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CALCULATING THE AIR FLOW VELOCITY IN AIR CAVITIES IN WALLS

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Abstract: *Moisture from walling m4sonry, in particular in historic and protected buildings, is often removed by means of air cavities located in walls under the ground level. The air flow should be, if possible, almost uninterrupted so that this method could be efficient. This paper discusses the calculation of the air flow velocity inside the wall cavities.*

Keywords: Wet masonry, Remediation of wet masonry, Air cavities, Air flow in air cavities.

1. Introduction

Air insulation methods have ranked among frequent methods used for removal of moisture from walling masonry. This is, in particular, the case of historic buildings. When designing, an empiric approach is often used for air cavities. The air flow should be, if possible, almost uninterrupted so that this method could be efficient in the air cavities. The calculation below is for an air cavity (see Fig. 1) with natural flow of air where both the suction and exhaust holes are located at the outside.

2. Calculating an Open Air Cavity

In order to calculate performance of an air cavity in a wall it is necessary to determine the velocity of air in the air cavity – w_x [m.s^{-1}], the temperature of the air which flows in the air cavity – t_x [°C], the partial pressure of water vapours in the air cavity – p_{dx} [Pa], the partial pressure of water vapours in the air cavity upon saturation – p''_{dx} [Pa], condensation of water vapours in the air cavity and a pressure drop of the air flow – Δp [Pa], the pressure loss – Δp_z [Pa], and to decide whether the air cavity works properly. This paper discusses only the calculation of the air flow velocity and the related development of temperature of the air flow in the air cavity.

3. Velocity of the Air Flow in the Air Cavity

In past, the relation for calculation of the air flow velocity in an open air layer (for instance, in calculation of two-layer roofs) was developed on the basis of generally known equations which are valid for the pressure drop of the air flow in the air cavity and the equations which describe the friction losses and resistance integrated into the air cavity. By analogy, this relation can be used in calculations of air cavities through which the air flows naturally. This relation is specified in (Řehánek, 1982). The equation for the air cavity is as follows:

$$w_x = \sqrt{\frac{2 \cdot h \cdot (\rho_v - \rho_x) + (A_n - A_z) \cdot w_v^{\,2} \cdot \rho_v}{\lambda \cdot \dfrac{l_x}{d_r} \cdot \rho_x + \Sigma\xi \cdot \rho_x}} \quad [\text{m.s}^{-1}] \qquad (1)$$

where: h [m] – is the difference in heights between the axis of the suction and exhaust holes.

ρ_v [kg.m^{-3}] – is the volumetric weight of the outdoor air.

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ρ_x [kg.m^{-3}] – is the volumetric weight of the air in the air cavity located in the x distance from the suction hole.

w_v [m.s^{-1}] – is the wind velocity which is determined either pursuant to ČSN 73 0540-3, (Řehánek, 1982) or on the basis of local meteorological data.

A_n [–] – is the aerodynamic coefficient on the wind side (A_a = 0.6).

A_z [–] – is the aerodynamic coefficient on the lee side (A_a = – 0.3).

The aerodynamic coefficients A_a and A_z are used only if the air cavity is connected to the outdoor air.

l_x [m] – is the length of the section in the air cavity.

ξ [–] – is the coefficient of integrated resistance (see ČSN 73 0540-3 and (Řehánek, 1982)).

d_r [m] – is the equivalent diameter of the air cavity located in the x distance from the suction hole:

$$d_r = \frac{2 \cdot v \cdot s}{v + s} \quad [\mathrm{m}] \tag{2}$$

where: v [m] – is the internal ground clearance of the air cavity.

s [m] – is the internal free width of the air cavity.

λ [–] – is the resistance coefficient which depends on the Reynolds number Re and is as follows:

a) for the laminar flow (Re ≤ 2320): $\lambda = \frac{64}{R_e}$ $[-]$ (3)

b) for the turbulent flow (Re > 2320): $\lambda = 0.0032 + \frac{0.221}{\mathrm{Re}^{0.237}}$ $[-]$ (4)

where: Re [–] is the Reynolds' number : $\mathrm{Re} = \frac{w_x \cdot d_r}{\nu}$ [–] (5)

where: d_r [m] – is the equivalent diameter of the air cavity.

w_x [m.s^{-1}] – is the air flow velocity in the air cavity.

ν [m^2.s^{-1}] – is the kinematic air viscosity which can be determined as follows:

$$\nu = \frac{\eta}{\rho_x} \quad \left[m^2 \cdot s^{-1}\right] \tag{6}$$

where: ρ_x [kg.m^{-3}] – is the volumetric weight of the air in the air cavity in the x point located out of the suction.

η [Pa.s] – is the dynamic air viscosity which can be determined as follows:

$$\eta = (17.2 + 0.047 \cdot t_x) \cdot 10^{-6} \quad [\mathrm{Pa.s}] \tag{7}$$

where: t_x [°C] – is the temperature of the air in the air cavity located in the x distance from the suction hole.

The most complicated part of the calculation is, however, the calculation of the velocity of the air which flows through the air cavity with natural air flow - w_x [m.s^{-1}] . The reason is that the velocity depends on two parameters:

1) on the resistance coefficient λ [–] which depends, in turn, on the air flow velocity w_x [m.s^{-1}] and on the Reynolds' number Re [–],
2) on the volumetric weight of the air in the air cavity in the distance x from the suction, ρ_x [kg.m^{-3}], which, in turns, depends on the temperature of the air t_x [°C] in the air cavity in the distance x.

This means, no explicit solution is possible for the velocity of the air flow in the air cavity - w_x [m.s^{-1}]. A numerical approach only can be used. It is advisable to use a software application.

4. Calculating the Temperature in the Air Cavity

The temperature in the air cavity is calculated using the balance equation:

$$Q = Q_1 + Q_2 + Q_3 + Q_4 \quad [W] \tag{8}$$

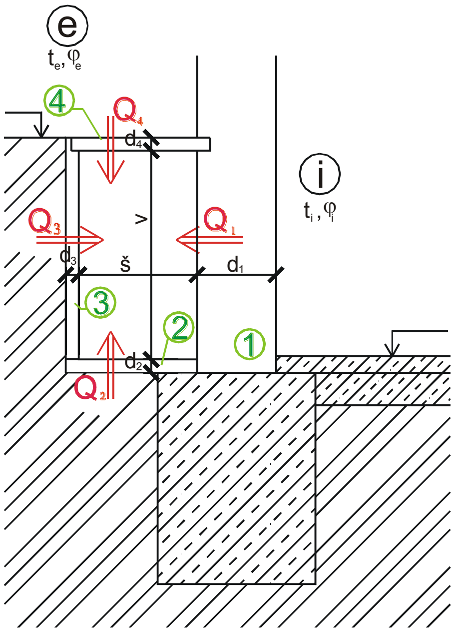

Fig. 1: Scheme of air cavity in wall.

where: Q [W] is the total heat transfer rate of the air which enters the air cavity.

Q_1 through Q_4 [W] is the quantity of the air which enters the air cavity through the structures No. 1 to No. 4.

$$Q_1 = v.U_1.(t_1 - t_x).dx \quad [W] \tag{9}$$

$$Q_2 = s.U_2.(t_2 - t_x).dx \quad [W] \tag{10}$$

$$Q_3 = v.U_3.(t_3 - t_x).dx \quad [W] \tag{11}$$

$$Q_4 = s.U_4.(t_4 - t_x).dx \quad [W] \tag{12}$$

where: v [m] is the internal ground clearance of the air cavity.

s [m] is the internal free width of the air cavity.

U_1 through U_4 [$W.m^{-2}.K^{-1}$] are the heat transfer coefficients No. 1 through 4 for the structure

t_1 through t_4 [°C] are the ambient air temperatures No. 1 through 4 for the structure outside

t_x [°C] – is the temperature of the air inside the air cavity located in the x distance from the suction hole.

The equation is as follows: $$Q = Q_t \quad [W] \tag{13}$$

where: Q_t [W] – is heat transfer rate which increases the temperature of the air in the air cavity by dt_x

$$Q_t = c\ M\ dt_x \quad [W] \tag{14}$$

$$M = \rho_x.s.v.w \quad [kg.s^{-1}] \tag{15}$$

where: w [$m.s^{-1}$] – is the air flow velocity in the air cavity.

ρ_x [$kg.m^{-3}$] – is the volumetric weight of the air in the air cavity located in the distance x from the suction hole.

v [m] – is the internal ground clearance of the air cavity.

s [m] – is the internal free width of the air cavity.

c [$J.kg^{-1}.K^{-1}$] – is the specific thermal capacity which results from the formula below:

$$c = 1010 + 0.12t_x \quad [J.kg^{-1}.K^{-1}] \tag{16}$$

where: t_x[°C] – is the temperature of the air inside the air cavity located in the x distance from the suction hole.

Because this equation can be used for calculation of the specific thermal capacity only if the air is dry, it is possible to use directly $c = 1010\ J.kg^{-1}.K^{-1}$ in the calculation.

Having substituted Q, Q_1, Q_2, Q_3 and Q_4 in (8) one obtains:

$$c.M.dt_x = v.U_1.(t_1 - t_x).dx + s.U_2.(t_2 - t_x).dx + v.U_3.(t_3 - t_x).dx + s.\ U_4.(t_4 - t_x).dx$$

$$c.M.dt_x = v.U_1.t_1.dx - v.U_2.t_x.dx + s.U_2.t_2.dx - s.U_2.t_x.dx + v.U_3.t_3.dx - v.U_3.t_x.dx + s.U_4.t_4.dx - s.U_4.t_x.dx$$

$$c.M.dt_x = v.U_1.t_1.dx + s.U_2.t_2.dx + v.U_3.t_3.dx + s.U_4.t_4.dx - v.U_1.t_x.dx - s.U_2.t_x.dx - v.U_3.t_x.dx - s.U_4.t_x.dx$$

$$c.M.dt_x = (v.U_1.t_1 + s.U_2.t_2 + v.U_3.t_3 + s.U_4.t_4).\ dx - (v.U_1 + s.U_2 + v.U_3 + s.U_4).\ t_x.dx$$

The terms in the brackets can be expressed as the constants A and B:

$$A = v.U_1.t_1 + s.U_2.t_2 + v.U_3.t_3 + s.U_4.t_4$$

$$B = v.U_1 + s.U_2 + v.U_3 + s.U_4$$

Then, one obtains:

$$c.M.dt_x = A.\ dx - B.\ t_x.dx$$

$$c.M.dt_x = (A - B.\ t_x).\ dx$$

After modification: $$\frac{dt_x}{A - B \cdot t_x} = \frac{d_x}{c \cdot M}.$$

After substitution: $$A - B.t_x = Z$$

$$dZ = -\ B.\ dt_x$$

$$dt_x = -\frac{d_z}{B}$$

Then: $$-\frac{d_z}{B.Z} = \frac{d_x}{c \cdot M}$$

or: $$\frac{B}{c \cdot M} \cdot d_x + \frac{d_z}{Z} = C.$$

Equation integral: $$\frac{B}{c \cdot M} \cdot \int d_x + \int \frac{d_z}{Z} = C$$

$$\frac{B}{c.M} \cdot x + \ln Z = C.$$

The integration constant is derived from a general boundary condition: x = 0, then $t_x = t_0$.

This means, the temperature of the air at the beginning of the air cavity is same as the temperature of the air at the exit, t_0. If x = 0 and the boundary condition above is fulfilled, the equation is as follows: $\ln Z = \ln(A - B.\ t_0) = C$

so: $$\ln(A - B \cdot t_x) = \ln(A - B \cdot t_0) - \frac{B}{c \cdot M} \cdot x.$$

After modification: $$A - B \cdot t_x = (A - B \cdot t_0) \cdot e^{\left(-\frac{B}{c \cdot M} \cdot x\right)},$$

then: $$-B \cdot t_x = (A - B \cdot t_0) \cdot e^{\left(-\frac{B}{c \cdot M} \cdot x\right)} - A$$

$$-B \cdot t_x = -(A - B \cdot t_0) \cdot e^{\left(-\frac{B}{c.M} \cdot x\right)} + A.$$

Resulting formula: $$t_x = \frac{A - (A - B \cdot t_0) \cdot e^{\left(-\frac{B}{c \cdot M} \cdot x\right)}}{B} \quad [°C], \qquad (17)$$

where: x [m] is the distance from the point from the start of the air cavity.

t_x [°C] is the temperature in x in the air cavity.

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