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THEORETICAL AND NUMERICAL SOLUTION OF A NEAR SONIC FLOW CONSIDERING THE OFF-DESIGN CONDITIONS

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Abstract: *The work deals with comparison of analytical design methods for transonic flows with modern computational fluid dynamics codes. The theory based on potential flow transformed into modified hodograph plane and simplified by near sonic flow conditions gives an exact solution of a cusped airfoil pointed into the sonic free stream. Results obtained from corresponding commercial ANSYS Fluent inviscid CFD code are compared with this theory on such profile. Because of considerable sensitivity of transonic flows, some off-design conditions in means of different free stream velocities are mentioned and analyzed to see the influence of small changes in boundary conditions on developed flow field.*

Keywords: Transonic flow, Hodograph, Airfoil, CFD, Simulation, Off-design conditions.

1. Introduction

Mathematical solution of transonic flows brings together classical hydraulic methods together with the wave propagation solutions to resolve the co-existence of both elliptic and hyperbolic differential equations. The subsonic field can be solved with methods related to conformal mapping and supersonic field then using method of characteristics. This theory allows us to design airfoil shapes with predicted flow behavior around them which can be easily compared with numerical simulations. Modern CFD codes can be used as an extension to this theory and provide fast response to theoretical results or confirm the influence of possible off-design conditions.

2. Theoretical Model

The mathematical model stands on the basis of potential flow. To avoid the non-linearity of basic system of equations we can transform the solution to modified hodograph plane replacing physical coordinates x,y with new, flow angle ϑ and Prandtl-Mayer angle υ. The fact that we are assuming only the near sonic flow, that means flow with only small perturbations to sonic flow, simplifies the equations enough to solve exactly the conformal subsonic zone and also characteristic supersonic zone.

This solution, deeply described in Stodůlka and Sobieczky (2014), can lead us to design of cusped airfoil pointed into the sonic free stream, see Fig. 1. This airfoil with sharp leading and trailing edge cuts the incoming flow which is then accelerated smoothly from sub to supersonic velocities past the airfoil forming oblique shocks at the trailing edge. Its shape is defined by thickness parameter τ and camber parameter ω.

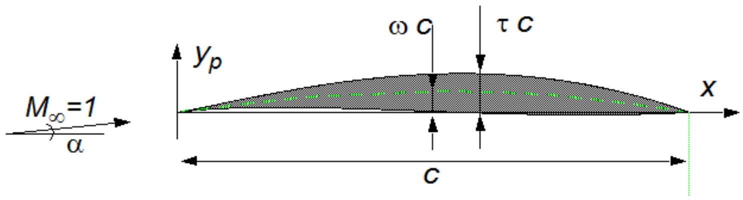

Fig. 1: Cusped airfoil parameters.

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Finding analytical solutions to above hodograph relations allows us to derive the formulae defining the shape, flow conditions and pressure coefficient for cusped airfoils in a uniform sonic flow $M_\infty = 1$ (Sobieczky, 1975). The camber/thickness parameter is given by:

$$P\left(\frac{\omega}{\tau}\right) = 2^{13/2}\cdot 3^{3/2}\cdot 5^{-7/2}\cdot\frac{\omega}{\tau}\left[1+2^{12}\cdot 3\cdot 5^{-6}\left(\frac{\omega}{\tau}\right)^2\right]^{-1/2} \qquad (1)$$

and using this parameter we can describe the angle of attack:

$$\alpha = \tau\cdot 2^{-9/2}\cdot 3^{-1/2}\cdot 5^{5/2}\cdot P\,\frac{1-2^{-1}\cdot 3^{-4}\cdot 5\cdot 13P^2}{(1-2^{-1}\cdot 3^{-2}\cdot 5P^2)^{3/2}} \qquad (2)$$

geometry data:

$$y_p(X) = \tau\cdot X(1-X)\left(2^2\cdot\frac{\omega}{\tau}\pm 2^{-2}\cdot 3^{-3/2}\cdot 5^{5/2}\cdot X^{1/2}\right) \qquad (3)$$

and finally the pressure coefficient:

$$c_p = \frac{(5^2\cdot\tau)^{2/3}}{\left[2^2\cdot 3\cdot(\gamma+1)\right]^{1/3}}\left[\frac{(1-2^{-2}\cdot 3^{-2}\cdot 5P^2)}{(1-2^{-1}\cdot 3^{-2}\cdot 5P^2)} - 2^{-1}\cdot 5X \mp \frac{2^{-1/2}\cdot 3^{-1}\cdot 5P\cdot X^{1/2}}{(1-2^{-1}\cdot 3^{-2}\cdot 5P^2)^{1/2}}\right] \qquad (4)$$

where γ is the specific heat capacity ratio.

Knowing these, we have a complete analytical solution of this problem of specified sharp cusped airfoil pointed into the sonic free stream.

3. Numerical Model

For numerical simulation a thin, lightly cambered airfoil was chosen with parameters τ = 0.05 and ω/τ = 0.02. The simulation was performed using inviscid ANSYS Fluent CFD model on quad mapped mesh with approx. 100000 elements with implemented AUSM numerical flux scheme with second order upwind. The only boundary condition was set as a pressure far field (ANSYS Inc. 2013).

Results in form of Mach number contours are shown on Fig. 2. It shows clearly shock free transonic character of the flow past airfoil ended by oblique shocks. The whole flow behavior corresponds also perfectly with the predicted flow field form theoretical analysis.

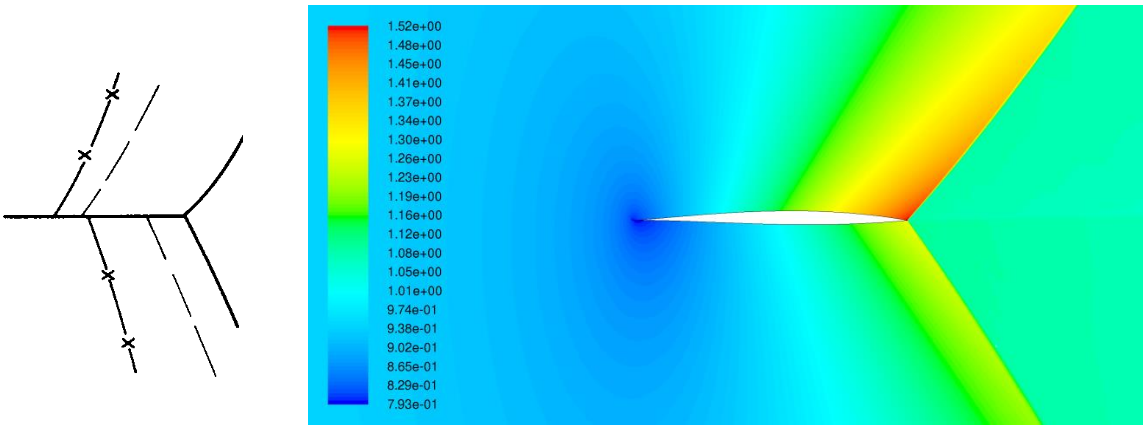

Fig. 2: Contours of Mach number.

Pressure coefficient distribution along the profile surface is compared on Fig. 3, with solid line representing numerical results and dashed the theoretical. Both lines are corresponding well in displayed range, but for thicker and more cambered airfoils reached Mach numbers rise and deviations start to appear in these regions, what is probably caused by deviation from near sonic flow theory assumptions. More about the numerical simulations can be found also in Stodulka and Sobieczky (2014).

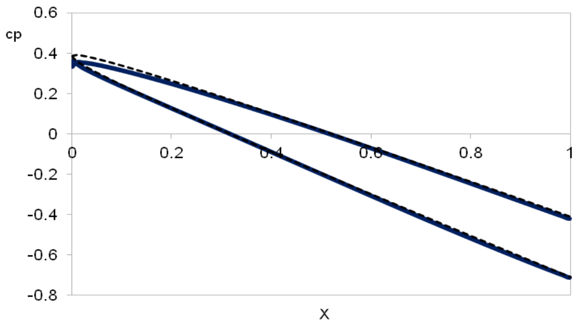

Fig. 3: Pressure coefficient distribution.

4. Off-Design Conditions

Previous chapter confirmed well the correspondence between theoretical and numerical results and ability of the CFD code to describe expected flow field. But such sensitive boundary condition as a sonic free stream holds this case strictly in an academic field. To get closer to real problems it is easy to imagine some off-design modifications with incoming speed oscillating around sonic value. For upcoming simulations was used the symmetrical cusp with thickness parameter τ = 0.05. The numeric setup remained same as in previous models and the only difference was in domain dimensions and computational mesh. Due to larger domain and size savings was used the unstructured adapted tri mesh.

If we will assume only small perturbations to sonic flow we can expect a creation of detached bow wave in front of the airfoil for slightly supersonic velocities. The location of the wave depends on free stream Mach number, the higher the velocity is, the closer is the distance between the leading edge and shock until it attaches. Behind the bow wave is the flow field similar to the sonic case with smooth acceleration and oblique shocks. Expected result and simulation is shown on Fig. 4.

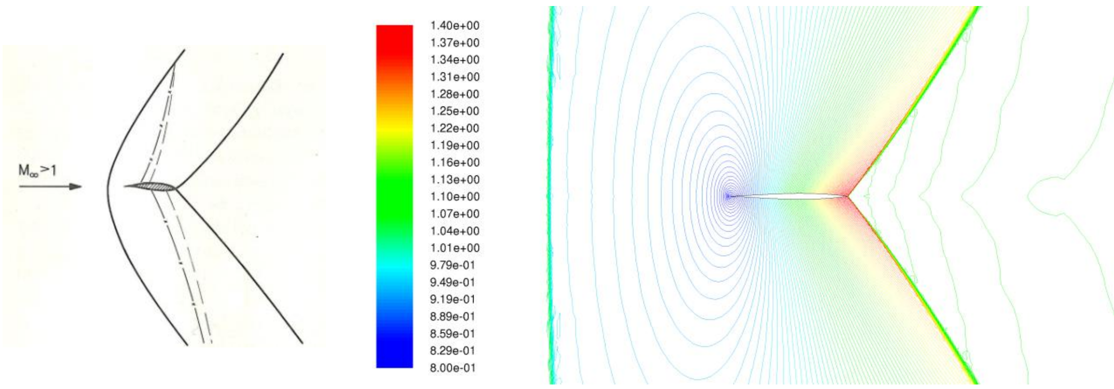

Fig. 4: Cusped airfoil in supersonic free stream, $M_\infty = 1.05$.

There are also some results explaining shape, strength and location of detached shock depending on the free stream Mach number (Sobieczky, 1974). Dependence of wave location and free stream velocity is shown on Fig. 5. Solid points are simulation data, crosses are data from DLR-τ code (Trenker and Sobieczky, 2001) and dashed line is theoretically exact asymptote.

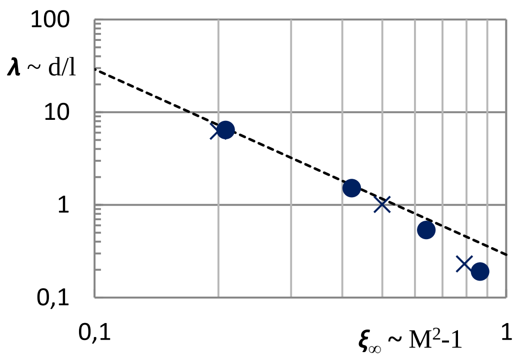

Fig. 5: Bow wave location for different Mach numbers.

The opposite problem appears when the free stream velocity will be subsonic. This case leads to creation of so called fish tail wave formed behind the cusp separating the local supersonic domain from the rest of the flow field. This configuration is showed on Fig. 6.

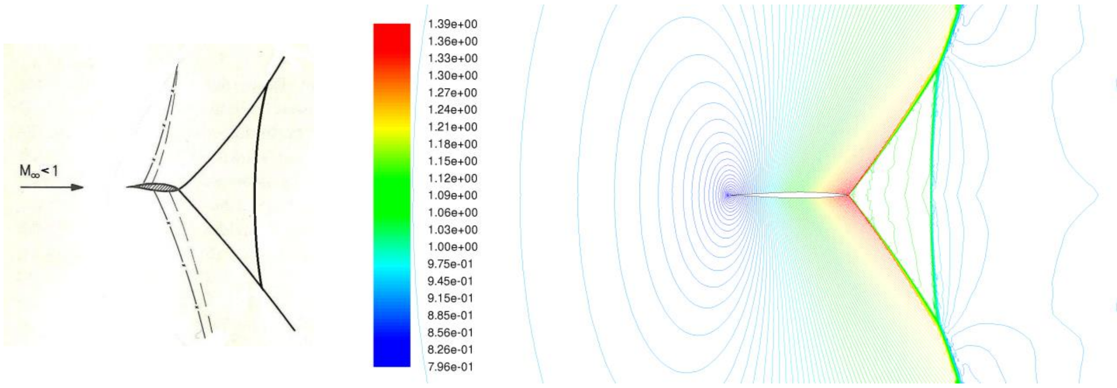

Fig. 6: Cusped airfoil in subsonic free stream, $M_\infty = 0.95$.

These two cases extend the problematic of developed sonic flow past sharp profile and describe the sensitivity of transonic problems as a whole. It is clear that the results from CFD code correspond well with expected flow field for both off-design cases and even here proved itself as fast and reliable tool for design and verification.

5. Conclusions

The near sonic flow theory using conformal together with characteristic mapping for both velocity regions gives a precise modified hodograph solution of the whole flow field past cusped airfoil in a sonic free stream. Due to availability of modern computational fluid dynamics codes, the numerical experiment enables to receive data on parameters and flow structures past these airfoils very fast and effectively to be compared with the theory. And obtained results confirmed well the accordance between numerical and exact analytical data by means of described near sonic flow theory. CFD results showed the described problem in a new light and turned out to be a powerful tool for fast verifications or sensitivity analysis giving new objects for theory solutions. Off-design condition setups added a bit more practical view on the problematic and proved that even here we are still able to predict and simulate the flow behavior correctly. Especially for cases described above is the numerical simulation the only way how to compare and discuss obtained results and theories. Posted results also confirmed significant sensitivity of transonic flows, especially when talking about sonic and near sonic problems. At the end it is necessary to say that more than any concrete practical value this work is aimed to show the possibility of using the already developed theories for creating first systematic designs that can be initial cases for optimized shapes for real prototypes and these cases in transonic regime are now easily comparable with results obtained from modern CFD codes.

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