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LATERAL-TORSIONAL BUCKLING OF I-SECTION BEAMS WITH INITIAL RANDOM IMPERFECTIONS

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Abstract: *The paper deals with a statistical analysis of load carrying capacity of a simply supported straight I-beam with equal end moment with cross-section IPE 220, solved by geometrically nonlinear solution influenced by lateral-torsional buckling. The beam was modelled applying the programme ANSYS on behalf of the element BEAM188. Imperfections were considered to be random quantities. The initial curvature and the axis rotation are considered to have the shape of one half-wave of the sine function. The correlation between the amplitudes of initial curvature and initial rotation of the axis is considered as the parameter of solution within the interval from -1 to 1. The influence of this correlation on the change of mean value and standard deviation of random load carrying capacity is studied, the other imperfections being considered to be random quantities resulting from experiments. Realizations of initial imperfections are simulated applying the Latin Hypercube Sampling method. The conclusion presents a discussion of need of paying attention to initial torsion of the axis, when creating stochastic computational models.*

Keywords: Lateral-torsional buckling, Load-carrying capacity, Imperfection, Beam, Steel, Slenderness.

1. Introduction

The paper presented deals with the stochastic analysis of load carrying capacity of a simply supported straight IPE 220 beam with equal end moment. The influence of lateral-torsional buckling on load carrying capacity of the beam the non-dimensional slenderness of which equals 1 is studied. The beam was solved by means of geometrically nonlinear solution so that it would be possible to take into consideration the influence of initial imperfections on load carrying capacity. The first initial geometrical imperfections are assumed to follow the shape of the first eigenmode pertaining to lateral-torsional buckling. This imperfection consists of lateral buckling of the beam in the direction perpendicular to the minor axis of cross-section, and of the rotation of the axis of rotation of cross-sections directed at the beam centre. The beam curvature according to its first eigenmode of lateral-torsional buckling supposes that the lateral-torsional buckling and the rotation of beam axis are functionally dependent. It is not clear to what extent this assumption corresponds with the results which would be obtained from experiments.

The majority of laboratory measurements pay attention more to measurement of initial curvature of the beam axis than to measurement of initial rotations of cross-sections (Fukumoto et al., 1976). However, in case of lateral-torsional buckling, both imperfections can be of importance. It is a question which correlation can be considered to exist between them. The consideration of the correlation by the value 1 need not correspond with the reality accurately. The correlation between them is primarily given by manufacturing processes. To get an idea about to how large extent the correlation value can influence statistical characteristics of load carrying capacity, this problem is studied on behalf of a nonlinear computational model in the present paper.

The computational model was realized out in the ANSYS programme, the random influence of all initial imperfections having been taken into consideration.

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2. Computational Model

A computational model of a two-hinged beam of IPE 220 profile was created. Its length L was calculated in dependence on non-dimensional slenderness   according to EUROCODE 3 as L = 3.323 m. The model was created applying the programme ANSYS applying the beam element BEAM188. This element is suitable for analysing slender beam structures. It is based on Timoshenko beam theory which includes shear-deformation effects and it is well-suited for linear, large rotation, and large strain nonlinear applications. BEAM188 has seven degrees of freedom at each node (these include translations in the x, y and z directions and rotations about the x, y and z directions, the seventh degree of freedom is a warping magnitude. At both ends, the model was loaded by bending moments of the same size, and of opposite sign. The beam diagram is in Fig. 1.

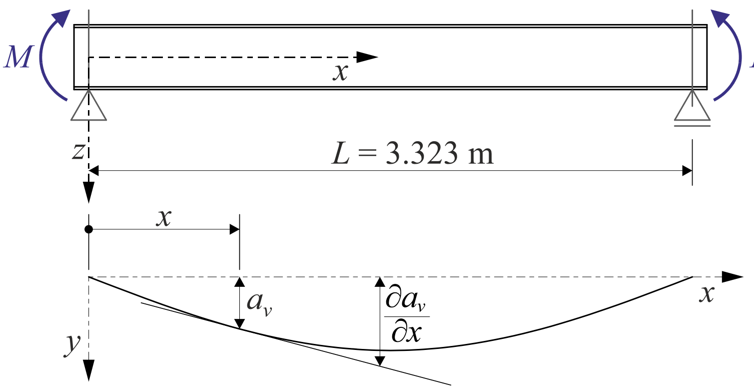


Fig. 1: Beam diagram.

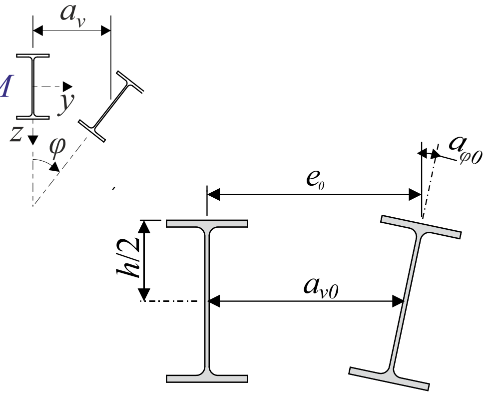


Fig. 2: Curvatures in the middle span.

2.1. Initial imperfections

The beam axis rotation in the direction of the major axis, i.e., in the plane xy, is described by the function:

$$a_{\mathrm{v}} = a_{\mathrm{v}0}\,\sin\left(\frac{\pi x}{L}\right), \tag{1}$$

and the rotation of cross sections along the beam length is given as

$$a_{\varphi} = a_{\varphi 0}\,\sin\left(\frac{\pi x}{L}\right), \tag{2}$$

a_{v0} and $a_{\varphi 0}$ being the amplitudes (Kala, 2013), see Fig. 2. If the beam is curved according to the first eigenmode, it is then valid that

$$a_{vo} = \frac{e_0}{1+\frac{h}{2}\frac{P_z}{M_{cr}}}, \tag{3}$$

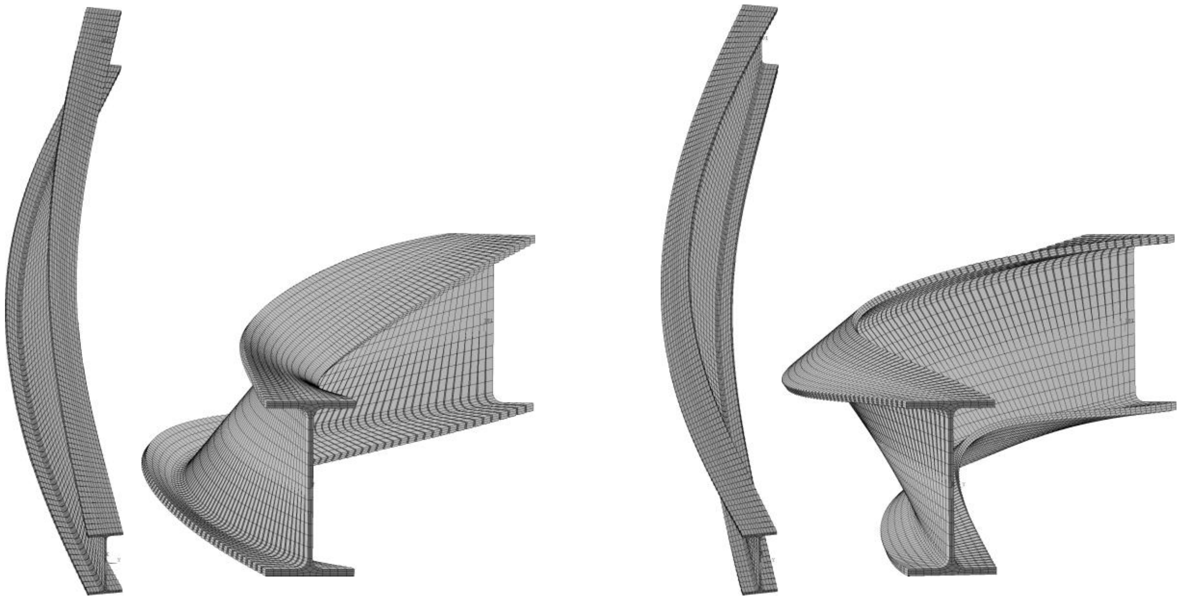
$$a_{\varphi 0} = a_{\mathrm{v}0}\,\frac{P_z}{M_{cr}}, \tag{4}$$

where e_0 is the amplitude of one half-wave of the sine function relating to the upper flange, and for force P_z, the following relation is valid:

$$P_z = \pi^2\,\frac{EI_z}{L^2}, \tag{5}$$

When generating the random quantities and subsequently creating the computational model, various values of correlations are considered between initial curvature e_0 and initial cross-section rotation $a_{\varphi 0}$, and namely within the interval -1 to 1 with the step 0.1. The initial curvature is simulated by random input quantity e_0, from which the axis curvature a_{v0} is calculated according to the formula (3). The random imperfection $a_{\varphi 0}$ is selected as being correlated with imperfection e_0. As there was not the information on standard deviation of initial rotation $a_{\varphi 0}$, this was calculated on behalf of (3) and (4) on condition that e_0 was a random quantity, and h, P_z, M_{cr} were deterministic quantities given by nominal geometrical characteristics of the cross-section. As the mean value of e_0 is zero, so the mean value of $a_{\varphi 0}$ is zero as well. Let us remark that for the calculation, the quantity $a_{\varphi 0}$ is not considered as the functionally depending on e_0, as it would be indicated by (3) and (4), but these formulae serve only for calculation of standard deviation of the amplitude od initial rotation of cross-section $a_{\varphi 0}$ as a random input quantity correlated with e_0.

Calculations of load carrying capacity are thus carried out for series of random realizations with 21 different correlations between both initial imperfections Let us remark that neither mean values nor standard deviations of initial imperfections e_0 a $a_{\varphi 0}$ change with the correlation change. Examples of initial curvature in combination with the rotation of cross-sections modelled applying the ANSYS are schematically presented in Fig. 3.



a) *b)*

Fig. 3: Diagram of computational model: a) Initial curvature combined with rotation of cross-sections with correlation -1; b) Initial curvature combined with station of cross-sections with correlation 1.

3. Stochastic Analysis of Load Carrying Capacity

3.1. Random input quantities

In general, the load carrying capacity M_d is a random quantity which is a function of random geometrical and material characteristics, and can be studied by applying simulation methods of the Monte Carlo type, see, e.g., (Gottvald & Kala, 2012; Kala, 2012). For the presented problem, 500 random realizations were simulated for each of the series of 21 correlations considered between input imperfections by the Latin Hypercube Sampling method (Iman, 1980), (McKey, 1979).

The dimensions of the profile IPE 220 (Fig. 4), material characteristics of the steel grade S 235(Melcher, 2004), and initial imperfections e_0 and $a_{\varphi 0}$ were random input quantities. The Gaussian distribution of probability density is considered for all input random quantities. Residual stresses were not considered. With the exception of initial imperfections e_0 and $a_{\varphi 0,}$ all the quantities are mutually statistically independent.

3.2. Random output quantities

As the load carrying capacity value $M_{d,}$ such value of the bending moment M (see Fig. 1) is considered at which the von Mises stress, at the most stressed point of the beam, is equal to yield strength f_y. The possibility of cross-section to plasticize is not considered, and thus, M_d is the value of elastic load-carrying capacity. The statistics of load carrying capacities is illustrated by the diagram in Fig. 4.

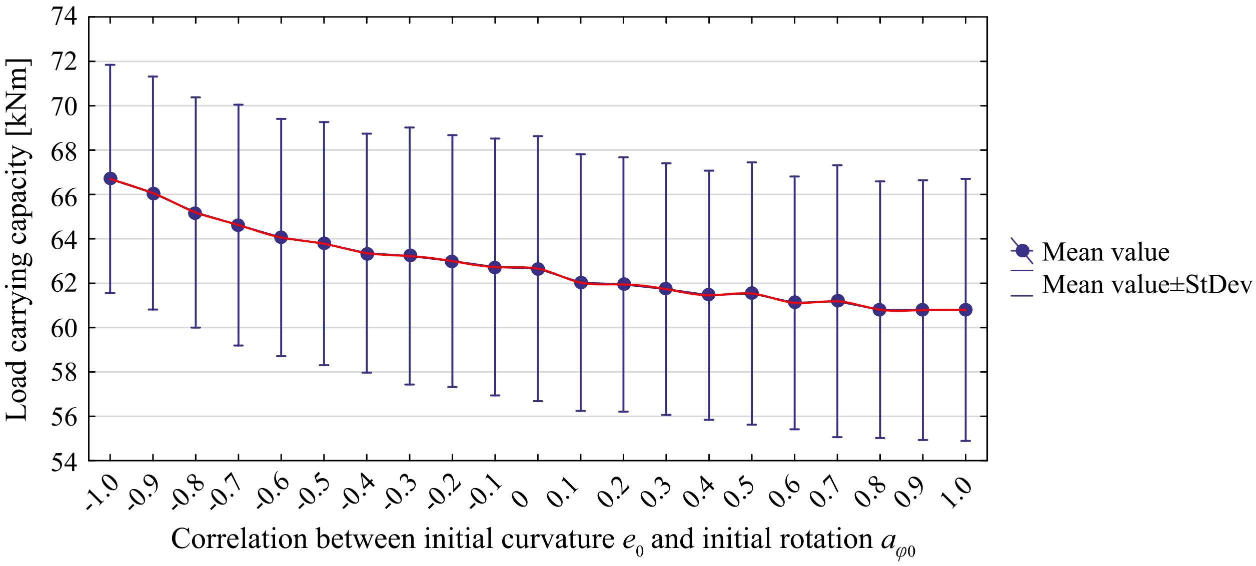


Fig. 4: Mean values with standard deviations of load carrying capacities for correlations considered.

4. Conclusions

It is evident from the diagram in Fig. 4 that the mean load carrying capacity value decreases with increasing correlation between initial curvature e_0 and initial rotation of cross-sections $a_{\varphi 0}$, whereas the standard deviation shows the increasing tendency. At the same time, the decrease in mean value shows a moderate nonlinear trend decreasing in the region in which correlations approach 1. The lowest mean value of load carrying capacity was obtained for the correlation 1, i.e., for complete functional dependence between these two imperfections. It confirms the fact that initial rotation of beams is an imperfection not to be neglected. For the correlation 1, not only the lowest value of average load carrying capacity but, at the same time, high value of standard deviation of load carrying capacity will be obtained. The mean value of load carrying capacity is, for this correlation, by approximately 9.84 % lower than the mean value of load carrying capacity for the correlation -1. If the design load carrying capacity were calculated as 0.1 percentile, the low mean value and the high standard deviation would lead to a low value of 0.1 percentile. The curvature of the beam axis according to the first eigenmode pertaining to lateral-torsional buckling (correlation 1) is conservative from the point of view of design reliability; to get more accurate calculation, it would be needed to know the real value of the correlation between e_0 and $a_{\varphi 0}$ found on the basis of large number of experiments.

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