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THE STABILITY INVESTIGATION OF VIBRATIONS OF FLEXIBLY SUPPORTED RIGID ROTORS DAMPED BY HYBRID MAGNETORHEOLOGICAL DAMPING ELEMENTS

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Abstract: *A usual technological solution how to reduce the time varying forces transmitted between the rotor and its casing consists in application of a flexible suspension and in adding the damping devices to the constraint elements. To achieve their optimum performance their damping effect must be controllable. For this purpose a concept of a hybrid damping device working on the principle of squeezing the layers of normal and magnetorheological oils have been developed. Here in this paper, there is investigated influence of the proposed damping element on stability of the rotor vibrations induced by its imbalance during the steady state operating regimes. The stability is evaluated by two approaches: by utilization of a transition matrix and the Floquet theorem and by the evolutive method. The former determines the stability by means of magnitude of the largest eigenvalue of the transition matrix set up over the span of time of one period, the later by time evaluation of eigenvalues of the system linearized in a small neighbourhood of the steady state phase trajectory of the rotor during one period and calculation of the corresponding damping ratio or logarithmic decrement.*

Keywords: ***rigid rotors, controllable damping, hybrid magnetorheological dampers, vibration stability, Floquet theorem, evolutive method.***

1. Introduction

Unbalance of rotating parts excites the time varying forces transmitted between the rotor and its casing. A usual technological solution how to reduce their magnitudes consists in application of a flexible suspension and in adding the damping devices to the rotor supports. A simple analysis shows that to achieve their optimum performance their damping effect must be controllable to be possible to adapt it to the current operation conditions. This is enabled by magnetorheological damping devices.

In rotating machinery the squeeze film dampers lubricated by Newtonian oils are widely used. Burrows et al., (1984) controlled the damping coefficients in the rotor support elements by changing the oil-supply pressure and examined effect of this manipulation on the rotor behaviour. The proposed approach reduced both the rotor vibration and the forces transmitted to the machine frame. A controllable squeeze film damping device based on changing the width of the conical gap by shifting the outer ring of the damping element in the axial direction was reported by Mu et al., (1991). A new concept of controlling the damping effects is represented by magnetorheological dampers. As resistance against the flow of magnetorheological liquids depends on magnetic induction, the damping force can be controlled by changing the magnitude of the electric current generating magnetic flux passing through the film of the magnetorheological liquid. Results of the experiments carried out with a squeeze film magnetorheological damper on a small test rotor rig were reported by Carmignani et al., (2005, 2006). Zapoměl et al., (2012) developed a mathematical model of a short squeeze film damper lubricated by magnetorheological liquid applicable for analysis of both the steady state and transient rotor vibrations.

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The results of the theoretical analysis show that to minimize the force transmission between the rotor and its stationary part the damping effect in the rotor supports must be high for lower rotor angular velocities and as low as possible for velocities higher than approximately the critical speed. Here in this paper there is investigated behaviour of a rigid rotor supported by hybrid magnetorheological squeeze film dampers with serial arrangement of the damping layers. These devices make it possible to reach lower damping effect at higher rotational speeds than their compact variant as discussed in Zapoměl et al., (2013). The principle objectives were to study influence of the damping devices on stability of the rotor vibrations. Results of the computational simulations were assessed by means of approaches based on application of the Floquet theorem and the evolutive method.

2. Mathematical model of the constraint element

The principal parts of the studied damping element (Figure 1) are three rings mutually separated by clearances filled with lubricating oils. The inner ring is coupled with the rotor journal by a rolling element bearing and with the damper's body by a squirrel spring. The outer ring is stationary, fixed to the damper housing. The lubricating films are concentric formed by normal (inner) and magnetorheological (outer) oils. Mutually they are separated by the middle ring which is coupled with the damper's housing by a soft spring element. From the physical point of view the lubricating films are in a serial arrangement. Lateral vibration of the rotor squeezes the oil films which produces the damping effect. The damper is equipped with an electric coil generating magnetic flux passing through the layer of the magnetorheological liquid. As resistance against its flow depends on magnetic induction, the change of the applied electric current can be used to control the damping force.

The mathematical model of the studied damping element is based on utilization of a classical theory of lubrication but with some modifications. The normal oil is considered to be Newtonian. The magnetorheological oil is represented by Bingham material with the yielding shear stress depending on magnetic induction. But if no magnetic field is applied, it behaves as normal Newtonian liquid. Further, it is assumed that the geometric and design parameters of the constraint element enable to considered it as short (length to diameter ratio of the rings is small, no or soft sealings are applied at the damper's ends).

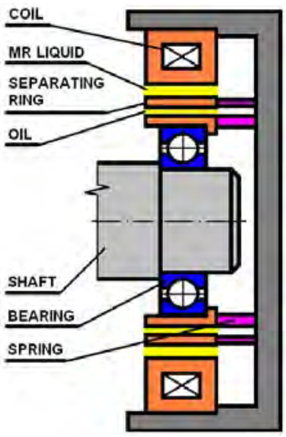

Fig. 1: Scheme of the studied damping device.

The pressure distribution in the layers of the normal and magnetorheological oils are governed by Reynolds equations, from which those referred to the magnetorheological lubricant was modified for Bingham material. The details on their derivation can be found in Krämer, (1993) and Zapoměl et al., (2012)

$$\frac{\partial^2 p_{CO}}{\partial Z^2} = \frac{12\eta}{h_{CO}^3}\dot{h}_{CO}, \tag{1}$$

$$h_{MR}^3 p_{MR}'^3 + 3\left(h_{MR}^2 \tau_y - 4\eta_B \dot{h}_{MR} Z\right) p_{MR}'^2 - 4\tau_y^3 = 0 \qquad \text{for} \qquad p_{MR}' < 0, \quad Z > 0. \tag{2}$$

p_{CO}, p_{MR} denote the pressure in the layers of the normal and magnetorheological oils, η is the normal oil dynamic viscosity, η_B is the Bingham viscosity, τ_y represents the yield shear stress, h_{CO}, h_{MR} are thicknesses of the classical and magnetorheological oil films, Z is the axial coordinate of the points in the lubricating films with the origin in the dampers' middle plane perpendicular to the shaft axis and ( ), (') denote the first derivatives with respect to time and coordinate Z.

Solving the modified Reynolds equation (2) yields the pressure gradient in the layer of the magnetorheological oil, which after the integration gives the pressure profile in the axial direction

$$p_{MR} = \int p'_{MR}\,\mathrm{d}Z\,. \tag{3}$$

The boundary conditions needed for calculation of the constants of integration of the Reynolds equation (1) and integral (3) express that the pressure at the damper's ends is equal to the pressure in the ambient space. More details on solving equation (2), determination of the yielding shear stress

$$\tau_y = k_d \left(\frac{I}{h_{MR}}\right)^{n_y} \tag{4}$$

and calculation of the thickness of the lubricating films at position given by the circumferential angular coordinate φ can be found in Zapoměl et al., (2010, 2012) and Krämer, (1993). n_y is the material constant of the magnetorheological liquid, k_d is the design parameter depending on the number of the coil turns and material properties of the magnetorheological liquid and I is the applied electric current.

In the areas where the thickness of the lubricating films rises with time ($\dot{h}_{CO} > 0$, $\dot{h}_{MR} > 0$) a cavitation takes place. It is assumed that pressure of the medium in cavitated regions remains constant and equal to the pressure in the ambient space. In noncavitated areas the pressure distribution is governed by solutions of the Reynolds equation (1) and integral (3) and the pressure gradient in the axial direction is always negative for $Z > 0$.

Consequently, components of the damping forces are calculated by integration of the pressure distributions around the circumference and along the length of the damping device taking into account the cavitation in the oil films.

$$F_{MRy} = -2\,R_{MR}\int_0^{2\pi}\int_0^{\frac{L}{2}} p_{DMR}\cos\varphi\,\mathrm{d}Z\,\mathrm{d}\varphi \quad , \qquad F_{MRz} = -2\,R_{MR}\int_0^{2\pi}\int_0^{\frac{L}{2}} p_{DMR}\sin\varphi\,\mathrm{d}Z\,\mathrm{d}\varphi\,, \tag{5}$$

$$F_{COy} = -2\,R_{CO}\int_0^{2\pi}\int_0^{\frac{L}{2}} p_{DCO}\cos\varphi\,\mathrm{d}Z\,\mathrm{d}\varphi \quad , \qquad F_{COz} = -2\,R_{CO}\int_0^{2\pi}\int_0^{\frac{L}{2}} p_{DCO}\sin\varphi\,\mathrm{d}Z\,\mathrm{d}\varphi\,. \tag{6}$$

F_{COy}, F_{COz}, F_{MRy}, F_{MRz} are the y and z components of the hydraulic forces produced by the layers of normal and magnetorheological oils respectively, R_{CO}, R_{MR} are the mean radii of the layers of normal and magnetorheological oils, L is the axial length of the damping element and p_{DCO}, p_{DMR} denote the pressure distributions (taking into account different pressures in cavitated and noncavitated regions) in the layers of normal and magnetorheological oils.

3. The investigated rotor

The investigated rotor (Figure 2) consists of a shaft and of one disc and is coupled with the frame by the studied constraint elements at both its ends. It turns at constant angular speed and is loaded by its weight and excited by the disc unbalance. The squirrel springs of the damping elements can be prestressed to be eliminated their deflection caused by the rotor weight. The whole system is symmetric relative to the disc middle plane.

In the computational model the rotor is considered as absolutely rigid and the investigated constraint devices are represented by springs and force couplings. The task was to analyse stability of the rotor vibrations for different magnitudes of the applied current.

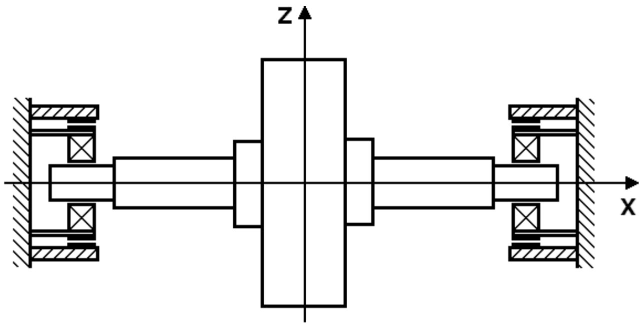

Fig. 2: Scheme of the investigated rotor system.

Lateral vibration of the rotor system is governed by a set of four nonlinear differential equations.

$$m_R \ddot{y}_R + b_P \dot{y}_R + 2k_R y_R = m_R e_T \omega^2 \cos(\omega t + \psi) + 2F_{COy}\,, \tag{7}$$

$$m_R \ddot{z}_R + b_P \dot{z}_R + 2k_R z_R = m_R e_T \omega^2 \sin(\omega t + \psi) - m_R g + 2F_{PSR} + 2F_{COz}\,, \tag{8}$$

$$m_{SR} \ddot{y}_{SR} + k_{SR} y_{SR} = -F_{COy} + F_{MRy}\,, \tag{9}$$

$$m_{SR} \ddot{z}_{SR} + k_{SR} z_{SR} = -F_{COz} + F_{MRz} - m_{SR} g + F_{PSSR}\,. \tag{10}$$

m_R is the rotor mass, b_P is the coefficient of the rotor external damping, k_R is the stiffness of the squirrel spring supporting the rotor, $m_{SR,}$ k_{SR} are the mass of the ring separating the lubricating layers and the stiffness of its support respectively, e_T is eccentricity of the rotor unbalance, ω is angular speed of the rotor rotation, ψ is the phase shift, t is the time, y_R, z_R, y_{SR}, z_{SR} are displacements of the rotor centre (centre of the rotor journal) and of the centre of the ring separating the lubricating layers in the horizontal and vertical directions, g is the gravity acceleration, F_{PSR}, F_{PSSR} are the forces prestressing the dampers' springs and ( ) denotes the second derivatives with respect to time.

The equations of motion (7) - (10) can be rewritten into a matrix form

$$\mathbf{M}\,\ddot{\mathbf{x}} + \mathbf{B}\,\dot{\mathbf{x}} + \mathbf{K}\,\mathbf{x} = \mathbf{f}_A + \mathbf{f}_H(\mathbf{x}, \dot{\mathbf{x}}) \tag{11}$$

where **M**, **B**, **K** are the mass, damping and stiffness matrices, $\mathbf{f}_A$, $\mathbf{f}_H$ are the vectors of the applied (unbalance, gravity, prestress) and hydraulic forces and **x** is the vector of displacements.

As the steady state solution of the equations of motion is assumed to be periodic with the period equal to the period of the rotor rotation, a trigonometric collocation method was applied for its determination. This requires to approximate it by a finite number of terms of a Fourier series

$$\mathbf{x} = \mathbf{x}_0 + \sum_{j=1}^{N_F} \mathbf{x}_{Cj}.\cos\left(j\frac{2\pi}{T}t\right) + \mathbf{x}_{Sj}.\sin\left(j\frac{2\pi}{T}t\right) \tag{12}$$

and to substitute it into the set of governing equations (11) for all collocation points of time

$$t_k = \frac{k-1}{2N_F + 1}T \qquad \text{for} \qquad k = 1, 2, ..., 2\,N_F + 1. \tag{13}$$

$\mathbf{x}_0$, $\mathbf{x}_{Cj}$, $\mathbf{x}_{Sj}$ are the vectors of the Fourier coefficients, N_F is the number of the used Fourier coefficients, and T is the period of the rotor vibration (rotation).

This manipulation yields a set on nonlinear algebraic equations

$$\mathbf{A}\,\mathbf{g} = \mathbf{r}(\mathbf{g}) \tag{14}$$

where **A**, **r**, **g** are the coefficient matrix, the right-hand side vector and the vector of the unknown Fourier coefficients.

More details on application of the trigonometric collocation method can be found e.g. in Zapoměl, (2006, 2007) or Zhao et al., (1994).

4. The stability investigations

The first approach to assessment of the vibration stability is based on a perturbation method and on application of the Floquet theorem. Substraction of the governing equation (11) related to undisturbed motion from those of the disturbed vibration

$$\mathbf{M}\left(\ddot{\mathbf{x}}+\Delta\ddot{\mathbf{x}}\right)+\mathbf{B}\left(\dot{\mathbf{x}}+\Delta\dot{\mathbf{x}}\right)+\mathbf{K}\left(\mathbf{x}+\Delta\mathbf{x}\right)=\mathbf{f}_A+\mathbf{f}_H\left(\mathbf{x}+\Delta\mathbf{x},\dot{\mathbf{x}}+\Delta\dot{\mathbf{x}}\right) \tag{15}$$

and expression of the vector of disturbed hydraulic forces by means of its expansion into the Taylor series in the neighbourhood of the phase trajectory taking into account only the linear terms

$$\mathbf{f}_H\left(\mathbf{x}+\Delta\mathbf{x},\dot{\mathbf{x}}+\Delta\dot{\mathbf{x}}\right)=\mathbf{f}_H\left(\mathbf{x},\dot{\mathbf{x}}\right)+\mathbf{D}_B\left(\mathbf{x},\dot{\mathbf{x}}\right)\Delta\dot{\mathbf{x}}+\mathbf{D}_K\left(\mathbf{x},\dot{\mathbf{x}}\right)\Delta\mathbf{x} \tag{16}$$

give the equation governing the time course of deviations of the rotor system displacements that after some modifications and transformation to the phase space reads

$$\begin{bmatrix}\Delta\ddot{\mathbf{x}}\\ \Delta\dot{\mathbf{x}}\end{bmatrix}=\begin{bmatrix}-\mathbf{M}^{-1}\left(\mathbf{B}-\mathbf{D}_B\right) & -\mathbf{M}^{-1}\left(\mathbf{K}-\mathbf{D}_K\right)\\ \mathbf{I} & \mathbf{O}\end{bmatrix}\begin{bmatrix}\Delta\dot{\mathbf{x}}\\ \Delta\mathbf{x}\end{bmatrix}. \tag{17}$$

$\mathbf{D}_B$, $\mathbf{D}_K$ are the Jacobi matrices of partial derivatives with respect to velocities and displacements, **I**, **O** are the unit and zero matrices and $\Delta\mathbf{x}$ is the vector of deviations of displacements.

As kinematic parameters of the rotor system are periodic functions of time, the coefficient matrix in (17) is also periodic with the same period of *T*. Utilizing the approach described in details in Zhao et al., (1994) stability of the rotor vibration can be assessed by means of the Floquet theorem. This requires to set up a transition matrix **H** over the span of time of one period

$$\mathbf{H}(T,0)=e^{(T-t_{N-1})\mathbf{W}_N}\, e^{(t_{N-1}-t_{N-2})\mathbf{W}_{N-1}}\,.....e^{(t_1-t_0)\mathbf{W}_1} \tag{18}$$

where

$$\mathbf{W}_\mathrm{k}=\frac{1}{t_\mathrm{k}-t_\mathrm{k-1}}\int_{t_{k-1}}^{t_k}\mathbf{W}(t)\,\mathrm{d}t\,. \tag{19}$$

W is the coefficient matrix of equation (17) and *N* is the number of the time intervals into which the period *T* is divided.

The rotor vibration is stable if magnitudes of all eigenvalues of the transition matrix **H** are less than 1.

The second approach to the stability evaluation is based on application of the evolutive method. This requires to linearize the nonlinear term represented by the vector of hydraulic forces in the equation of motion (11) in the neighbourhood of the rotor phase trajectory. The substitution of (16) in (15) and expressing the vector of displacements **y** of the disturbed vibration

$$\mathbf{y}=\mathbf{x}+\Delta\mathbf{x} \tag{20}$$

give after some manipulations the equation of motion of the linearized system

$$\mathbf{M}\,\ddot{\mathbf{y}}+\left[\mathbf{B}-\mathbf{D}_B(\mathbf{x},\dot{\mathbf{x}})\right]\dot{\mathbf{y}}+\left[\mathbf{K}-\mathbf{D}_K(\mathbf{x},\dot{\mathbf{x}})\right]\mathbf{y}=\mathbf{f}_A+\mathbf{f}_H(\mathbf{x},\dot{\mathbf{x}})-\mathbf{D}_B(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}}-\mathbf{D}_K(\mathbf{x},\dot{\mathbf{x}})\mathbf{x}\,. \tag{21}$$

If all its eigenvalues have only negative real parts at any moment of time, then the vibration is stable.

The eigenvalues λ are calculated by solving the characteristic equation

$$\left|\lambda^2\mathbf{M}+\lambda\left(\mathbf{B}-\mathbf{D}_B\right)+\mathbf{K}-\mathbf{D}_K\right|=0\,. \tag{22}$$

Because the Jacobi matrices of partial derivatives $\mathbf{D}_B$, $\mathbf{D}_K$ are functions of the displacements and velocities of the undisturbed vibration, the calculated eigenvalues λ are time dependent. The stability rate can be then characterized by the damping parameters (damping ratio, logarithmic decrement, etc.) that can be determined from the real and imaginary parts of the system eigenvalues.

5. Results of the computational simulations

The technological parameters of the investigated rotor system are: mass of the rotor 450 kg, stiffness of the squirrel spring 5 MN/m, length of the constraint elements 60 mm, widths of the clearances filled by the normal and magnetorheological oils 0.2 mm, 1.0 mm, middle radii of the layers of the normal and magnetorheological oils 55 mm, 75 mm, dynamical and Bingham viscosities of the normal and magnetorheological oils 0.004 Pas, 0.3 Pas, eccentricity of the rotor centre of gravity 0.1 mm, exponential material constant of the magnetorheological oil 2 and the design parameter 0.001 N/A^2. The force prestressing the squirrel springs is not sufficiently high. Its magnitude has only 70% of those needed for the complete compensation of the springs deflection caused by the rotor weight.

The steady state orbits of the centres of the rotor journal and of the separating ring for three magnitudes of the current (0.0 A, 0.7 A, 1.4 A) are drawn in Figure 3 and time histories of the horizontal and vertical time varying components of the force transmitted by one constraint element during the span of time of two periods are evident from Figure 4. The results show that due to the insufficient prestressing of the squirrel springs the orbits are slightly non-circular and are shifted in the vertical direction. The rising current reduces the size of the orbits but slightly increases magnitude of the transmitted force. This implies a suitable setting of the current makes it possible to reach an optimum compromise between minimizing the transmitted force and supressing amplitude of the vibration.

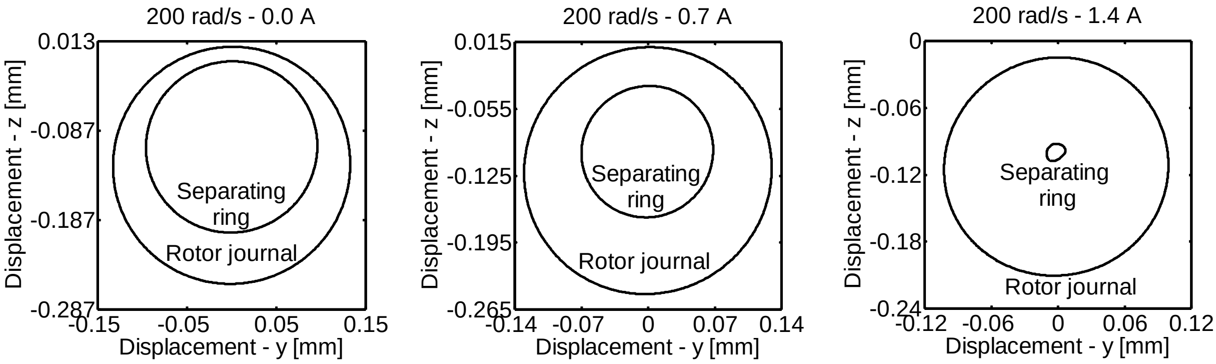

Fig. 3: Orbits of the rotor journal and the separating ring.

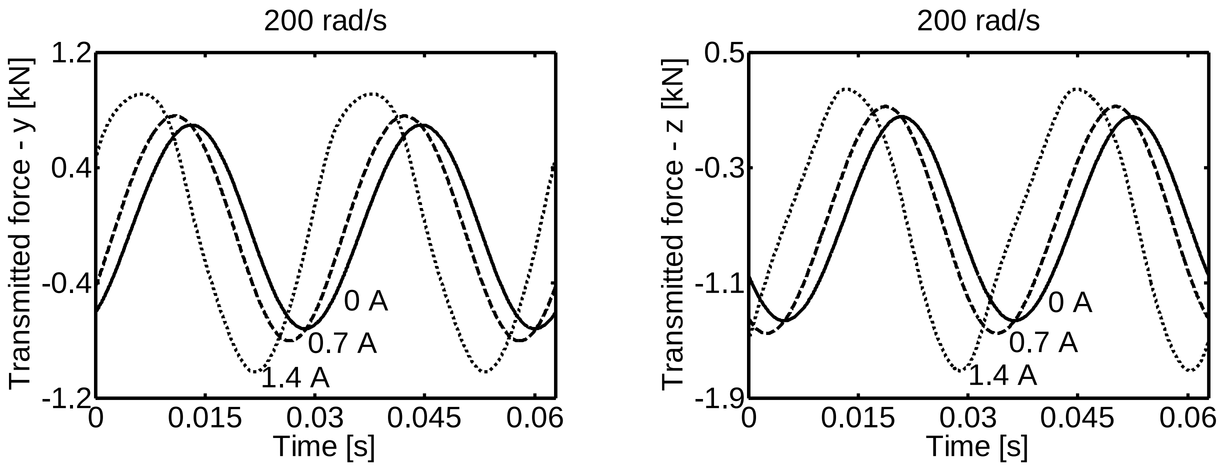

Fig. 4: Courses of the time varying components of the force transmitted by the constraint element.

Figure 5 shows the distribution of eigenvalues of the transition matrix set up over the span of time of one period for three magnitudes of the applied current in the Gauss plane. It is evident that moduli of all eigenvalues are less than 1 which implies that the rotor vibrations are stable for the investigated operating conditions. It is also evident that the rising current contributes to increase of the stability rate of the oscillations (moduli of the eigenvalues are getting smaller).

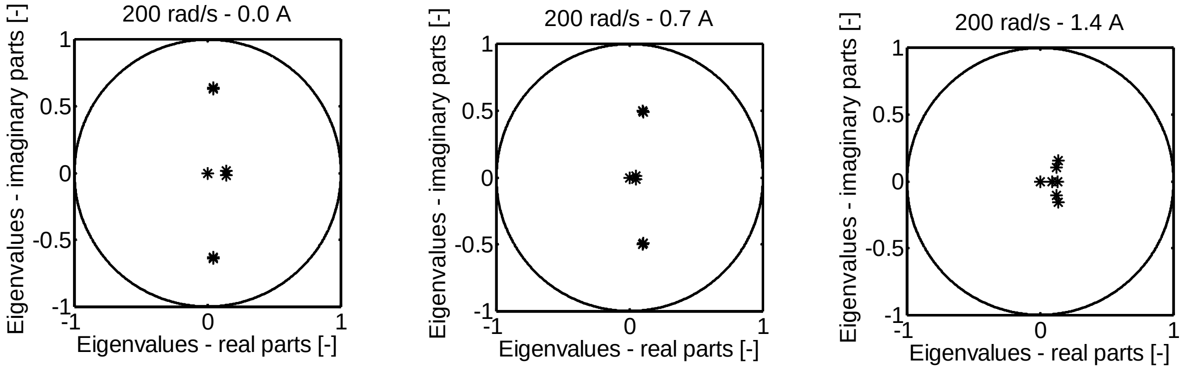

Fig. 5: Distribution of eigenvalues of the transition matrix in the Gauss plane.

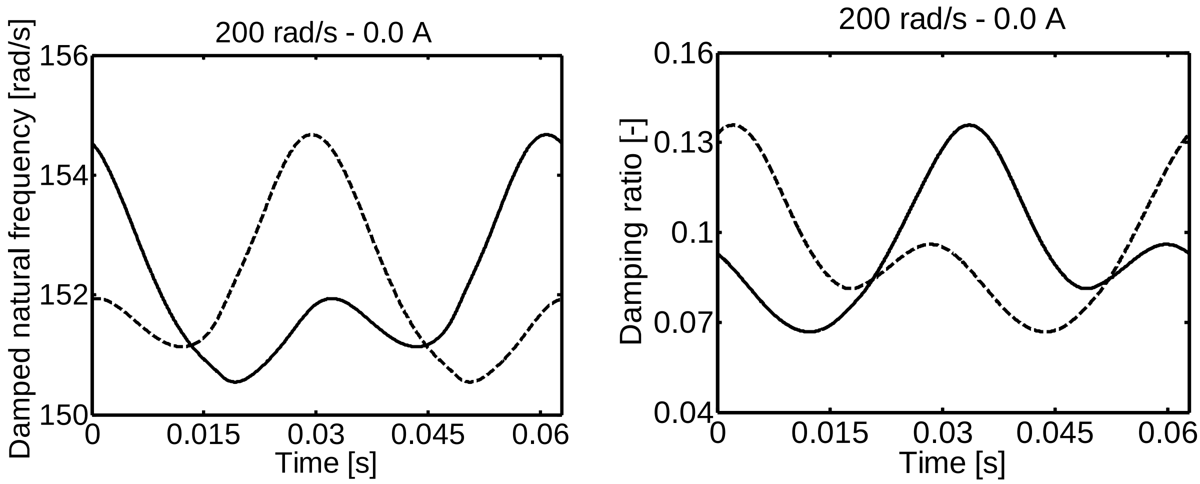

Fig. 6: Time evolution of the system frequency and damping parameters (current 0.0 A).

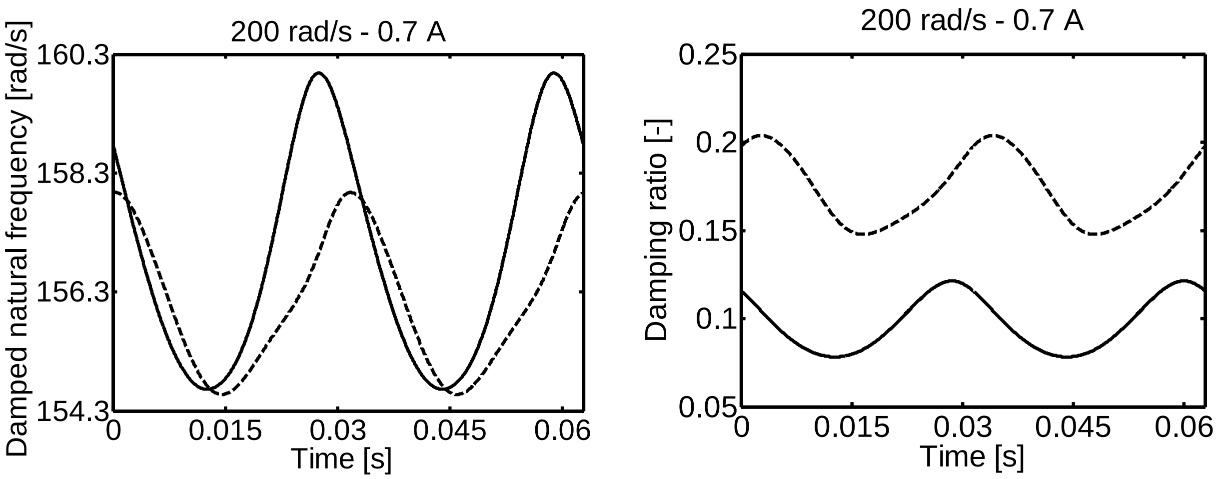

Fig. 7: Time evolution of the system frequency and damping parameters (current 0.7 A).

Analysis of the rotor steady state vibrations by means of the evolutive method gives the time histories of eigenvalues of the system linearized in the neighbourhood of the phase trajectory. The results show that the real parts of all of them are negative and this confirms that the rotor oscillations are stable. In Figures 6, 7 and 8 there are depicted the time evolutions of the damped natural frequencies and corresponding damping ratios that belong to two partial motions having the oscillatory character during the span of time of two periods. It is evident that their values are not constant but that they changes. This is caused by insufficient prestress of the squirrel springs leading to the shift of the journal centres in the constraint elements and consequently to anisotropical properties of the lubricating layers. Based on the rotor design parameters the undamped natural frequency of the system is 149 rad/s. As evident from Figures 6 - 8, presence of the damping device increases the natural frequencies which implies that the lubricating films are not only the sources of dissipation of mechanical energy but that they also contribute to rising the system stiffness. The results also show that the rising current increases values of the damping coefficient. This means that the stability ratio rises which is in accordance with the results obtained by application of the Floquet theorem.

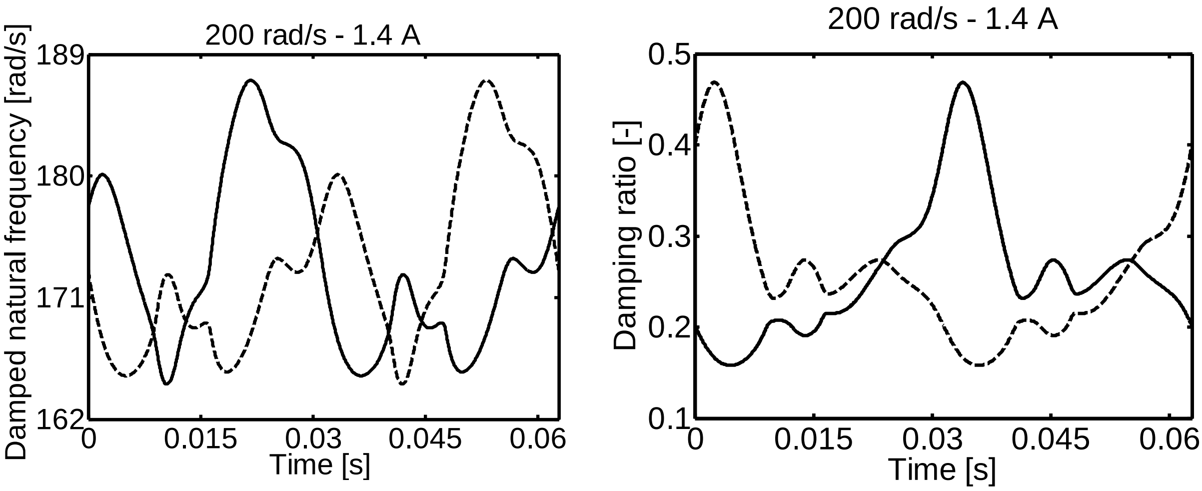

Fig. 8: Time evolution of the system frequency and damping parameters (current 1.4 A).

6. Conclusions

The carried out computational simulations show that the studied magnetorheological damping element makes it possible, due to the current control, to achieve the optimum compromise between reducing magnitude of the force transmitted to the rotor casing and minimizing amplitude of its vibration for the given operating conditions. Stability of the induced oscillations can be evaluated by means of two different approaches based on application of the Floquet theorem and the evolutive method which provides information on the time course and changes of the system spectral and modal parameters. Both methods enable to determine the stability rate which expresses how far from the stability limit the system finds itself. Both approaches can be used for investigation of sensitivity of stability of the rotor system vibrations on different design or operating parameters.

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