20th International Conference
ENGINEERING MECHANICS 2014
Svratka, Czech Republic, 12 – 15 May 2014

APPLICATION OF THE LATTICE BOLTZMAN METHOD FOR THE MODELLING OF PULSATILE FLOW IN IDEALISED BYPASS GEOMETRIES

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Abstract: *This paper is focused on numerical simulations of pulsatile laminar blood flow using the lattice Boltzmann method. Compared to the traditional methods like the finite volume or finite element methods, the relatively new approach of the lattice Boltzmann method is simpler, faster and less memory-intensive. Because of these desirable properties, this method can be seen as an appropriate candidate for numerical simulations in the field of biomedicine, where large and complex flow problems have to be solved in short time. In this work, the lattice Boltzmann method is tested on a reference idealised bypass model. The obtained numerical results are compared with those computed using the finite volume method.*

Keywords: Lattice Boltzmann method, Finite volume method, Incompressible Newtonian fluid, Pulsatile blood flow, Idealised bypass geometry.

1. Introduction

The lattice Boltzmann method represents a relatively new approach for the modelling of fluid flow problems (Succi, 2001), (Chen et al., 1998). The principle of this method is based on the mesoscopic fluid description unlike the classical methods that utilise the macroscopic description. The lattice Boltzmann method originates from the lattice gas cellular automata (LGCA) representing a simplified molecular dynamics. However, unlike the LGCA, the lattice Boltzmann method operates with virtual particles. This approach makes it possible to solve various complex flow problems such as multiphase flow and free surface flow. Due to its simplicity, accuracy and computational efficiency, the lattice Boltzmann method has become very popular during the last decade.

Because any extra knowledge of the local hemodynamics prior to surgery can bring benefits not only to vascular surgeons in their pre-operative planning, but also to patients, the mathematical modelling of blood flow in patient-specific geometries has become very popular in recent years. A certain drawback of this approach is the need for very fast calculations in complex computational domains that are usually reconstructed from CT scans. From this point of view, the lattice Boltzmann method appears to be an appropriate numerical method for this type of computationally-demanding calculations. Its main advantages include robustness and ability to perform blood flow simulations directly in grids obtained from CT scans.

The aim of the present study is to compare the computational capabilities of the lattice Boltzmann method with those of the finite volume method for the numerical simulations of pulsatile Newtonian blood flow in a reference idealised bypass model taken from (Vimmr et al., 2012). Further numerical results for other bypass models with either tapered or widening grafts will be presented and discussed in detail at the conference.

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2. Methods

The lattice Boltzmann method is derived from a special discretisation of the Boltzmann equation, describing the time evolution of the particle distribution function $f(\boldsymbol{x}, \boldsymbol{v}; t)$ in the phase space,

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\nabla f = \frac{1}{\tau}(f^{eq} - f), \tag{1}$$

where the right hand side of Eq. (1) is the Bhatnagar-Gross-Krook model of the collision operator (Bhatnagar et al., 1954), the function $f^{eq}(\rho, \boldsymbol{v})$ is an equilibrium distribution function and τ is the single relaxation time. After a special space-time discretization of Eq. (1), the lattice Boltzmann equation is obtained

$$f_i(\boldsymbol{x} + \boldsymbol{e}_i\Delta t, t + \Delta t) = f_i(\boldsymbol{x}, t) - \frac{1}{\tau}\left(f_i(\boldsymbol{x}, t) - f_i^{eq}(\boldsymbol{x}, t)\right), \tag{2}$$

where $f_i, i = 1, 2, \ldots, 19$ are the distribution functions corresponding to the i-th particle velocity $\boldsymbol{e}_i$. The spatial distribution of the particle velocities $\boldsymbol{e}_i$ in 3D, which is denoted D3Q19, is shown in Fig. 1. In according to (Succi, 2001), the equilibrium functions $f_i^{eq}(\boldsymbol{x}, t)$ are defined as

$$f_i^{eq}(\boldsymbol{x}, t) = f_i^{eq}(\rho(\boldsymbol{x}, t), \boldsymbol{v}(\boldsymbol{x}, t)) = w_i\rho\left(1 + 3(\boldsymbol{e}_i.\boldsymbol{v}) + \frac{9}{2}(\boldsymbol{e}_i.\boldsymbol{v})^2 - \frac{3}{2}(\boldsymbol{v}.\boldsymbol{v})\right), \tag{3}$$

where w_i are weights, $\rho = \rho(\boldsymbol{x}, t)$ is the density and $\boldsymbol{v} = \boldsymbol{v}(\boldsymbol{x}, t)$ is the velocity vector.

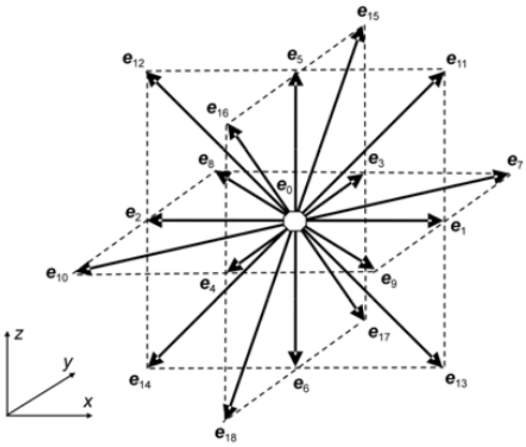

Fig. 1: D3Q19 lattice (3D space, 19 velocities).

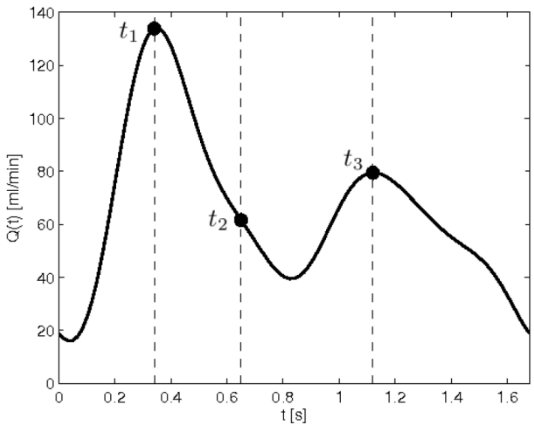

Fig. 2: Time-dependent inlet flow rate Q(t).

The macroscopic variables $\rho(\boldsymbol{x}, t)$ and $\boldsymbol{v}(\boldsymbol{x}, t)$ can be recovered from the distribution functions $f_i(\boldsymbol{x}, t)$ as

$$\rho(\boldsymbol{x}, t) = \sum_{i=1}^{19} f_i(\boldsymbol{x}, t), \qquad \boldsymbol{v}(\boldsymbol{x}, t) = \sum_{i=1}^{19} f_i(\boldsymbol{x}, t)\boldsymbol{e}_i.$$

The pressure $p(\boldsymbol{x}, t)$ is connected with the density $\rho(\boldsymbol{x}, t)$ through the following relationship

$$p = \frac{1}{3}\rho.$$

Using the Chapman-Enskog expansion (Succi, 2001), the nonlinear system of incompressible Navier-Stokes equations

$$\nabla\boldsymbol{v} = 0, \tag{4}$$

$$\rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\nabla\boldsymbol{v}\right) + \nabla p = \mu\nabla^2\boldsymbol{v}, \tag{5}$$

can be recovered from the lattice Boltzmann equation (2). To demonstrate the computational capabilities of the lattice Boltzmann method, we compare the obtained results with those computed using the pressure-based second order finite volume method described in (Vimmr et al., 2012).

3. Numerical results

The numerical simulations of pulsatile Newtonian blood flow are performed in an idealised bypass model, consisting of both proximal and distal anastomoses. The geometry of this model with average values shown in Fig. 3 is taken from (Vimmr et al., 2012). At the inlet, a time-dependent velocity magnitude corresponding to the inlet flow rate, Fig. 2, is prescribed. At the outlet, the constant pressure boundary condition is considered. Figs. 4 and 5 show velocity profiles at selected cross-sections of the reference bypass model computed by the two numerical methods considered in this study at the time instant $t_3 = 1.12$ s. The comparison of longitudinal sections of the velocity profiles from Figs. 4 and 5 is displayed in Fig. 6. Tab. 1 lists the number of elements and CPU times necessary for the completion of two full cardiac cycle periods.

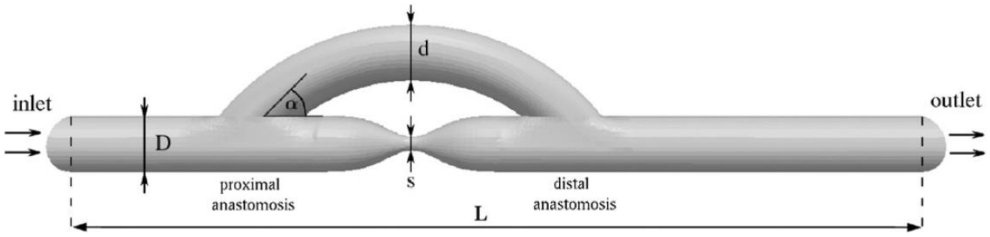

Fig. 3: Geometry of the reference 3D bypass model.

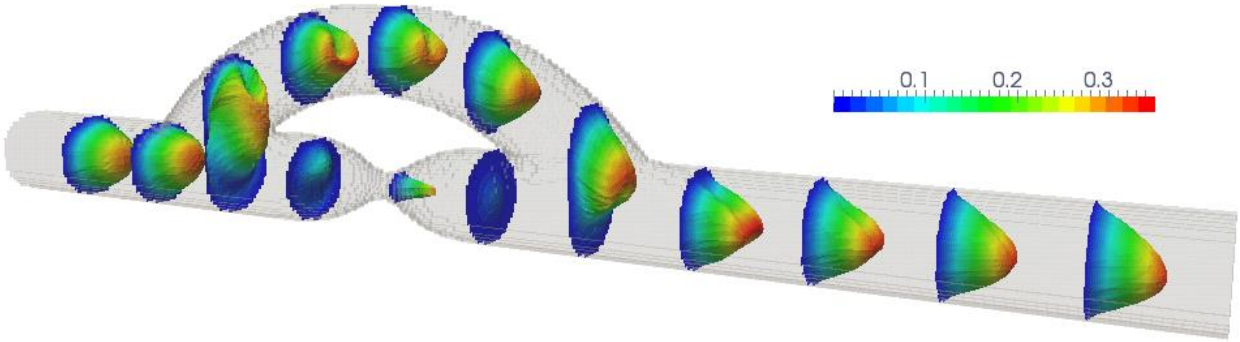

Fig. 4: Velocity profiles obtained using the lattice Boltzmann method at the time $t_3 = 1.12$ s.

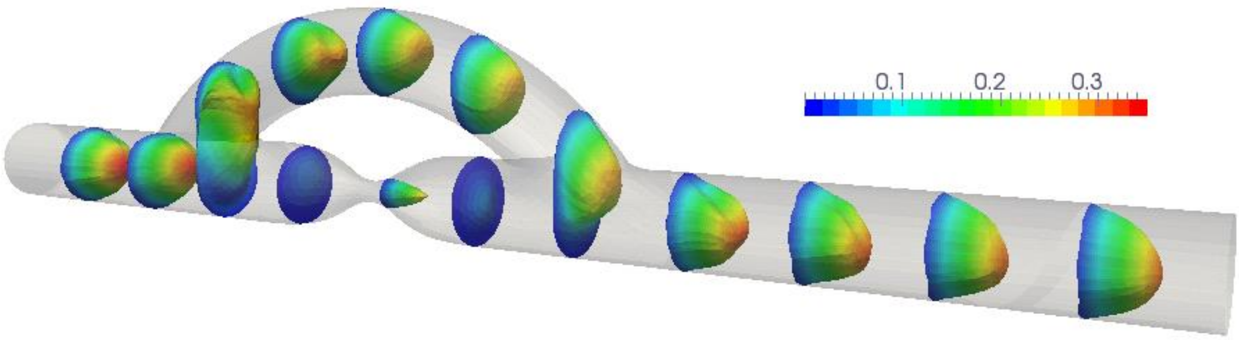

Fig. 5: Velocity profiles obtained using the finite volume method at the time $t_3 = 1.12$ s.

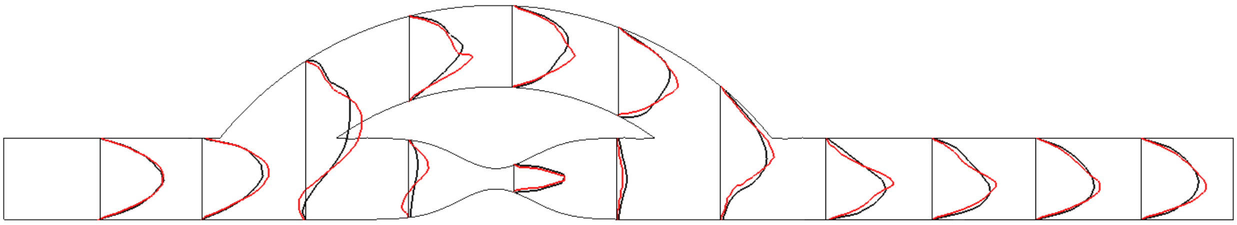

Fig. 6: Comparison of velocity profiles in the middle of the reference bypass model computed by the lattice Boltzmann method (red) and the finite volume method (black) at the time $t_3 = 1.12$ s.

The results obtained with the lattice Boltzmann method are in a relatively good qualitative agreement with those computed using the finite volume method, although several differences are present. The main differences are primarily caused by dissimilar computational meshes, see Fig. 7. Namely, the computational mesh used for the numerical computation with the finite volume method consisted of tetrahedral elements, which were able to precisely fit the boundary of the computational domain. By contrast, the computational grid used for the lattice Boltzmann method consisted of hexahedral bricks, which could only approximately fit the boundary of computational domain. This mesh difference results in different distributions of blood flow within the graft and the stenosed native artery.

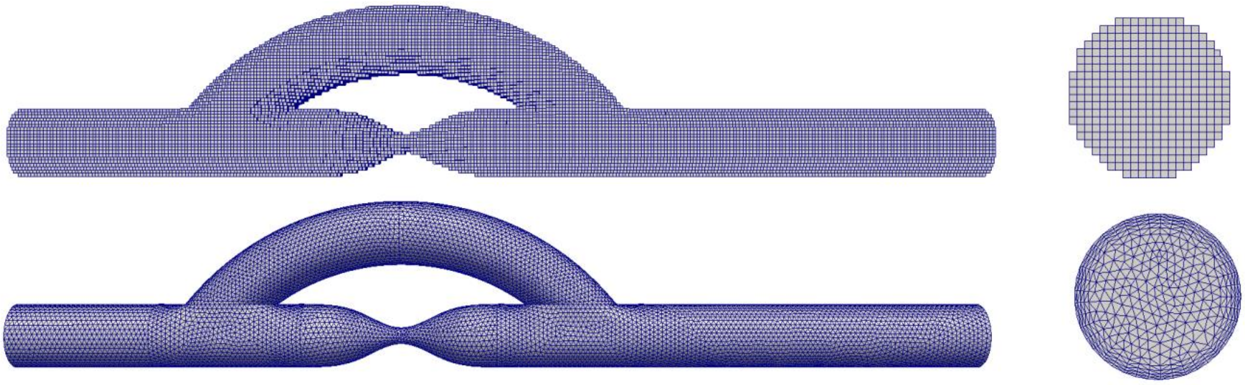

Fig. 7: Computational meshes used for the lattice Boltzmann method (top) and the finite volume method (bottom).

Tab. 1: CPU times for the two methods considered in this study. The listed time indicates the end of the computational process after reaching two full cardiac cycle periods.

method	number of cells	CPU time [s]
FVM	256461	174294
LBM	162779	36598

4. Conclusions

The lattice Boltzmann method represents an excellent tool for the solution of large and complex flow problems because of its simplicity, accuracy and high performance, Tab. 1. Some difficulties can be encountered when it is necessary to precisely capture the boundary of the computational domain, as shown in this study. In such cases, it is recommended to implement local grid refinement or apply a special boundary condition corresponding to the curvature of the boundary.

Acknowledgement

This work was supported by the European Regional Development Fund (ERDF), project 'NTIS - New Technologies for the Information Society', European Centre of Excellence, CZ.1.05/1.1.00/02.0090 and by the internal student grant project SGS-2013-036 of the University of West Bohemia.

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