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BIFURCATION LOAD OF A COLUMN COMPOSED OF PIPE AND ROD WITH TWO-PARAMETRIC ELASTIC ELEMENT

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Abstract: *In this paper the slender system subjected to Euler's load has been presented. Considered system is composed of two elements: pipe and rod. The rod is mounted concentrically with the pipe; in such way that the deflection and angle of deflection of pipe and rod are identical. The investigated system is hinged on both ends. In the column between pipe and rod the two-parametric elastic element has been placed. Both transversal and rotational displacements are being limited by means of this element. In the work static problem of considered column was formulated on the basis of minimum total potential energy principle (static criterion of instability). Rectilinear form of static equilibrium is only considered in this paper. On the basis of static criterion of instability the bifurcation load (the biggest load at which system is still in the rectilinear equilibrium) has been determined in relation to system's parameters. The parameters of the system are as follows: stiffness of translational and rotational springs (elastic layer), flexural rigidity asymmetry factor (pipe and rod), parameter of location of elastic layer.*

Keywords: Column, Divergence system, Euler's load, Two-parametric elastic layer.

1. Introduction

Slender systems subjected to both conservative and non-conservative type of loads were the subject of many investigations. Euler's load (Euler, 1744; Uzny, 2011; Sokół, 2014), at which external force does not change the direction of action is the most popular one in slender systems. In literature the others types of external load of columns loads can be found. Generalized load (Bochenek and Życzkowski, 2004), a load with force directed towards the positive or negative pole (Gajewski and Życzkowski, 1969), characteristic load (Tomski's load) (Tomski and Uzny, 2008, 2013a) can be considered as a conservative ones. Generalized Beck's load (Beck, 1952; Tomski and Uzny, 2013b; Langthjem and Sugiyama, 2000) and generalized Reut's (Langthjem and Sugiyama, 2000; Nemat-Nasser and Herrmann, 1966) load can be qualified as a non-conservative loads.

Slender systems can be divided into constructions composed of the identical elements or elements with different bending and compression stiffness (Uzny, 2011; Tomski and Uzny, 2008). The second group can be characterized by two forms of static equilibrium: rectilinear and curvilinear. The magnitude of external load, at which rectilinear form occurs, is changing from zero up to value of bifurcation load. Curvilinear form of static equilibrium occurs between bifurcation and critical force. In this type the local and global instability phenomenon occurs (Uzny, 2011; Tomski and Uzny, 2008). The regions of local and global instability depend on flexural rigidity asymmetry factor. Slender system composed of two elements: pipe and rod with different stiffness was researched in work (Uzny, 2011), where a single-parametric elastic layer was placed between pipe and rod. By means of this element an increase in bifurcation load has been achieved (especially in the area of local instability). Additionally in this system the change of buckling form; region of local instability is decreasing at greater stiffness of single-parametric elastic layer.

The main purpose of this work is to study an influence of two-parametric elastic layer on stability of system composed of pipe and concentrically installed rod.

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2. Problem Formulation and Solution

Considered column composed of pipe and rod is presented in Fig. 1b (column C2E). The rod is mounted concentrically with the pipe; in such way that the deflection and angle of deflection of pipe and rod are identical. System is loaded by compressive external force with constant line of action (Euler's force). The investigated system is hinged on both ends as shown in Fig. 1a. In the column between pipe and rod the two-parametric elastic element has been placed. Elastic layer was modeled by means of spring system which consist of transversal spring (with rigidity C_T) and rotational one (with rigidity C_R). Location of elastic layer is defined by the parameter ζ. In this study, it is assumed that total flexural rigidity of system is constant ($(EJ)_1 + (EJ)_2 = (EJ) = idem$). Flexural rigidity asymmetry factor is a variable parameter, which is defined as follows:

$$\mu_a = \frac{(EJ)_2}{(EJ)_1} \tag{1}$$

Bifurcation force of complex system will be compared to critical load of the system composed from the pipe only (column C1E) – Fig. 1c. Mathematical model of the investigated complex system (C2E) is presented in Fig. 1a. This model consists of four elements. Elements marked with subscripts 11 and 12 correspond to pipe whereas subscripts 21 and 22 model the rod. Bending rigidities in the mathematical model are marked as follows: $(EJ)_{11} = (EJ)_{12} = (EJ)_1$ and $(EJ)_{21} = (EJ)_{22} = (EJ)_2$.

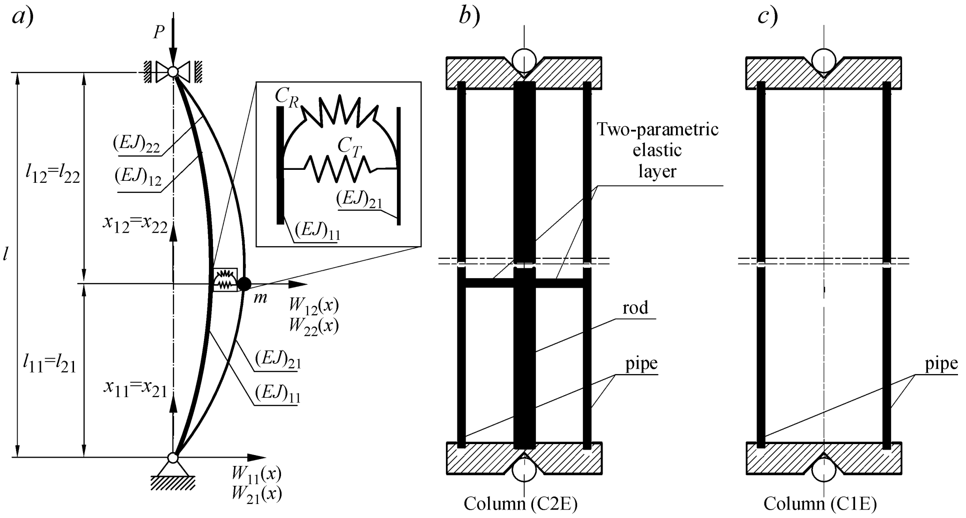

Fig. 1: Considered systems.

The problem has been formulated on the basis of the minimum total potential energy principle.

$$\delta V = 0 \tag{2}$$

Potential energy of considered system is as follows:

$$V = \frac{1}{2}\sum_i \sum_j (EJ)_{ij} \int_0^{l_{ij}} \left(W_{ij}^{II}(x_{ij})\right)^2 dx_{ij} - \frac{1}{2}\sum_i \sum_j S_{ij} \int_0^{l_{ij}} \left(W_{ij\,ij}^{I}(x_{ij})\right)^2 dx_{ij} + \\ + \frac{1}{2} C_T \left(W_{11}(l_{11}) - W_{21}(l_{21})\right)^2 + \frac{1}{2} C_R \left(W_{11}^I(x_{11})\Big|^{x_{11}=l_{11}} - W_{21}^I(x_{21})\Big|^{x_{21}=l_{21}} \right)^2 \tag{3}$$

Internal forces in individual elements of mathematical model are determined as (Uzny, 2011):

$$S_{11} = S_{12} = P\frac{(EA)_{11}}{(EA)_{11} + (EA)_{21}}, \; S_{21} = S_{22} = P - S_{11} \tag{4}$$

In equations (4) longitudinal rigidities of individual units are marked as $(EA)_{ij}$. Geometrical boundary conditions of considered column are present below:

$$W_{11}(0) = W_{21}(0) = W_{12}(l_{12}) = W_{22}(l_{22}) = 0 \,, W_{11}(l_{11}) = W_{12}(0) \,, W_{21}(l_{21}) = W_{22}(0) \tag{5a-h}$$

$$W_{11}^I(x_{11})\Big|_{x_{11}=0} = W_{21}^I(x_{21})\Big|_{x_{21}=0}, \; W_{12}^I(x_{12})\Big|^{x_{12}=l_{12}} = W_{22}^I(x_{22})\Big|^{x_{22}=l_{22}} \tag{5i, j}$$

$$W_{11}^{I}(x_{11})\Big|^{x_{11}=l_{11}} = W_{12}^{I}(x_{12})\Big|_{x_{12}=0}, \; W_{21}^{I}(x_{21})\Big|^{x_{21}=l_{21}} = W_{22}^{I}(x_{22})\Big|_{x_{22}=0} = 0 \qquad (5k, l)$$

After substitution of (3) into minimum potential energy principle (2) and taking into account geometrical boundary conditions (5), the differential equation for unknown static displacement (6) and natural boundary conditions (7) were obtained:

$$(EJ)_{ij} W_{ij}^{IV}(x_{ij}) + S_{ij} W_{ij}^{II}(x_{ij}) = 0 \qquad (6)$$

$$\sum_i (EJ)_{i1} W_{i1}^{II}(x_{i1})\Big|_{x_{i1}=0} = 0, \; \sum_i (EJ)_{i2} W_{i2}^{II}(x_{i2})\Big|^{x_{i2}=l_{i2}} = 0 \qquad (7a, b)$$

$$(EJ)_{11} W_{11}^{III}(x_{11})\Big|^{x_{11}=l_{11}} - (EJ)_{12} W_{12}^{III}(x_{12})\Big|_{x_{12}=0} - C_T\left(W_{11}(l_{11}) - W_{21}(l_{21})\right) = 0 \qquad (7c)$$

$$(EJ)_{21} W_{21}^{III}(x_{21})\Big|^{x_{21}=l_{21}} - (EJ)_{22} W_{22}^{III}(x_{22})\Big|_{x_{22}=0} + C_T\left(W_{11}(l_{11}) - W_{21}(l_{21})\right) = 0 \qquad (7d)$$

$$-(EJ)_{11} W_{11}^{II}(x_{11})\Big|^{x_{11}=l_{11}} + (EJ)_{12} W_{12}^{II}(x_{12})\Big|_{x_{12}=0} - C_R\left(W_{11}^{I}(x_{11})\Big|^{x_{11}=l_{11}} - W_{21}^{I}(x_{21})\Big|^{x_{21}=l_{21}}\right) = 0 \qquad (7e)$$

$$-(EJ)_{21} W_{21}^{II}(x_{21})\Big|^{x_{21}=l_{21}} + (EJ)_{22} W_{22}^{II}(x_{22})\Big|_{x_{22}=0} + C_R\left(W_{11}^{I}(x_{11})\Big|^{x_{11}=l_{11}} - W_{21}^{I}(x_{21})\Big|^{x_{21}=l_{21}}\right) = 0 \qquad (7f)$$

The solutions of (6) can be presented in the following form:

$$W_{ij}(x_{ij}) = A_{ij} \cos\left(\sqrt{\frac{S_{ij}}{(EJ)_{ij}}} x_{ij}\right) + B_{ij} \sin\left(\sqrt{\frac{S_{ij}}{(EJ)_{ij}}} x_{ij}\right) + C_{ij} x_{ij} + D_{ij} \qquad (8)$$

Substitution of solutions (8) into boundary conditions (5), (7) one obtains the system of equations for which the matrix determinant is equated to zero; a transcendental equation for bifurcation force P_b.

3. Results of Numerical Calculations

The sample of results of numerical calculations is presented in Fig. 2.

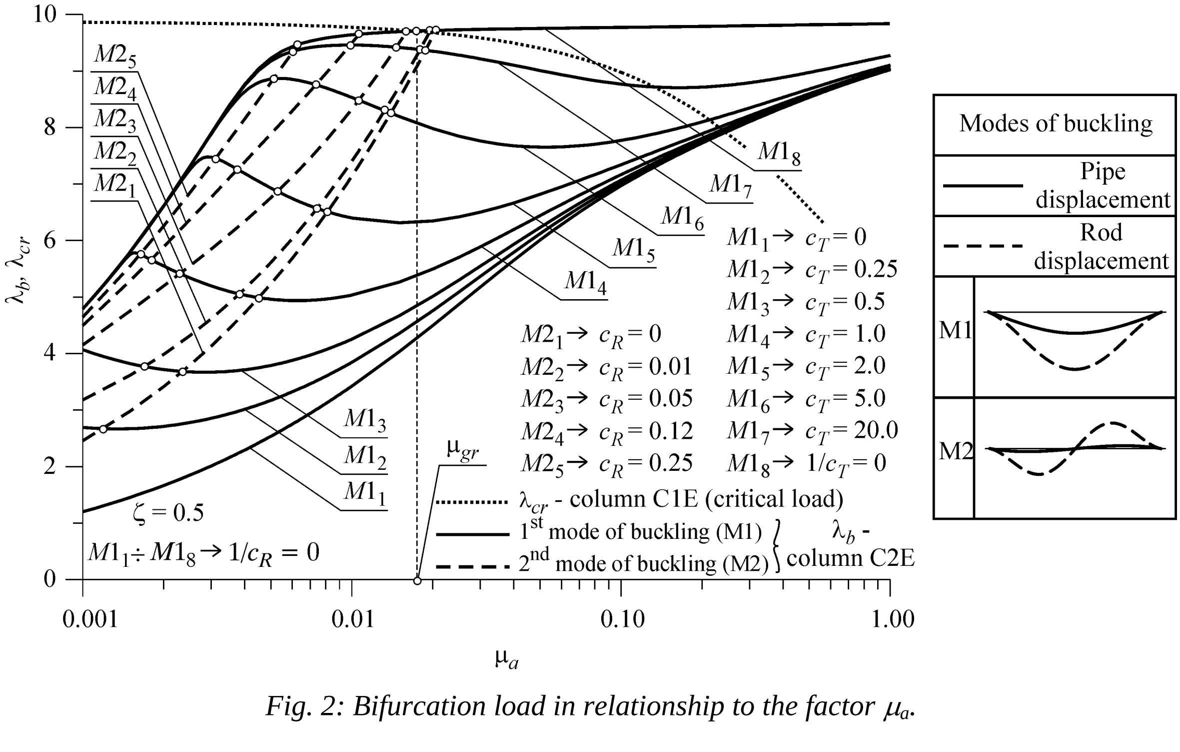

Fig. 2: Bifurcation load in relationship to the factor μ_a.

Bifurcation load was determined as dependent on flexural rigidity asymmetry factor μ_a. Calculations were performed for different values of transversal C_T and rotational C_R rigidities with ζ = 0.5 (elastic layer is placed on the half of system's length). Non-dimensional parameters, used for more general presentation of results of numerical calculations (Fig. 2), are defined as follows:

$$c_T = \frac{C_T (l)^3}{(EJ)}; \quad c_R = \frac{C_R l}{(EJ)}; \quad \lambda_b = \frac{P_b (l)^2}{(EJ)}; \quad \lambda_{cr} = \frac{P_{cr} (l)^2}{(EJ)}, \quad \zeta_A = \frac{l_{11}}{l} \tag{9a-e}$$

The bifurcation load of column C2E and critical load of C1E system are plotted in Fig. 2. The different types of lines were used in order to distinguish buckling modes. The magnitude of flexural rigidity asymmetry factor, at which critical force of column C1E is higher than bifurcation force of column C2E, corresponds to local instability of considered system.

4. Conclusions

In this paper an influence of two-parametric elastic layer on bifurcation load of a slender system composed of pipe and rod is presented. It was demonstrated that the increase of transversal rigidity as well as rotational one causes an increase of the bifurcation load. Influence of rotational rigidity on bifurcation load is noticeable even at smaller value of the flexural rigidity asymmetry factor. Parameters of rigidity of the elastic layer have also an influence on buckling mode (curves for column C2E – Fig. 2). In the future considered system can be further researched and developed (especially an influence of two-parametric layer on vibration frequency).

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