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SOME PROBLEMS WITH THE ESTIMATION OF PROJECTILE TRAJECTORY PERTURBATIONS

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Abstract: *The basis for the projectile trajectory analysis is the analysis of trajectories calculated for the standard conditions (Position, Material, and Weather). Trajectories calculated under the non-standard conditions are considered to be perturbed trajectories. The tools utilized for the analysis of perturbed trajectories are weighting factor functions (curves) – WFFs. The most important curves describe consequences of perturbation of meteorological conditions (WFF – wind, WFF – air density, WFF – virtual temperature, WFF – air pressure). The weighting factors (WFs) are calculated from the WFFs and obtained WFs are used for calculation of (meteorological) ballistic elements μ_B (ballistic wind w_B, density ρ_B, virtual temperature τ_B, pressure p_B). The tabular firing tables are prepared in such a way that the calculation of ballistic elementsμ_B is required for the follow-up calculation. Ballistic elements μ_B are presented in the ballistic meteorological messages. The basis for creation of STANAG 4061 in 1957 were chosen WFFs. The original materials are not available today and therefore the corresponding WFFs were reconstructed to create conditions for the accuracy analysis of procedures based on the use of STANAG 4061. The article shows results of reconstructions – corresponding WFFs. In detail the WFF-virtual temperature will be analyzed.*

Keywords: Perturbation of the projectile trajectory, Weighting factor function (curve), Weighting factor, (meteorological) Ballistic elements, Firing Tables.

1. Introduction

The analysis of the procedure that was used for the reconstruction of the weighting factor functions (curves) – WFFs was presented in the article Cech et al. (2014). The analysis was limited to the type of met message: Surface-to-surface fire (K = 3).

The use of Weighting Factors $q(\mu)$ (particular magnitudes are presented in e.g. STANAG 4061) is based on linearization of the solution of projectile trajectories for non-standard trajectories that are close to standard ones. In practice the perturbation methods described in Molitz, H. and Strobel, R. (1963) can be used. The first step is always calculation of corresponding weighting (factor) function (WFF) $r(\mu)$.The weighting factors $q(\mu)$ are consequently derived from this function. The procedures of WFFs calculations gradually developed, see e.g. Curti (1945), Molitz and Strobel (1963). In the end the procedure proposed by Garnier (1929) and modified by Bliss (1944) became the most widespread. The procedure for WFFs calculation used by authors of this article is another modification of Garnier – Bliss procedure,e.g. Kovalenko and Shevkunov (1975), Logvin and Aleksandrov (1977), Petrovic (2005, 2006).

The reason for dealing with this issue is specific for the current time and countries that at the same time use the tabular firing tables in format defined in STANAG 4119 demanding the use of standard ballistic meteorological messages described in STANAG 4061 and also the tabular firing tables created in the former USSR and its satellites since the second half of the fifties of the last century. Their use requires utilisation of the Reference Height of Trajectory – RHT from corresponding WFFs and standardized met message

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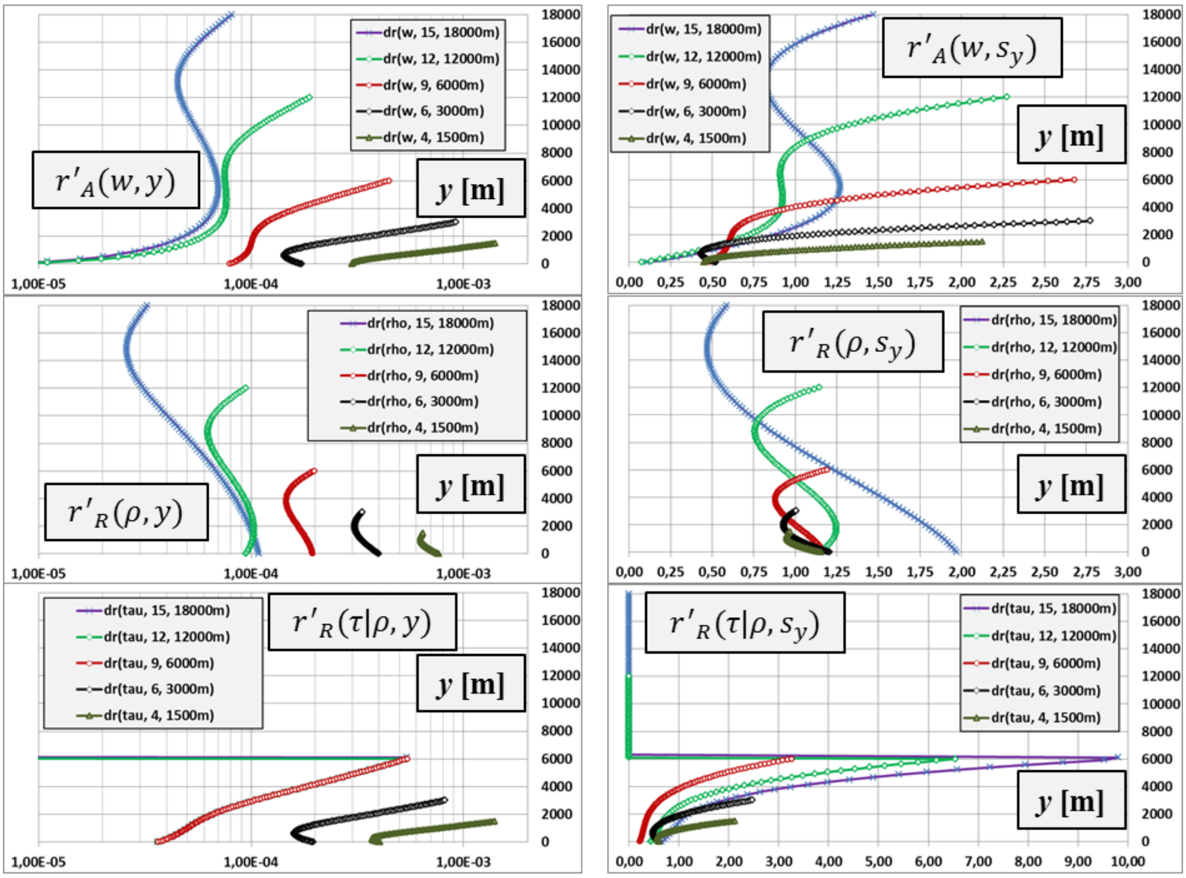

Fig. 1: Reconstructed derivatives of Weighting factor functions – WFFs with respect to y or relative height s_y – normalized form.

Meteo-11 or Meteo-44. The key is study of possibility of mutual conversion WFFs, WFs and RHTs and creation of methodology for mutual evaluation of accuracy of both procedures.

2. Weighting Factor Function

The following definitions hold true, e.g. Kovalenko, V. V. and Shevkunov, V. I. (1975)

$$\Delta\mu_B = \int_0^{Y_S} \Delta\mu(y) \cdot r'_A(y) \cdot dy = \int_0^1 \Delta\mu(s_y) \cdot r'_A(s_y) \cdot ds_y, \qquad (1)$$

$$\delta\mu_B = \int_0^{Y_S} \delta\mu(y) \cdot r'_R(y) \cdot dy = \int_0^1 \delta\mu(s_y) \cdot r'_R(s_y) \cdot ds_y, \qquad (2)$$

where:

$s_y = y/Y_S$ – relative magnitude of y coordinate,

Y_S – vertex height of trajectory [m],

$\Delta\mu(y) = \mu(y) - \mu_{STD}(y)$ - absolute deviation (index A) of met element μ at height y,

$\delta\mu(y) = \Delta\mu(y)/\mu_{STD}(y)$ - relative deviation (index R) of met element μ at height y,

$\mu(y)$ – real or measured magnitude of met parameter μ at height y,

$r'_A(y) = \frac{\partial r_A}{\partial y}$ – partial derivative of WFF $r_A(\mu, y)$ with respect to y,

$r'_R(y) = \frac{\partial r_R}{\partial y}$ – partial derivative of WFF $r_R(\mu, y)$ with respect to y.

It is obvious that the ballistic elements $\Delta\mu_B$ and $\delta\mu_B$ are calculated as weighted mean from the measured values $\mu(y)$. The conversion relation between WFF $r_A(\mu, y)$ and $r_R(\mu, y)$ is not presented here. It is rather common to show the graphs of WFF $r_A(\mu, s_y)$ and $r_R(\mu, s_y)$ – see Fig. 2.

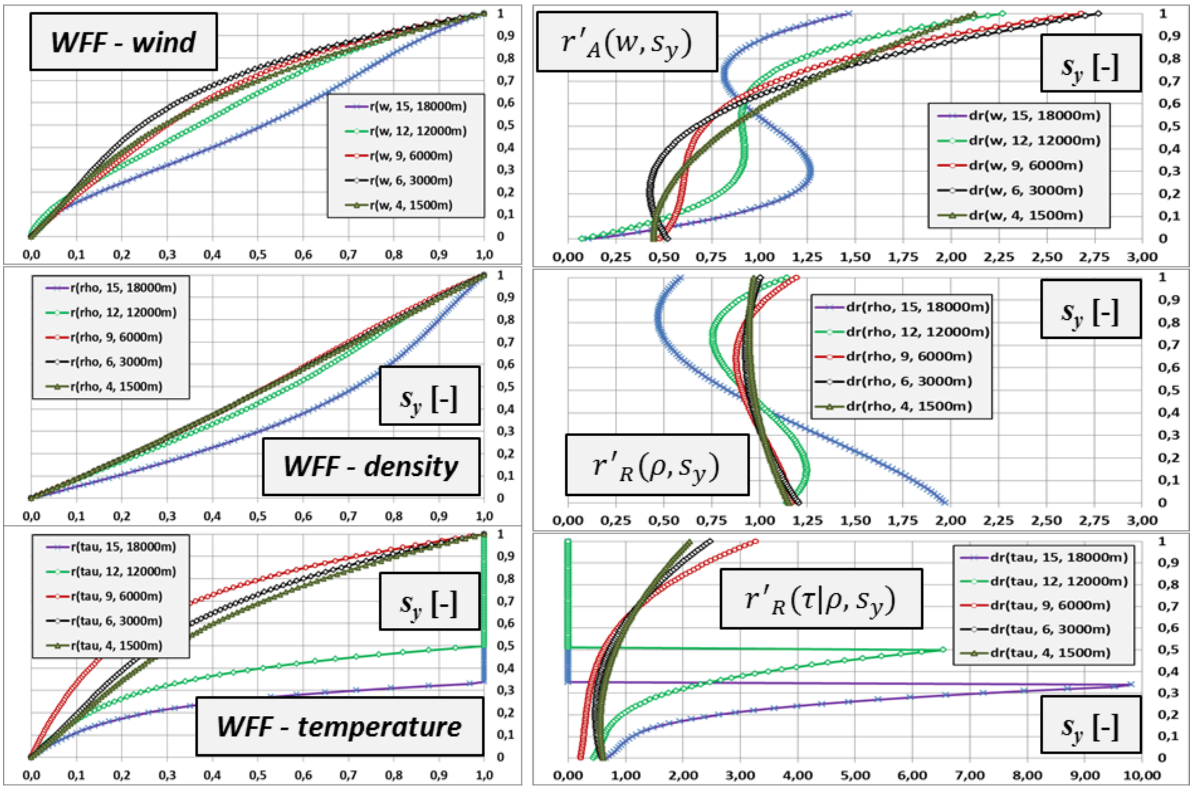

Fig. 2: Reconstructed Weighting factor functions (curves) – WFFs and their derivatives with respect to relative heights s_y – normalized form that is often used by number of authors.

Results of calculations – reconstructed WFFs and their derivatives – are presented in Fig. 1 and 2. The STANAG 4061 divides the atmosphere into height layers n = 1, 2, …, 15. Upper limit of the n-th zone is $y(n) = Y_S(n)$. The illustrative cases are chosen for layers n = 4, 6, 9, 12, 15 and their corresponding heights $y(n)$ = 1500, 3000, 6000, 12000 and 18000 m.

3. Weighting Factor Function of the Virtual Temperature

The analysis of this issue concerning of WFF of the wind has already been done in Cech et al. (2014).

The untypical course of WFF of the virtual temperature (WFF - temperature) and its derivatives for zones n = 10 - 15 (illustrative case is for n = 12 and 15) is apparent from Fig. 1 and Fig. 2. This phenomena is known from the twenties of the last century and is sometimes labeled as “norm-effect”. Roth and Sägner (1962) dealt with this issue in detail but simulation that has been carried out by authors imply that the issue is more complicated than these authors present.

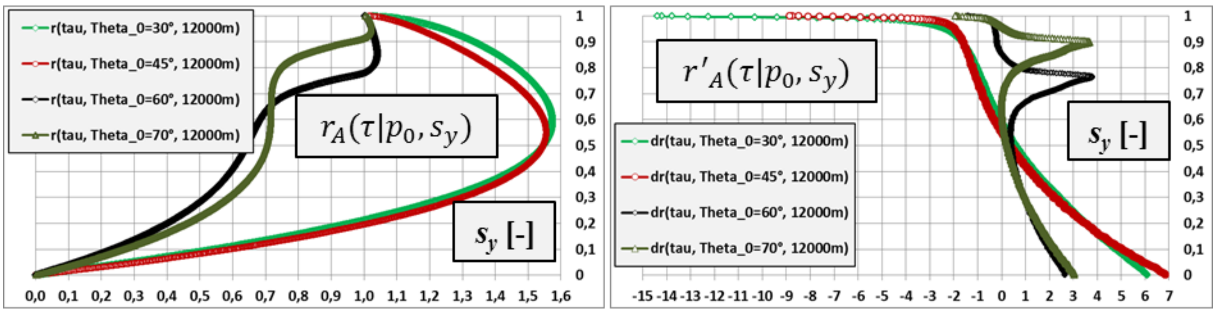

Fig. 3: Weighting factor functions – WFFs - temperature and their derivatives with respect to relative height s_y – normalized form for Y_S = 12000 m, c_{43} = 0.43 m^2/kg and Θ_0 = 30°, 45°, 60° and 70° (v_0 = 1668, 1040, 802 and 718 m/s).

Fig. 3 shows WFFs and their derivatives for n = 12 and $y(n)$ = 12000 m, ballistic coefficient c_{43} = 0.43 m^2/kg, angles of departure Θ_0 = 30°, 45°, 60° and 70° and corresponding initial velocities v_0. It is apparent that shapes of WFFs strongly depend on the angle of departure Θ_0.

Following comparison based on the WFFs - temperature curves presented in Fig. 1, Fig. 2 and Fig. 3 is conditional and follows from the general analogies valid for the projectile trajectories. Immediate comparison is not possible because WFFs in Fig. 1 and Fig. 2 respecting STANAG 4061 suppose complementary WFFs-density that is expressed by notation $(\tau \mid \rho)$. Presented results of authors calculations suppose complementary met parameter air-pressure at the muzzle level p_0, which is expressed by notation $(\tau \mid p_0)$. Deeper explanation can be find in Molitz and Strobel (1963) and also Cech et al. (2014).

As it has already been presented for the WFF-wind, the STANAG 4061 shows only one WFF curve for the given height $y(n)$ ("mean" WFF). It is apparently weighted mean of several WFFs curves. But used departure angles Θ_0 and ballistic coefficient c_{43} of these WFFs are not known just as used weights.

This "mean" WFF-temperature exceeds magnitude 1 and authors of STANAG 4061 simplified the problem by introducing the condition that for $r_R(\tau, s_y) > 1$ the magnitude of $r_R(\tau, s_y) = 1$. Unfortunatelly this simplification leads to distorted derivatives of WFFs courses with respect to y (compare Fig. 2 and Fig. 3). It follows from the relations (1) and (2) that the calculated relative deviation of ballistic virtual temperature $\delta\tau_B$ will be significantly different from the correct or more accurate magnitude.

4. Conclusions

It follows from the analysis of reconstructed WFFs and follow-up calculations that the weighting factors (WFs) presented in STANAG 4061 are sufficiently accurate as "average magnitudes" determined from "average" WFFs only for very narrow domain of combinations of magnitudes of departure angles and ballistic coefficients, i.e. particular weapons and projectiles. For the other combinations the sufficient accuracy in determination of ballistic elements μ_B is not guaranteed. The larges inaccuracies are caused by very simplified courses of "mean" WFFs-temperature for $Y_S > 6000$m. Further research will be focused especially on WFFs - temperature.

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STANAG 4044 MET Adoption of a Standard Atmosphere.

STANAG 4061 MET Adoption of a Standard Ballistic Meteorological Message (METBKQ).

STANAG 4119 Adoption of a Standard Cannon Artillery Firing Table Format.