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MODELING AND OPTIMIZATION OF SHAPES AND INTERFACES IN VIBROACOUSTIC

E. Rohan[*], V. Lukeš[**], Z. Novotný[***]

Abstract: *We consider acoustic wave propagation in waveguides which are equipped with perforated plates in a form of single or multi-plate panels. Recently we developed mathematical models of the acoustic or vibro-acoustic interaction on such perforated structures using the method of homogenization. These models describe transmission conditions imposed on flat interfaces representing the panels in the global acoustic problems. In view of vast application in the structural optimization, we developed the sensitivity analysis of the acoustic fields with respect to the perforation design. In this paper we report on methods of modeling and optimization of the waveguides designed by the shape, position of the panels and their design.*

Keywords: Vibro-acoustic Transmission, Micro-perforated Panel, Homogenization, Optimization, Sensitivity Analysis.

1. Introduction

The paper deals with design sensitivity analysis and optimization of perforated plates or multi-plate panels which interact with acoustic waves. Transmission conditions for coupling the acoustic pressure fields on both sides of the interface were derived using the homogenization of vibroacoustic interactions in a fictitious layer in which the deforming plates are embedded, see (Rohan and Lukeš, 2010) and (Rohan and Lukeš, 2011, Rohan and Lukeš, 2013). This modeling approach allows one to reduce significantly complexity of finite element meshes needed for computing acoustic response in global domains (the waveguides), since the complicated geometries describing the perforated panels are handled when solving local problems in representative volumes. By virtue of the homogenization method, the "macroscopic" parameters involved in the non-local transmission conditions take into account specific shapes of the holes in the plate and the geometrical arrangement of the panels containing multiple plates.

The problem of minimization or maximization of transmission losses in a waveguide containing the plate can be solved to find piecewise periodic design of the plate perforation the shape of which is described with a few optimization parameters. Moreover we consider (simultaneously, or separately) optimization of the waveguide shape. The design sensitivity analysis of objective functions with respect to the design parameters is based on the shape derivatives and the adjoint equation technique.

2. Model of the Acoustic Waveguide with Homogenized Perforated Panels

We consider the global problem of the wave propagation in a waveguide $\Omega^G \subset R^3$ filled by the acoustic fluid. Ω^G is subdivided by perforated plate Γ_0 in two disjoint subdomains Ω^+ and Ω^-, so that $\Omega^G = \Omega^+ \cup \Omega^- \cup \Gamma_0$, see Fig. 1. The acoustic pressure field p is discontinuous in general along Γ_0. Distribution of p in Ω^G is described by the following equations, where ω is the wave frequency (i.e. $\omega = \kappa \cdot c$ where c is the sound speed and κ is the wave number) and $g\kappa^2$ is the transversal acoustic velocity

[*] Prof. Dr. Ing. Eduard Rohan, DSc.: European Centre of Excellence, NTIS New Technologies for Information Society, Faculty of Applied Sciences, University of West Bohemia, Univerzitní 8, 30614, Pilsen, CZ, rohan@kme.zcu.cz

[**] Ing. Vladimir Lukeš, PhD.: Faculty of Applied Sciences, University of West Bohemia in Pilsen, Univerzitní 22, 306 14 Pilsen; CZ, vlukes@kme.zcu.cz

[***] Bc. Zdeněk Novotný: Faculty of Applied Sciences, University of West Bohemia in Pilsen, Univerzitní 22, 306 14 Pilsen; CZ

$$c^2\nabla^2 p + \omega^2 p = 0 \quad in\ \Omega^+ \cup \Omega^-,$$

$$\left.\begin{aligned} \wp(\omega,[p]_{\Gamma_0}, g^0) = 0 \\ c^2 \partial p/\partial n^\pm = \pm \mathrm{i}\omega g \end{aligned}\right\} \quad on\ \Gamma_0,$$

$$r\mathrm{i}\omega p + c^2 \partial p/\partial n = s 2\mathrm{i}\omega \bar{p} \quad on\ \partial\Omega, \tag{1}$$

where $\wp$ is an abstract function describing the homogenized transmission conditions, s,r and $\bar{p}$ are given data, $[\cdot]_{\Gamma_0}$ is the jump across Γ_0 and $\partial p^\pm / \partial n^\pm$ is the normal derivative on Γ_0 computed using the traces of pressures $p^\pm$ defined in domains Ω^+ and Ω^-, respectively. Boundary $\partial\Omega = \Gamma_w \cup \Gamma_{in} \cup \Gamma_{out}$ of the waveguide is split into walls and the input/output parts; by the constants r,s in (1) different conditions on $\partial\Omega$ are respected: $r = s = 0$ on the waveguide walls Γ_w, whereas $r = s = 1$ on Γ_{in} (waveguide input) and $r = 1$, $s = 0$ on Γ_{out} (waveguide output).

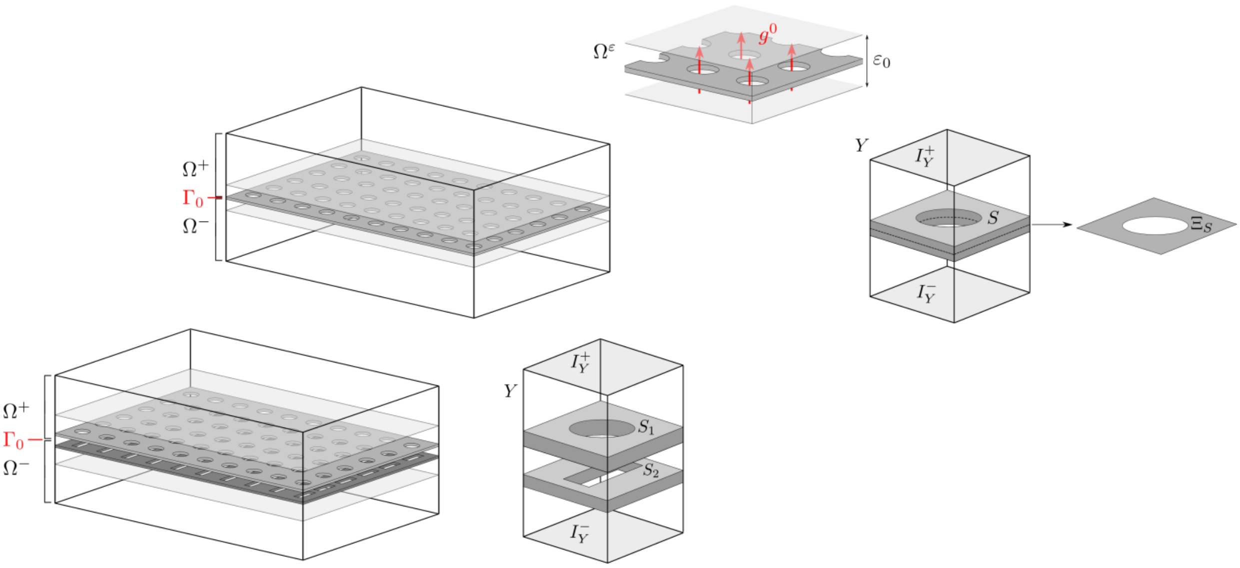


Fig. 1: Top: The global domain Ω^G and the transmission layer in which the plate is situated. The representative periodic cell (RPC) Y where the solid part S represents the perforated plate. Bottom: The double plate panel and its RPC.

The homogenized transmission conditions on Γ_0, see (1), describe the vibro-acoustic transmission on the perforated plates and involve internal variables describing the plate displacements, deflection and rotations (in the case of the Reissner-Mindlin type models). There are two cases to follow:

Single plate panel. The homogenized Reissner-Mindlin thick plate is tailor-made for perforation by general cylindrical holes with axes orthogonal to the mid-plane of the plate (cf. (Rohan and Miara, 2011), an extension for the phononic plates), the transmission problem leads to two subproblems; the membrane "in-plane" modes of the plate vibrations coupled with surface tangent acoustic waves. The transmission condition involving g is related to the deflection-rotation modes of the plate vibrations coupled with the exterior acoustic fields.

Double-plate panel. Each of the plate is presented by thin plates. According to (Rohan and Lukeš, 2010), in general, there is coupling between transverse and surface acoustic waves, however, this phenomenon vanishes when the plate perforation preserves the transverse isotropy (e.g. the case of cylindrical holes). However, the coupling effect is retained also in a case of panels composed of parallel plates with "mutually shifted" cylindrical holes (possibly of different shapes) which, thus, break the transverse symmetry of the panel. We develop a homogenized vibro-acoustic transmission condition which is employed to couple two external acoustic fields on the both sides of the panel.

Here we describe the simpler of the two cases, involving only the simple plate model. For a given design (see below), the Galerkin formulation of (1) is to find $\mathbf{U} := (p,g,\omega,\boldsymbol{\theta}) \in \mathcal{U}$, such that

$$\Psi(\mathbf{U},\mathbf{V}) = f(\mathbf{V}) \quad \forall \mathbf{V} = (q,\psi,z,\boldsymbol{\vartheta}) \in \mathcal{U}_0 = \mathcal{U}\,, \tag{2}$$

where (using the inner products $(\cdot,\cdot)_\Omega$ and $\langle\cdot,\cdot\rangle_{\Gamma_0}$)

$$\begin{aligned}
\Psi(\mathbf{U},\mathbf{V}) \quad &:= c^2(\nabla p,\nabla q)_\Omega - \omega^2(p,q)_\Omega + c\langle p,q\rangle_{\Gamma_{in-out}} - iwc^2\langle g,q^+ - q^-\rangle_{\Gamma_0} \cdot \\
&+ \omega^2\left[C(w,\psi) + C(z,g) - F(g,\psi) - N(w,z) - M(\boldsymbol{\theta},\boldsymbol{\vartheta})\right] \\
&+ E(\overline{\nabla}^S\boldsymbol{\theta},\overline{\nabla}^S\boldsymbol{\vartheta}) + S\left(\overline{\nabla}w - \boldsymbol{\theta},\overline{\nabla}z - \boldsymbol{\vartheta}\right) - i\omega\frac{1}{\varepsilon_0}\langle\psi, p^+ - p^-\rangle_{\Gamma_0}\,, \\
f(\mathbf{V}) \quad &:= 2i\omega\langle\overline{p},q\rangle_{\Gamma_{in}}\,,
\end{aligned} \tag{3}$$

and $\mathcal{U}$ is the set of admissible solutions. Above the bilinear forms C,F,N,M,E,S involve homogenized vibro-acoustic transmission coefficients depending on the perforation design which is given by distributed parameters $\mathbf{d}$. Recall the discontinuity of p and q (the test functions) on Γ_0; by $\varepsilon_0 > 0$ the real thickness of the plate is respected.

3. Optimal Design Problem

The design variables $\mathbf{d}$ describe the shape of holes within the RPC, see Fig. 1. It is given by the design curve Γ_S which determines the simple cylindrical type of perforations. In a more general setting of "nearly-periodic" perforations, the geometry of holes can vary with the global coordinate $x \in \Gamma_0$. The second group of the design parameters $\mathbf{a}$ describe the shape of waveguide $\partial\Omega_G$ in a standard way, see e.g. (Haslinger and Neittaanmäki, 1996). In our study both $\mathbf{d}$ and $\mathbf{a}$ are the selected control points of spline-boxes in which the microstructures (the RVE of the transmission layer), or the global domain Ω_G of the waveguide are embedded.

The optimal waveguide shape and perforation design problem can be defined as follows:

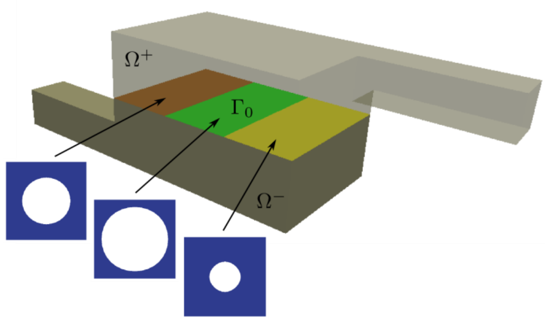
$$\begin{aligned}
&\min_{d\in D_{adm}, a\in A_{adm}} \Phi(\mathbf{U}), \qquad \Phi(\mathbf{U}) = \int_{\Omega_G} |p - \hat{p}|^2 \,. \\
&\textit{subject to:} \quad \mathbf{U} \textit{ solves } (2)
\end{aligned} \tag{4}$$

Above $\hat{p}$ is the desired acoustic field, D_{adm} is the set of admissible designs, constraining the shape regularity of the perforation and some other constraints (e.g. the area of the plate perforation); A_{adm} constraints shapes of Ω_G.

We report an example; the aim was to achieve a desired average acoustic pressure in the waveguide Ω_G. For simplicity, the plate is subdivided in 3 parts, each one is perforated by circular holes. Thus, only 3 optimization variables parameterize the design. The optimal designs are displayed in Fig. 2. The target average magnitude of the acoustic pressure was 317.14 m^2/s.

4. Conclusions

The proposed modeling approach allows for an efficient treatment of the acoustic transmission on perforated plates. We considered various objective functions and optional constraints to optimize the acoustic field in the waveguide. Recently we developed a model of the double-plate panels which can better influence the acoustic transmission.



initial design:

diameters of the holes $0.6\varepsilon_0, 0.6\varepsilon_0, 0.6\varepsilon_0$

optimal design:

diameters of the holes $0.29\varepsilon_0, 0.49\varepsilon_0, 0.58\varepsilon_0$

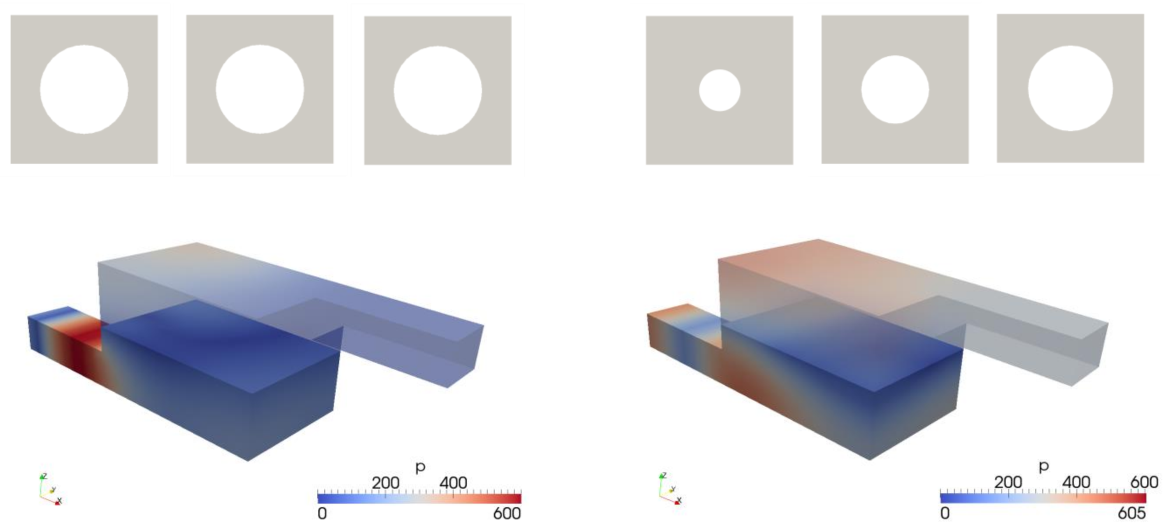


Fig. 2: Top-Right: The global domain Ω^G with a perforated plate, 3 sizes of holes in different parts are considered. Top-Left: Initial and optimized design of the three perforations. Bottom: The overall acoustic fields for the initial (left) and optimized (right) designs.

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References

Haslinger, J., Neittaanmäki, P. (1996) Finite Element Approximation for Optimal Shape, Material and Topology Design, 2nd ed. J. Wiley & Sons, Chichester, U.K..

Rohan, E., Lukeš, V. (2010) Homogenization of the acoustic transmission through perforated layer, J. of Comput. and Appl. Math., 234, pp. 1876-1885.

Rohan, E., Lukeš, V. (2011) Homogenized perforated interface in acoustic wave propagation – modeling and optimization, In: Proc. 10th International Conference on Vibration Problems, ICOVP 2011 (J. Náprstek et al, eds.), Springer Proceedings in Physics, 139, pp. 321-327.

Rohan, E., Lukeš, V. (2013) Optimal design in vibro-acoustic problems involving perforated plates. In: Proc. 11th International Conference on Vibration Problems (ICOVP-2013), Z. Dimitrovová et.al. (eds.) Lisbon, Portugal, AMPTAC, article, pp. 1-10, ISBN: 978-989-96264-4-7.

Rohan, E., Miara, B. (2011) Band gaps and vibration of strongly heterogeneous Reissner-Mindlin elastic plates, Comptes Rendus Mathematique, 349, pp. 777-781.