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INFLUENCE OF THE BENDING RIGIDITY FACTOR ON VIBRATION AND INSTABILITY OF A COLUMN WITH INTERNAL CRACK

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Abstract: *In this paper, the results of studies on transverse vibrations and instability of a geometrically nonlinear column with internal crack subjected to Euler's load are presented. The investigated column is composed of two members. The bending rigidity stiffness between members is described by the bending rigidity ratio. The internal member consists of two elements, connected by a pin and a rotational spring of stiffness C. The rotational spring stiffness C shows the size of crack. The boundary problem has been formulated on the basis of the Hamilton's principle. Due to the geometrical nonlinearity of the system, the solution of the problem was performed by means of the perturbation method. The natural vibration frequencies were computed after obtaining the equations from the first power of the small parameter ε. The results of numerical calculation illustrate the influence of the bending rigidity factor and crack size on vibration frequency and critical loading of the system.*

Keywords: Crack, Column, Vibration, Non-linear system, Instability.

1. Introduction

The types of connection between elements of the structure as well as physical and geometrical features have great influence on dynamic behavior of the system. The failure of the structure may be caused by the crack propagation. The vibration monitoring and crack detection are needed to prevent system failure. The knowledge of crack effect on static and dynamic behavior is important issue in practical applications.

The problems of analysis of the structures with cracks (cracks can be divided into always open and breathing ones), dynamic characteristics of systems and mathematical models have been discussed in past years by Anifantis (1981), Chondros and Dimarogonas (1989), Lee and Bergman (1994), Chondros (2001), Binici (2005).

In this paper the massless rotational spring represents the crack. The spring stiffness coefficient depends on the crack depth. The natural boundary conditions satisfy the continuity of transversal and longitudinal displacements, bending moments and shear forces in the point of location of rotational spring (Uzny, 2011). In many scientific papers authors are focused only on vibration analysis of cracked single columns (Arif Gurel, 2007). In this paper the two member column with internal crack has been investigated. Additionally, the influence of the different magnitudes of bending rigidity factor between the elements of the system on dynamic behavior is also taken into the account.

The investigated system due to its geometrical features is treated as a slender one. Main objective of this work is monitoring of the structure's dynamic behavior. The monitoring is based on vibration frequency and shape mode analysis. The critical force magnitude as a function of a cracked system is also presented. The obtained magnitudes are compared to the uncracked system. The results of numerical calculations allow to predict the possible crack initiation in the cantilever two member column loaded by axially applied external force with constant line of action. Furthermore, the investigated wide range of magnitudes of bending rigidity factor gives a scope on dynamic behavior of the system in different configurations.

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2. Problem Formulation

In the Fig. 1 the investigated cantilever column is presented. Rods (2) and (3) are connected by the pin and rotational spring of stiffness *C* (the smaller magnitude of *C* the greater crack size). The system is loaded by the external force *P* applied in the point of connection of rods (1) and (3). Rods have the lengths l_1, l_2, l_3 respectively. The physical model of the investigated system may be composed of two coaxial tubes, tube and rod or be a flat frame.

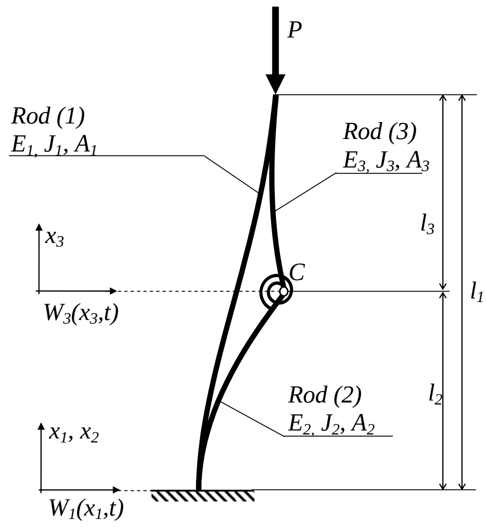

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Fig. 1: Bent axes diagram of the investigated system.

The boundary problem has been formulated on the basis of the Hamilton's principle:

$$\delta\int_{t_1}^{t_2}(\mathrm{E}^k-\mathrm{E}^p)dt=0 \tag{1}$$

where the kinetic E^k and potential E^p energies are expressed as follows:

$$\mathrm{E}^k=\frac{1}{2}\sum_{i=1}^{3}\int_0^{l_i}\rho_i A_i\left(\frac{\partial W_i(x_i,t)}{\partial t}\right)^2 dx \tag{2}$$

$$\begin{aligned}\mathrm{E}^p=\frac{1}{2}\Bigg\{&\sum_{i=1}^{3}\int_0^{l_i}E_iJ_i\left[\frac{\partial^2 W_i(x_i,t)}{\partial x_i^2}\right]^2dx+\int_0^{l_i}E_iA_i\left[\frac{\partial U_i(x_i,t)}{\partial x_i}+\frac{1}{2}\left(\frac{\partial W_i(x_i,t)}{\partial x_i}\right)^2\right]^2dx+\\&+\frac{1}{2}C\left(\left.\frac{\partial W_3(x_3,t)}{\partial x_3}\right|_{x_3=0}-\left.\frac{\partial W_2(x_2,t)}{\partial x_2}\right|_{x_2=l_2}\right)^2\Bigg\}+PU_1(l_1,t),\end{aligned} \tag{3}$$

where the following notation is used: E_i – Young modulus, J_i – moment of inertia, A_i – cross section area ρ_i – material density C – rotational spring stiffness, P – external load. Substitution of equations (2) and (3) into (1) leads to among alia the equations of motion (4)

$$E_iJ_i\frac{\partial^4 W_i(x_i,t)}{\partial x_i^4}-S_i(t)\frac{\partial^2 W_i(x_i,t)}{\partial x_i^2}+\rho_iA_i\frac{\partial^2 W_i(x_i,t)}{\partial t^2}=0 \qquad i=1,2,3 \tag{4}$$

and natural boundary conditions. The geometrical and natural boundary conditions are as follows:

$$
W_2(l_2,t)=W_3(0,t),\; W_1(l_1,t)=W_3(l_3,t),\; E_1J_1\frac{\partial^2 W_1(x_1,t)}{\partial x_1^2}\bigg|^{x_1=l_1}+E_3J_3\frac{\partial^2 W_3(x_3,t)}{\partial x_3^2}\bigg|^{x_3=l_3}=0,\; S_1+S_2=P
$$

$$
E_1J_1\frac{\partial^3 W_1(x_1,t)}{\partial x_1^3}\bigg|^{x_1=l_1}+P\frac{\partial W_1(x_1,t)}{\partial x_1}\bigg|^{x_1=l_1}+E_3J_3\frac{\partial^3 W_3(x_3,t)}{\partial x_3^3}\bigg|^{x_3=l_3}=0,\; W_2(0,t)=\frac{\partial W_2(x_2,t)}{\partial x_2}\bigg|_{x_2=0}=0
$$

$$
E_2J_2\frac{\partial^3 W_2(x_2,t)}{\partial x_2^3}\bigg|^{x_2=l_2}+S_2\frac{\partial W_2(x_2,t)}{\partial x_2}\bigg|^{x_2=l_2}-E_3J_3\frac{\partial^3 W_3(x_3,t)}{\partial x_3^3}\bigg|_{x_3=0}-S_3\frac{\partial W_3(x_3,t)}{\partial x_3}\bigg|_{x_3=0}=0,\quad S_2=S_3 \qquad \text{(5a-n)}
$$

$$
-E_3J_3\frac{\partial^2 W_3(x_3,t)}{\partial x_3^2}\bigg|_{x_3=0}+C\left[\frac{\partial W_3(x_3,t)}{\partial x_3}\bigg|_{x_3=0}-\frac{\partial W_2(x_2,t)}{\partial x_2}\bigg|^{x_2=l_2}\right]=0,\; W_1(0,t)=\frac{\partial W_1(x_1,t)}{\partial x_1}\bigg|_{x_1=0}=0
$$

$$
E_2J_2\frac{\partial^2 W_2(x_2,t)}{\partial x_2^2}\bigg|^{x_2=l_2}-C\left[\frac{\partial W_3(x_3,t)}{\partial x_3}\bigg|_{x_3=0}-\frac{\partial W_2(x_2,t)}{\partial x_2}\bigg|^{x_2=l_2}\right]=0,\; \frac{\partial W_1(x_1,t)}{\partial x_1}\bigg|^{x_1=l_1}=\frac{\partial W_3(x_3,t)}{\partial x_3}\bigg|^{x_3=l_3}
$$

3. Results of Numerical Calculations

The results of numerical calculations shown in Figs. 2 and 3 are presented in the non-dimensional form, where

$$
p=\frac{Pl^2}{E_1J_1+E_2J_2},\; c=\frac{Cl}{E_1J_1+E_2J_2},\; \omega=\sqrt{\Omega^2\,\frac{(\rho_1A_1+\rho_2A_2)l^4}{E_1J_1+E_2J_2}},\; r_m=\frac{E_2J_2}{E_1J_1},\; r_m=\frac{E_3J_3}{E_2J_2},\; d_i=\frac{l_i}{l}.
$$

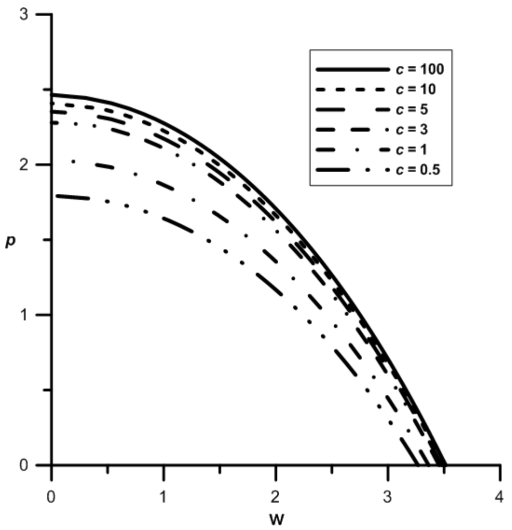

Fig. 2: External load vs. vibration frequency for different crack size (d_2 = 0.5, r_m= r_w = 1).

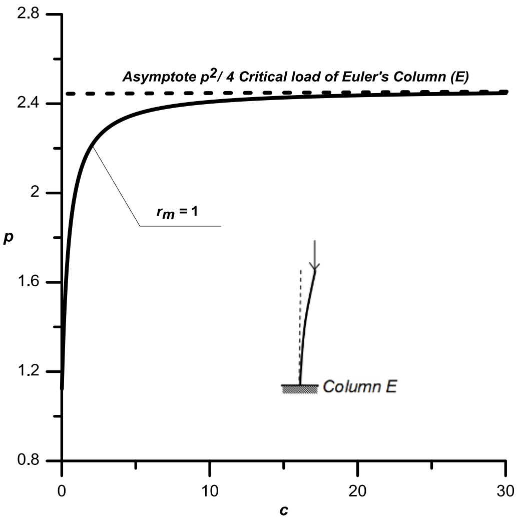

Fig. 3: Influence of crack size on maximum load magnitude (d_2 = 0.5, r_w = 1).

In the Fig. 2 an influence of crack size localized in the middle of the column is presented. When the crack is small (*c* = 10, 100) the magnitudes of maximum load are comparable. With the reduction of the rotational spring stiffness (*c = 5, 3, 1, 0.5*) the decrease of natural vibration frequency and critical loading occurs. It can be concluded that the critical force strongly depends on crack size, as illustrated on – Fig. 3. The critical force of the system without crack (the investigated system with great magnitude of *c* can be treated as a particular case which corresponds to Euler's column) is

$$
p_{cr}=\frac{\pi^2}{4}.
$$

In the case when the bending rigidity factor ratio between rods (1) and (2) is changing regardless to crack size the natural vibration frequency and maximum loading are varying. If r_m parameter is greater than 1

(E_2J_2 > E_1J_1), then with the growing crack size the reduction of vibration frequency and maximum loading appears (Fig. 4). When $r_m < 1$ the opposite situation takes place. In the Fig. 5 the curves on the plane p - r_m for different crack size have been plotted.

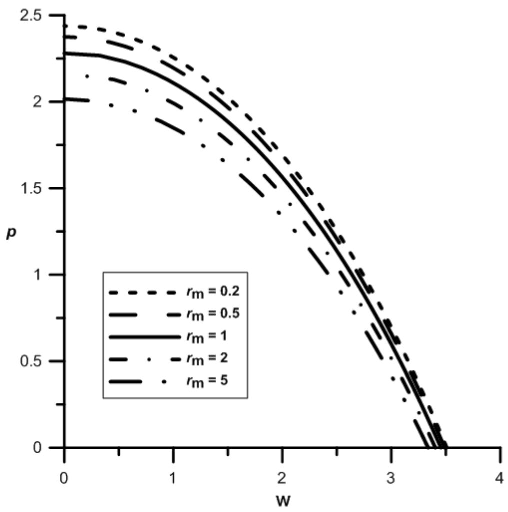

Fig. 4: External load vs. vibration frequency for different rigidity factors (d_2 = 0.5, c = 3, r_w = 1).

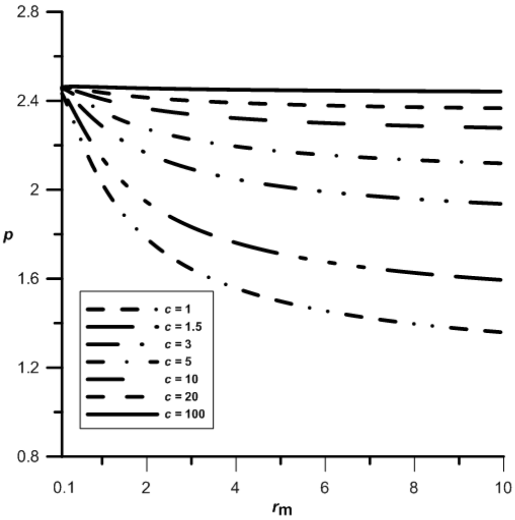

Fig. 5: Rigidity factor as a function of external load for different crack size (d_2 = 0.5, r_w = 1).

The reduction of r_m regardless to crack size causes an increase of maximum loading capacity of the column. With the $r_m > 1$, the growing crack size corresponds to rapid decrease of p magnitude. The change of maximum force magnitude results in vibration frequency change.

4. Conclusions

In this paper the influence of the bending rigidity factor ratio between rods (1) and (2) on the dynamic behavior of the cantilever column with internal crack have been investigated. The results of numerical calculations can be used to identify crack size on the basis of natural vibration frequency analysis. It can be concluded that crack size has significant influence on vibration frequency and maximum load magnitude. For larger crack size the lower critical load was obtained. For $r_m > 1$ the influence of the crack size on investigated parameters is significant. There exists crack size above which the change in r_m parameter on maximum load and vibration frequency is very small. The deviation in the dynamic behavior of the investigated system might be considered as a possible indicator of crack initiation.

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