety factor for masonry is considered $\gamma_M = 2,0$.

3.2 Ultimate limit state (ULS)

A uniform normal stress distribution (f_d) in the compression zone of the cross-section is assumed in ULS, the tensile zone is neglected. The maximum height h_{cu} of the compression zone is 0.2h, i.e. maximum design value of eccentricity e_u must be less then 0,4h (see Fig. 5). The initial eccentricity e_{init} is assumed according to EN 1996.

The design resistance of masonry cross-section is given by formulas:

$$N_{Rd,\max} = f_d \cdot b \cdot (h - 2e_u) \qquad M_{Rd,\max} = N_{Rd,\max} \cdot e_u \qquad V_{Rd,\max} = f_{vd} \cdot b \cdot (h - 2e_u)$$

$$h_{cu} \geq \frac{h}{5} \rightarrow e_u \leq \frac{2h}{5} \qquad e_u = \frac{M_{Ed}}{N_{Ed}} + e_{init} = \frac{M_{Ed}}{N_{Ed}} + \frac{L_{cr}}{450} \tag{6}$$

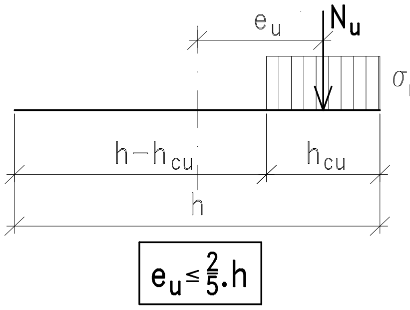

Fig. 5: Normal stress distribution in the cross-section in ULS

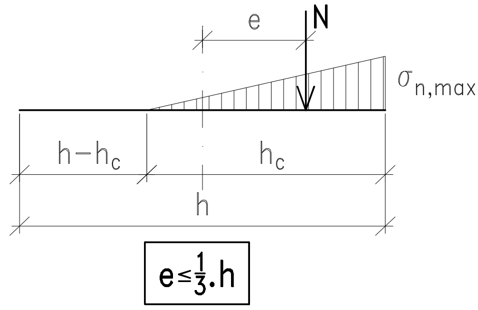

Fig. 6: Normal stress distribution in the cross-section in SLS

3.3 Serviceability limit state (SLS)

The linear normal stress distribution (σ_n) in the compression zone of the cross-section is assumed in SLS, the tensile stress is neglected. In addition, the maximum compressive stress value is $\sigma_n = 0,45 f_k$. The maximum height of the compression zone h_c is 0.5h, i.e. maximum design value of the eccentricity e must be less then h/3 (see Fig. 6). The initial eccentricity e_{init} is assumed according to EN 1996.

The design resistance of masonry cross-section is given by formulas:

$$\sigma_{n,\max} = \frac{N_{Ek}}{3b \cdot (h - 2e)} \leq 0{,}45 f_k \qquad h_c \geq \frac{h}{2} \rightarrow e \leq \frac{h}{3}$$

$$e = \frac{M_{Ek}}{N_{Ek}} + e_{init} = \frac{M_{Ek}}{N_{Ek}} + \frac{L_{cr}}{450} \tag{7}$$

4. Load-bearing capacity

Determination of the load-bearing capacity is composed of two basic tasks: finding the critical loading position of basic (unit) vehicle and finding the critical loading value (vehicle weight).

The position of critical loading depends on the loading (vehicle) arrangement and can be found by common linear analysis. The usual decisive criterion is achieving the maximum tensional stress in the critical cross-section.

The value of the critical loading is calculated by non-linear analysis in each limit state. The loading of the structure (in the critical position) is increased until one of the conditions defined by the European and national standards for the appropriate limit state is reached.

5. Worked example

To explain the developed method a typical buried masonry arch bridge was analyzed. The dimensions of the structure are shown in Fig. 7, the width of the cross-secrion is assumed 1,0 m. The force *F* at middle span represents one wheel of rear axis of a standard vehicle. The task is to find the magnitude of force *F* or vehicle weight respectively.

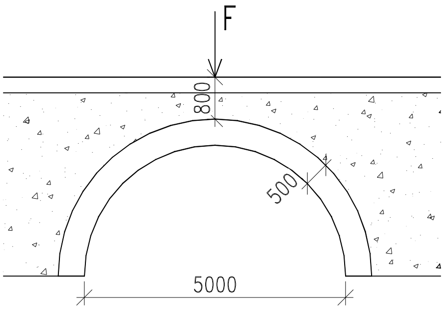

Fig. 7: Structural scheme in longitudinal section

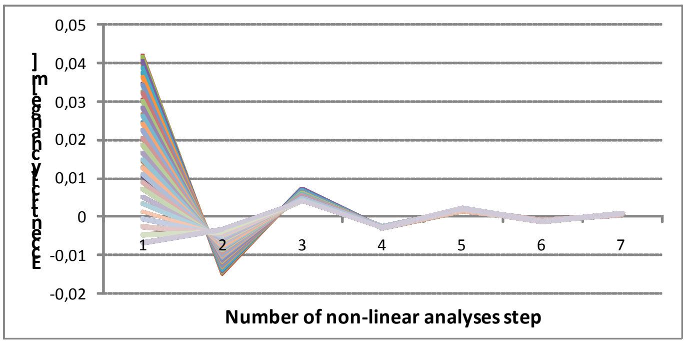

Fig. 8: The relationship of the eccentricity changes on number of the steps of the non-linear analysis

The effect of the backfill is determined according to chapter 2.2; the material parameters are shown in Tab.1. The distribution of traffic loading on the surface of the arch is obtained by a separate calculation based on soil mechanics principles and assuming fully rigid arch behavior.

Tab. 1: Properties of the structural materials

Masonry	Backfill
Density $\rho = 25{,}0$ kN/m^3	Density $\rho = 19{,}0$ kN/m^3
Modulus of elasticity $E = 7{,}5$ GPa	Angle of internal friction $\varphi = 30$ °

The analysis of arch structure was performed according to principles defined above. After seven steps of the non-linear analysis, all standards requirements (see chapter 3) were reached and the magnitude of force *F* = 135 kN was set. The course of the normal force eccentricity *e* along the structure length at the beginning (step 0) and at the end (step 7) of iteration process is shown in Fig.9. The course of analysis convergence is shown in Fig. 8.

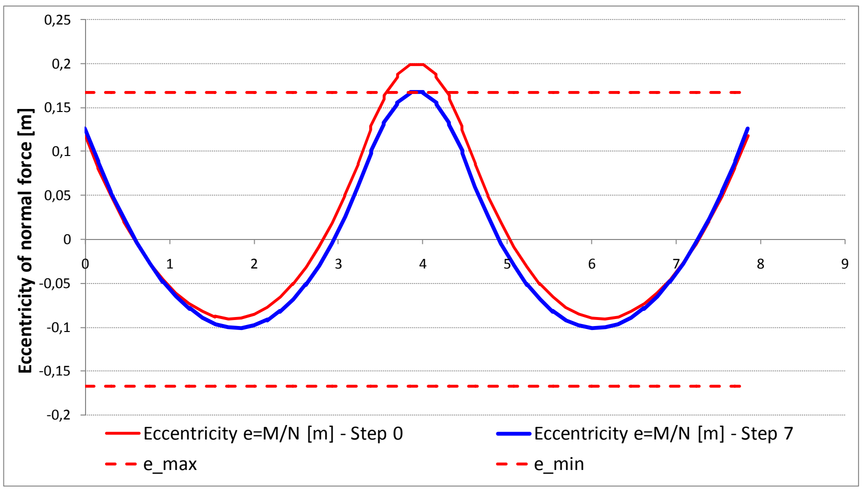

Fig. 9: The course of the normal force eccentricity e along the structure length (at the beginning (0) and at the end (7) of iteration process)

Assuming dynamic ratio δ = 1,4 and weight of rear axis equal 75% of vehicle weight, the maximum vehicle weight is :

$$M_{veh} = \frac{135 \cdot 2}{1{,}4 \cdot 0{,}75} = 25{,}7t$$

6. Conclusions

The new non-linear engineering method for load-bearing capacity determination according to the European standards was developed. Currently, the method is being verified in engineering practice.

Acknowledgement

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