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KINEMATICALLY EXCITED NON-LINEAR VIBRATION OF A BEAM ON ELASTIC SUPPORTS WITH CLEARANCES

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Abstract: *The paper deals with nonlinear vibration of a beam excited by harmonic movement of frame which the beam is fixed in. Except for supports of the ends of the beam, another supports along axis of the beam are considered. In these supports, there are clearances which cause nonlinear vibration of the system. A mathematical model of the beam is obtained using finite element method and decomposition of such a system is used to obtain a model including kinematical excitation. As an application, nonlinear vibration of a guide thimble in spacer grid of nuclear fuel assembly is shown. To get solution, numerical integration in time domain is used and quality of vibration is shown using orbits and phase trajectories of representative dynamical quantities.*

Keywords: Nonlinear vibration, Kinematical excitation, Clearance, Beam.

1. Introduction

In many engineering applications, non-linear vibration of beam type components appears. The non-linearity given by beam supports with clearances may cause that the vibro-impact motion occurs. This topic is very actual and it is widely studied with respect to structure reliability, see (Chena et al., 2014). Particularly, nuclear fuel assemblies consist of a large number of beam type components. The aim of this paper is to describe vibration of such a system in general and to define the admissible states of the beam motion. Let us suppose the fuel assembly components are fixed in the support plates and are kinematically excited. In the fuel assembly, there are rods (fuel rods and guide thimbles) which are fit into spacer grids of load-bearing skeleton (frame) with a clearance. Therefore, impacts occur between beam and the frame in levels of spacer grids which generate large impact forces during vibration which can lead to material stress increase and to degradation of surface of rods.

2. Mathematical Model of Kinematically Excited Beam on Elastic Supports with Clearances

A radial symmetric flexible fixed-ended beam is considered (see Fig. 1). One end of the beam is fixed in a rigid frame which moves harmonically with frequency ω and the second end moves harmonically as well with the same frequency ω but generally with different amplitude and with phase shift φ. Along axis of the shaft, there are m elastic supports with stiffnesses $k_i, i = 1 \dots m$, and there is a radial clearance Δ between the beam and a ring of support.

To get mathematical model of the beam, finite element method (FEM) is used. The beam is supposed to be one dimensional continuum satisfying Euler-Bernoulli theory – it is radially uncompressible and there is no deplanation of a cross section in deformed position, see (Byrtus et al., 2010). One finite element has two nodes and in every one node, there are six degrees of freedom; displacement in axial direction u, lateral displacements v, w in sense of axes x, y, respectively, torsional rotation angle φ, and flexure rotation angles ψ, ϑ. The beam can then be divided to $N-1$ elements with N nodes and it is chosen in the

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way that there is a node in each nonlinear support. The vector of generalized coordinates of whole system $\mathbb{q}(t)$ can be written in the form

$$\mathbb{q}(t) = [\dots, u_i, v_i, w_i, \varphi_i, \vartheta_i, \psi_i, \dots]^T \in \mathcal{R}^{6N},\ i = 1 \dots N. \tag{1}$$

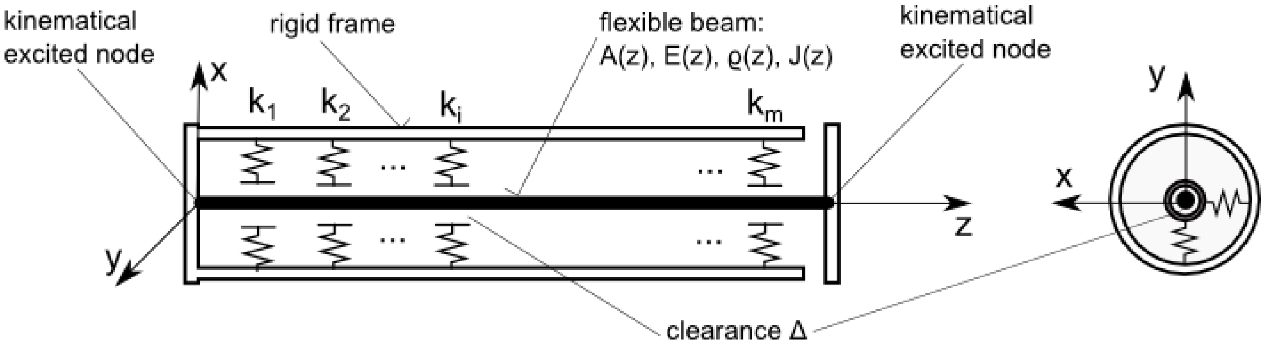

Fig. 1: System of kinematically excited rigid frame with flexible beam on elastic supports with clearances.

First, conservative mathematical model of the system will be derived and subsequently it will be extended to full nonlinear model. Conservative model can be derived in the following form

$$\mathbb{M}\ddot{\mathbb{q}}(t) + \mathbb{K}\mathbb{q}(t) = \mathbb{O}, \tag{2}$$

where $\mathbb{M},\ \mathbb{K} \in \mathcal{R}^{6N,6N}$are mass matrix and stiffness matrix of the beam, respectively. To include kinematic excitation, the vector of generalized coordinates of the system in vertical position can be divided into three subvectors $\mathbb{q}_p, p = L, F, U$, where index L corresponds to lower kinematically excited node (fixed to the frame), index F corresponds to all free nodes and index U corresponds to generalized coordinates of upper kinematically excited node (fixed to the frame). System can be then written in decomposed form

$$\begin{bmatrix} \mathbb{M}_L & \mathbb{M}_{L,F} & \mathbb{O} \\ \mathbb{M}_{F,L} & \mathbb{M}_F & \mathbb{M}_{F,U} \\ \mathbb{O} & \mathbb{M}_{U,F} & \mathbb{M}_U \end{bmatrix} \begin{bmatrix} \ddot{\mathbb{q}}_L(t) \\ \ddot{\mathbb{q}}_F(t) \\ \ddot{\mathbb{q}}_U(t) \end{bmatrix} + \begin{bmatrix} \mathbb{K}_L & \mathbb{K}_{L,F} & \mathbb{O} \\ \mathbb{K}_{F,L} & \mathbb{K}_F & \mathbb{K}_{F,U} \\ \mathbb{O} & \mathbb{K}_{U,F} & \mathbb{K}_U \end{bmatrix} \begin{bmatrix} \mathbb{q}_L(t) \\ \mathbb{q}_F(t) \\ \mathbb{q}_U(t) \end{bmatrix} = \mathbb{O}. \tag{3}$$

After rewriting second row of (3), mathematical model is

$$\mathbb{M}_F\ddot{\mathbb{q}}_F(t) + \mathbb{K}_F\mathbb{q}_F(t) = -\mathbb{M}_{F,L}\ddot{\mathbb{q}}_L(t) - \mathbb{M}_{F,U}\ddot{\mathbb{q}}_U(t) - \mathbb{K}_{F,L}\mathbb{q}_L(t) - \mathbb{K}_{F,U}\mathbb{q}_U(t). \tag{4}$$

Now, the model (4) can be extended considering damping and nonlinear forces generated in supports with clearances. Damping is supposed to be proportional, so $\mathbb{B}_F = \alpha\mathbb{M}_F + \beta\mathbb{K}_F$, where $\alpha, \beta \in \mathcal{R}^+$ can be determined from estimation of damping ratio of first two eigenmodes. After inclusion of damping matrix and using vector of nonlinear forces $\mathbb{f}_N(\mathbb{q}_F)$, the model (4) can be completed in the form

$$\mathbb{M}_F\ddot{\mathbb{q}}_F(t) + \mathbb{B}_F\dot{\mathbb{q}}_F(t) + \mathbb{K}_F\mathbb{q}_F(t) = -\mathbb{M}_{F,L}\ddot{\mathbb{q}}_L(t) - \mathbb{M}_{F,U}\ddot{\mathbb{q}}_U(t) - \mathbb{K}_{F,L}\mathbb{q}_L(t) - \mathbb{K}_{F,U}\mathbb{q}_U(t) + +\mathbb{f}_N(\mathbb{q}_F). \tag{5}$$

The vector $\mathbb{f}_N(\mathbb{q}_F)$ depends on generalized coordinates of free nodes and it is given as a sum of vectors of nonlinear forces in all supports

$$\mathbb{f}_N(\mathbb{q}_F) = \sum_j \mathbb{f}_N^{(j)}(\mathbb{q}_F),\ j = 1 \dots m. \tag{6}$$

Vector $\mathbb{f}_N^{(j)}(\mathbb{q}_F)$ includes only lateral forces generated in j-th support and it can be written in a form

$$\mathbb{f}_N^{(j)}(\mathbb{q}_F) = \left[\dots, F_{Nx}^{(j)}, F_{Ny}^{(j)}, \dots\right]^T, \tag{7}$$

where forces $F_{Nx}^{(j)}$ and $F_{Ny}^{(j)}$ are placed in positions corresponding to transversal displacement of supported node „j“ in direction x, y, respectively. These forces are given in the form

$$F_{Nx}^{(j)} = k_{jx}(u_{xj} - \Delta)\mathcal{H}(u_{xj} - \Delta), \tag{8}$$

$$F_{Ny}^{(j)} = k_{jy}(u_{yj} - \Delta)\mathcal{H}(u_{yj} - \Delta), \tag{9}$$

where k_{jx}, k_{jy} are contact stiffnesses in direction x, y in j-th support, u_{xj}, u_{yj} are relative displacements of center of the beam with respect to the frame in direction x, y in j-th node and $\mathcal{H}(.)$ is Heaviside function.

3. Application to Vibration of Guide Thimble of Fuel Assembly of a Nuclear Reactor

The application part of the paper is aimed at vibration of fuel assembly TVSAT of the nuclear WWER 1000 type reactor which is in detail described in Sýkora (2009). Mathematical model of the reactor is built up in Hlaváč & Zeman (2013). The above theory was applied to nonlinear vibration of guide thimbles, which together with fuel rods, center tube and load-bearing skeleton (frame) with eight spacer grids make a fuel assembly. The guide thimbles are at both ends fixed in two support plates by means of lower and upper piece. All the rods are linked by the spacer grids which are transversal to their axes and the guide thimbles are inserted into spacer grids with a small clearance. One guide thimble is shown in the Fig. 2 with both lower and upper pieces. Further, only one guide thimble at the position r in 1st segment (s=1) will be considered. A generally spatial movement of the plates is given in configuration space $\mathbb{q}_X(t) = \left[x, y, z, \varphi_x, \varphi_y, \varphi_z\right]_X^T, X = L, U$ as shown in the Fig. 2 and vibration of a guide thimble in configuration space which is radially-tangential in x, y plane according to Zeman & Hlaváč (2011). Between these two systems, there is transformation which can be described by

$$\mathbb{q}_{r,X} = \mathbb{T}_{r,X}^{(s)} \mathbb{q}_{X}, \ X = L, U, \tag{10}$$

where $\mathbb{T}_{r,X}^{(s)}$ is transformation matrix which transforms movement of lower and upper plates $\mathbb{q}_X$ to movement of kinematical excited nodes of the guide thimble $\mathbb{q}_{r,X}$. It is dependent only on geometrical parameters, see Zeman & Hlaváč (2011). Only one chosen guide thimble is considered and indexes r and s will be omitted further.

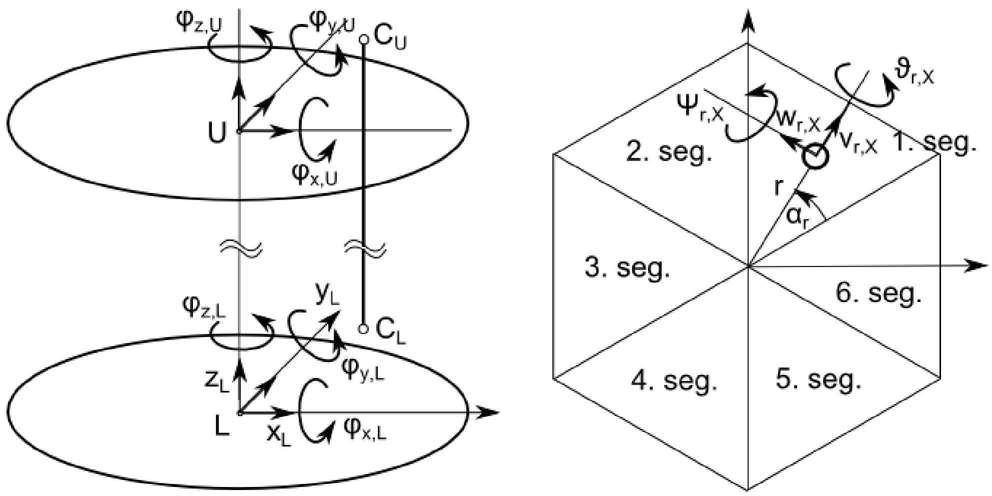

Fig. 2: Guide thimble in lower and upper piece and coordinate systems.

The considered guide thimble was divided into 18 finite elements. The lower C_L and upper C_U nodes are supposed to be ideally fixed and in every even free node, there are supports (spacer grids) with the same clearance considered. The geometry of spacer grids is shown in Hlaváč & Zeman (2013). In the first phase, only harmonic excitation with frequency ω and with phase shift $\varphi = 0$ was implemented. Mathematical model of the system can be written in the form

$$\mathbb{M}_F \ddot{\mathbb{q}}_F(t) + \mathbb{B}_F \dot{\mathbb{q}}_F(t) + \mathbb{K}_F \mathbb{q}_F(t) = (\mathbb{M}_{F,L}\omega^2 - \mathbb{K}_{F,L})\mathbb{T}_L(\mathbb{q}_L^S \sin \omega t + \mathbb{q}_L^C \cos \omega t) +$$
$$+(\mathbb{M}_{F,U}\omega^2 - \mathbb{K}_{F,U})\mathbb{T}_U(\mathbb{q}_U^S \sin \omega t + \mathbb{q}_U^C \cos \omega t) + \mathbb{f}_N(\mathbb{q}_F), \tag{11}$$

where $\mathbb{q}_L, \mathbb{q}_U$ are vectors of amplitudes of harmonic excitation. To get solution of the system, numerical implementation in MATLAB was accomplished. To integrate mathematical model (11), fourth order Runge-Kutta method with adaptive step in time domain was implemented using ode45 MATLAB built-in function. For the simulation, important parameters are clearance $\Delta = 1$ mm, and frequency $\omega = 2\pi f$,

$f = 3$ Hz, contact stiffness in all spacer grids $k_j = 0.179 \cdot 10^6$ N/m, $j = 1 \dots 8$. Vectors of kinematical excitation are supposed in the form

$$\mathbb{q}_L^S = \mathbb{q}_U^S = [0{,}01;\ 0;\ 0;\ 0;\ 0;\ 0],\ \mathbb{q}_L^C = \mathbb{q}_U^C = [0;\ 0{,}0075;\ 0;\ 0;\ 0;\ 0]. \tag{12}$$

Numerical integration results are shown in the Fig. 3. Orbits in plane x, y in chosen nodes of guide thimble are depicted (Fig. 3, on the left). It is evident, that boundary nodes which are close to kinematically excited nodes vibrate quasiharmonically. Nodes in the middle of the guide thimble vibrate with the largest amplitude and that is why impacts occur the most often there. There are phase trajectories depicted as well (Fig. 3, on the right). The impact motion can be well identified from these phase portraits. The impacts correspond to points at the trajectories where large changes in the velocity occur.

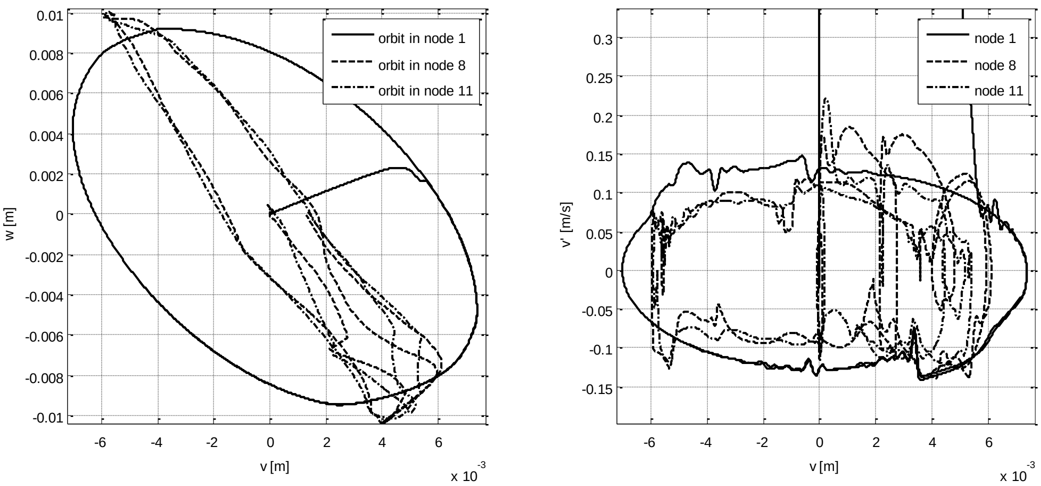

Fig. 3: Orbits in chosen nodes of guide thimbles (on the left) and example of phase trajectories.

4. Conclusion

The paper focuses on mathematical modelling of kinematically excited beam supported by nonlinear supports. The theory described above shows an approach to modelling of such a system. Particular application to guide thimble of nuclear fuel assembly is implemented in MATLAB and typical dynamical response of the system to harmonic kinematical excitation is shown.

Future analyses will be focused on optimization of parameters (clearances, contact stiffnesses, etc.) to reach lower impact forces between guide thimble and the spacer grids. Except for that, derived mathematical model and the display of impact motion are wide enough to represent dynamical response caused by another kind of excitation than only the harmonic one. Seismic excitation can be considered and a response of the system can be analyzed simulating seismic event in nuclear power plant.

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