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# IDENTIFICATION OF THE AEROELASTIC PROFILE BASED ON OPTICAL MEASUREMENT

# Š. Chládek<sup>\*</sup>, I. Zolotarev<sup>\*</sup>

**Abstract:** This paper introduces an identification of an aeroelastic profile as n - degrees of freedom linear system. The common identification is based on the excitation of the system with an impact hammer and the measurement of the response by an acceleration sensor and using the transfer functions the modal properties are evaluated. This approach gives a disadvantage in the sense of influence of the system structural properties. In this paper the identification of the dynamical system is based on the optical measurement of the system response. The great advantages of this approach are both the low influence of the measuring devices to the system properties and the high precision of the measurement. The theory has been verified on the aeroelastic profile NACA 0012 with 2 degrees of freedom and the results are presented.

Keywords: System identification, Aeroelastic profile, Optical measurement.

## 1. Introduction

The identification of the real object means to derive its mathematical model. The mathematical model has to fulfill certain criterion; the most important one is that the model describes the object properties as close as possible. Identification of the dynamical system is widely spread field of study, when the most methods are based on the system excitation and its response measurement and using the transfer function the modal properties are evaluated, see Kozánek (1982). There have to be performed more measurements for the complete system description. This approach gives a disadvantage in the sense of added mass (weight of the sensor) and added damping (the sensor's wire attachment). The mentioned disadvantages influence the profile's modal and structural properties and these changes can cause a difference between numerical simulation and real experiment results. This paper describes an identification method based on the optical measurement of the system response. The great advantage of the optical measurement is a minimal impact on the system properties and the requirement of one measurement only.

## 2. System Identification

The linear system can be described by the equation of motion

$$\mathbf{M}\ddot{\mathbf{z}}(t) + \mathbf{B}\dot{\mathbf{z}}(t) + \mathbf{K}\mathbf{z}(t) = \mathbf{F}(t), \tag{1}$$

where z(t) is the position vector, F(t) is the excitation vector and M, B, K are the mass, damping and stiffness matrices, respectively. The acceleration, velocity and displacement responses can be expressed using the modal transformation

$$\ddot{\mathbf{z}}(t) = \mathbf{V}\ddot{\mathbf{q}}(t), \ \dot{\mathbf{z}}(t) = \mathbf{V}\dot{\mathbf{q}}(t), \ \mathbf{z}(t) = \mathbf{V}\mathbf{q}(t),$$
(2)

where **V** is the matrix of modal vectors. Coupling the equations (1), (2), using the equalities  $\mathbf{M}^{-1}\mathbf{B} = diag(2b_{ri}\Omega_{0i})$ ,  $\mathbf{M}^{-1}\mathbf{K} = diag(\Omega_{0i}^2)$ , i = 1, 2, ..., n, where *n* is the number of degrees of freedom and the assumption the modal vectors matrix is orthogonal, the resulting equation of motion in modal coordinates can be written in the form

$$\ddot{q}_{i}(t) + 2b_{ri}\Omega_{0i}\dot{q}_{i}(t) + \Omega_{0i}^{2}q_{i}(t) = V_{j,i}^{T}F_{i}(t),$$
(3)

<sup>&</sup>lt;sup>\*</sup> Ing. Štěpán Chládek, Ing. Igor Zolotarev, CSc.: Institute of Thermomechanics AS CR, v. v. i., Dolejškova 1402/5, 182 00 Prague, CZ, chladek@it.cas.cz, igor@it.cas.cz

where  $b_{ri}$ ,  $\Omega_{0i}$  is the damping ratio and natural frequency, respectively. The modal coordinate  $q_i(t)$  can be derived from the natural coordinates using a suitable data processing, i.e. appropriate data filtration. The relation between the structural matrices and modal properties can be expressed in the form

$$\mathbf{M} = \mathbf{V}^{-1}, \mathbf{B} = diag(2b_{ri}\Omega_{0i})\mathbf{V}^{-1}, \mathbf{K} = diag(\Omega_{0i}^2)\mathbf{V}^{-1}$$
(4)

where i = 1, 2, ..., n denotes the number of degrees of freedom. The damping ratios  $b_{ri}$ , undamped natural frequencies  $\Omega_{0i}$  and modal vector matrix V are determined from the modal coordinates. The natural frequencies  $\Omega_i$  of the damped system are calculated using the Fourier transformation. The Hilbert transformation of the modal coordinate  $q_i$  is used for calculation of  $b_{ri}$  in the sense of relation

$$b_{ri} = \sqrt{\frac{1}{1 + \left(\frac{\Omega_i}{k_i}\right)^2}} \tag{5}$$

where  $k_i$  is the slope of the envelope of the modal coordinate  $q_i$  when it is depicted in semi logarithmic coordinates. Similar approach based on Hilbert-Huang transformation is described in Yang et at. (2003). The natural frequency of undamped system is determined as

$$\Omega_{0i} = \sqrt{\frac{\Omega_i}{b_{ri}^2 - 1}} \tag{6}$$

The element  $V_{i,j}$  of the matrix V is calculated from the mean value of the ratio of the modal coordinate  $q_i$  in the position 1 and j.



Fig. 1: Scheme of the profile NACA 0012 attached to the frame. Points P1, P2 denote the positions of targets, which have been recorded by camera.CG means center of gravity, EA denotes the elastic axis.

#### 3. Experimental Measurement

It has been chosen a profile NACA 0012 with two degrees of freedom for the theory verification. The length of the chord is 100 mm and the span of the wing is as well 100 mm. The wing is attached with two leaf springs to the frame, see Chládek et al., (2012). The scheme of the profile is in the Fig. 1.

There has been attached a white paper with two black points P1, P2 on one side of the profile. The black points have been chosen to increase the contrast to the white background and these points have been used for the instantaneous position determination. The number of black points has to be equal to the number of

degrees of freedom. The paper with the weight of less than 0.2 g is the only one influence to the profile. The position of the points has been captured with high frequency camera Dantec NanoSense. The sampling frequency has been set up to  $f_s = 2000 Hz$  and the record time as non-optional parameter has been calculated as t = 11.3 s. One picture from the camera is shown in the Fig. 2. The profile has been deflected from its equilibrium position and its response has been measured. Based on the Fourier transform of the signal the natural frequencies  $\Omega_i$  have been computed and the digital filters have been designed. It has been chosen a bandpass type filter with the finite impulse response.



Fig. 2: Picture of the targets recorded by the high frequency camera with the points P1 (right), P2 (left).



Fig. 3: Eigenmodes related to the 1st natural frequency (left) and to the 2nd natural frequency (right).

Tab. 1 Comparison of modal properties of the system evaluated from the time record of the position measured in the points P1 and P2.

	Record point P1	Record point P2
f <sub>01</sub> [Hz]	31.62	31.62
f <sub>02</sub> [Hz]	38.88	38.88
f <sub>1</sub> [Hz]	31.62	31.62
f <sub>2</sub> [Hz]	38.88	38.88
b <sub>r1</sub> [-]	323 x 10 <sup>-5</sup>	324 x 10 <sup>-5</sup>
b <sub>r2</sub> [ - ]	190 x 10 <sup>-5</sup>	190 x 10 <sup>-5</sup>



*Fig. 4: Comparison of experiment (solid line) and numerical simulation (discrete points) in time record of the profile movement.* 

The modal coordinates have been derived applying the suggested filters to the original signals. The modal properties have been computed using (5), (6) and they are listed in the Tab. 1. This table has two columns due to the fact that all modal properties have been calculated twice from the signal measured at the point P1 and P2 as well. The eigenmodes related to the natural frequencies are plotted in the

Fig. 3. The structural matrices have been calculated using (4). The possible way for the matrices verification is the computation of (1) with the initial conditions corresponding to the real experiment and the excitation vector  $\mathbf{F}(t) = \mathbf{0}$ . The numerical integration has been performed with the time  $t = (t_0, t_1)$  and the results are compared with the experiment in the Fig. 4.

### 4. Conclusions

Identification method based on the optical measurement of the system displacement has been introduced. There have been performed both numerical simulation and experiments for the theory verification. The results have demonstrated high accordance of the real system with its mathematical model. Considering the Tab. 1 it can be concluded that the measurement has been performed with very high precision, because the differences of calculated modal properties are negligible. The most important results of introduced identification process are the correct structural matrices, which have been proved by comparing the real experiment and the numerical simulation of system response to the given initial conditions. The major disadvantage of the submitted method is the requirement of an expensive high frequency camera while the widespread used acceleration sensors are significantly cheaper.

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