

AN IMPROVED THERMAL-STRUCTURAL FINITE ELEMENT MODEL FOR MANUFACTURING PROCESSES WITH HEAT GENERATION

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Abstract: *In this paper a universal mathematical model capable of predicting thermo-mechanical behaviour of various types of metal during their manufacturing using fully coupled thermal-structural finite element analysis is presented. The model takes into account the internal damping of the material, elastic and plastic heating and it can be used in wide range of strain rates that accompany the deformation of the body. In the model finite element implementation an improved heat equation, the updated Lagrangian formulation, the NoIHKH material model and the Jaumann rate in the form of the Green-Naghdi rate in the co-rotational Cauchy's stress tensor integration have been used. Cyclic tension of a notched aluminium specimen has been studied using prescribed axial deformation and a sine function with linearly increasing amplitude and 2 Hz frequency in the run-up stage of the test and a sine function with constant amplitude and linearly increasing frequency in the operation stage of the test. A few selected analysis results are presented and briefly discussed. The analysis results are very positive and the authors believe that the model might open new perspectives in the study and numerical simulations of metals.*

Keywords: Thermal-structural analysis, Strong coupling, Finite strain elastoplasticity, Internal damping, Elastic, Plastic and internal damping induced heating, Wide range of strain rates.

1. Introduction

Many manufacturing processes are accompanied by significant heat generation in real situations. Depending on the particular manufacturing process itself, the heat might stem from various sources. In the presented research we are interested in the study of such processes, where the heat originates from mechanical work. A few typical examples are machining of metallic materials, in which cutting a piece of raw metal into a desired shape and size is accompanied by severe heat generation, or friction-stir welding (FSW), in which the amount of heat generated between the tool and the welded material softens the nearby metal, which when intermixed at the presence of mechanical pressure, joins the two pieces of the welded metal. Numerical simulation of the aforementioned manufacturing processes represents a real challenge in contemporary computational mechanics, as it involves in it the simulation of contact between the workpiece and the tool, the heat generation, which either originates in friction or deformation, where the latter is large and usually strain rate dependent, thus the simulation requires rezoning and remeshing of the spatially discretized body.

The aim of this work is to present a universal mathematical model capable of predicting thermo-mechanical behaviour of various types of metal during their manufacturing.

2. Methods

In the study a fully coupled thermal-structural finite element (FE) analysis has been carried out using large strain / large deformation formulation, which employed an improved heat equation with elastic heating, plastic heating and internal damping induced heating (Écsi et al., 2012a).

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The model is applicable to wide range of strain rates and it bases on our two former models utilizing an enhanced weak form for FE analysis that accounts for the strong coupling between the deformation field and the temperature field on the boundary of the body when convective or radiative heat transfer takes place on the boundary (Écsi et al., 2012b; Écsi et al., 2009). In the internal / material damping mathematical formulation we employed a modified Kelvin-Voight model which is capable of imitating the viscosity of the material during both, elastic and plastic deformations (Écsi et al., 2012b). The extended NoIHKH material model (Écsi et al., 2006), based on the original NoIHKH material model for cyclic plasticity of metal (Lemaitre, 2001), has been adapted to large strains / large deformations utilizing the updated Lagrangian formulation, the J_2 plasticity and the Jaumann rate in the form of the Green-Naghdi rate in the Cauchy's stress tensor integration. Using the co-rotational formulation the constitutive and evolution equations of the model are given with the following equations:

$${}^{n+1}\hat{\boldsymbol{\sigma}} = {}^{n+1}\hat{\boldsymbol{\sigma}}^{\text{el}} + {}^{n+1}\hat{\boldsymbol{\sigma}}^{\text{damp}}, \quad (1)$$

$${}^{n+1}\hat{\boldsymbol{\sigma}}^{\text{el}} = \Delta\hat{\boldsymbol{\sigma}}^{\text{el}} + {}^n\hat{\boldsymbol{\sigma}}^{\text{el}}, \quad (2)$$

$$\Delta\hat{\boldsymbol{\sigma}}^{\text{el}} = {}^{n+\frac{1}{2}}\hat{\mathbf{C}}^{\text{VJ}} : \left[\Delta t \left({}^{n+\frac{1}{2}}\hat{\mathbf{d}} - {}^{n+\frac{1}{2}}\hat{\mathbf{d}}^{\text{th}} \right) - x\Delta\tilde{\lambda} \frac{\partial f}{\partial \hat{\boldsymbol{\sigma}}} \right], \quad (3)$$

$${}^{n+1}\hat{\boldsymbol{\sigma}}^{\text{damp}} = {}^{n+1}\hat{\mathbf{C}}^{\text{VJdamp}} : \left[{}^{n+1}\hat{\mathbf{d}} - (1-x) \frac{\Delta\tilde{\lambda}}{\Delta t} \frac{\partial f}{\partial \hat{\boldsymbol{\sigma}}} \right], \quad (4)$$

$$f = \sigma_{\text{eq}} - \sigma_y - R \leq 0, \quad (5)$$

$$\sigma_{\text{eq}} = \sqrt{\frac{3}{2}(\hat{\boldsymbol{\Sigma}} - \boldsymbol{\Sigma}\hat{\mathbf{X}}) : (\hat{\boldsymbol{\Sigma}} - \boldsymbol{\Sigma}\hat{\mathbf{X}})}, \quad \hat{\boldsymbol{\Sigma}} = \hat{\boldsymbol{\sigma}} - \frac{1}{3}\text{tr}(\hat{\boldsymbol{\sigma}})\mathbf{I}, \quad \boldsymbol{\Sigma}\hat{\mathbf{X}} = \hat{\mathbf{X}} - \frac{1}{3}\text{tr}(\hat{\mathbf{X}})\mathbf{I}, \quad (6)$$

$$R = Q(1 - e^{-be^n}), \quad (7)$$

$${}^{n+1}\hat{\mathbf{X}} = \Delta t \cdot \left[{}^{n+\frac{1}{2}}\hat{\mathbf{C}}^{\text{VJcycl}} : {}^{n+\frac{1}{2}}\hat{\mathbf{d}}^{\text{pl-el}} - \gamma \left({}^{n+\frac{1}{2}}\boldsymbol{\varepsilon}^{\text{pl-el}} \right) {}^{n+\frac{1}{2}}\hat{\mathbf{X}} - {}^{n+\frac{1}{2}}\dot{\boldsymbol{\varepsilon}}^{\text{pl-el}} \right] + {}^n\hat{\mathbf{X}}, \quad (8)$$

$$\gamma(\boldsymbol{\varepsilon}^{\text{pl-el}}) = \gamma_{\infty} - (\gamma_{\infty} - \gamma_0)e^{-\omega\boldsymbol{\varepsilon}^{\text{pl-el}}}, \quad (9)$$

$${}^{n+\frac{1}{2}}\hat{\mathbf{d}}^{\text{pl-el}} = x \cdot \Delta\tilde{\lambda} \cdot \sqrt{\frac{3}{2}} \cdot \hat{\mathbf{N}}, \quad {}^{n+1}\hat{\mathbf{d}}^{\text{pl-damp}} = (1-x) \cdot \frac{\Delta\tilde{\lambda}}{\Delta t} \cdot \sqrt{\frac{3}{2}} \cdot \hat{\mathbf{N}}, \quad \hat{\mathbf{N}} = \frac{\hat{\boldsymbol{\Sigma}} - \boldsymbol{\Sigma}\hat{\mathbf{X}}}{\sqrt{(\hat{\boldsymbol{\Sigma}} - \boldsymbol{\Sigma}\hat{\mathbf{X}}) : (\hat{\boldsymbol{\Sigma}} - \boldsymbol{\Sigma}\hat{\mathbf{X}})}}, \quad (10)$$

$${}^{n+\frac{1}{2}}\mathbf{R} = \exp \left[\frac{\Delta t}{2} {}^{n+\frac{1}{2}}\mathbf{W} \right] \bullet {}^n\mathbf{R}, \quad {}^{n+1}\mathbf{R} = \exp \left[\Delta t {}^{n+\frac{1}{2}}\mathbf{W} \right] \bullet {}^n\mathbf{R}, \quad (11)$$

$$f \leq 0, \quad \Delta\tilde{\lambda} \geq 0, \quad f \cdot \Delta\tilde{\lambda} = 0. \quad (12)$$

In Eqns. (1)-(12) the left superscripts $n, n+1/2, n+1$ denote the physical quantity value at discrete times, corresponding to previous, mid and current configurations of the body within the current time step Δt . Here $\hat{\boldsymbol{\sigma}} = \mathbf{R}^T \cdot \boldsymbol{\sigma} \cdot \mathbf{R}$, $\hat{\boldsymbol{\sigma}}^{\text{el}}$, $\hat{\boldsymbol{\sigma}}^{\text{damp}}$ are the co-rotational Cauchy's stress tensor, its elastic part and its damping part, $\hat{\mathbf{X}} = \mathbf{R}^T \cdot \mathbf{X} \cdot \mathbf{R}$ is the co-rotational backstress tensor, in which objective integration there was employed the Jaumann rate in the form of the Green-Naghdi rate (De Souza Neto et al., 2008). We used an additive decomposition of the co-rotational strain rate tensor $\hat{\mathbf{d}} = \mathbf{R}^T \cdot \dot{\mathbf{d}} \cdot \mathbf{R}$ into an elastic part $\hat{\mathbf{d}}^{\text{el-el}}$, a thermal part $\hat{\mathbf{d}}^{\text{th}} = \alpha \dot{T} \mathbf{1}$ and a plastic part $\hat{\mathbf{d}}^{\text{pl-el}} = x(\Delta\tilde{\lambda} / \Delta t)(\partial f / \partial \hat{\boldsymbol{\sigma}})$ in Eqn. (3), and an additive decomposition of the co-rotational strain rate tensor $\hat{\mathbf{d}}$ into an elastic part $\hat{\mathbf{d}}^{\text{el-damp}}$ and a plastic part $\hat{\mathbf{d}}^{\text{pl-damp}} = (1-x)(\Delta\tilde{\lambda} / \Delta t)(\partial f / \partial \hat{\boldsymbol{\sigma}})$ in Eqn. (4), where x is the ratio of ductile and total damage, α is the

coefficient of thermal expansion, T is the absolute temperature, $\Delta\tilde{\lambda}$ is the plastic multiplier, $\mathbf{1}$ stands for a second order unit tensor and $\hat{\mathbf{C}}^{\nabla J_{damp}}$, $\hat{\mathbf{C}}^{\nabla J_{damp}}$, $\hat{\mathbf{C}}^{\nabla J_{cycl}}$ denote the fourth order co-rotational elastic material tensors of the model in the current configuration of the body along with the definitions of the accumulated plastic strain and the effective plastic strain as the measures of ductile and total damage (Écsi, L. & Élesztős, P., 2012a; Écsi, L. & Élesztős, P., 2012b). The corresponding rotation tensors (11) are expressed in terms of the time step size and a second order spin tensor \mathbf{W} . The extended NoIHKH material model for cyclic plasticity of metals that uses combined isotropic (7) and kinematic (8) hardening has been adapted for large strain elastoplasticity Eqns. (5)-(9). The loading/unloading criterions are defined with the discrete Khun-Tucker optimality conditions Eqn. (12).

2.1. Numerical experiment – Cyclic tension of a notched specimen

In our numerical experiment a 2024-T3 aluminium alloy notched specimen was studied using cyclic tension and zero stress ratio $R = 0$. One end of the specimen was fixed while there was a prescribed axial deformation applied to the second end of the specimen employing a sine function. The amplitude of the sine function increased linearly during the run-up stage of the test until the maximum amplitude, corresponding to 20% of the specimen length was reached, while the circular frequency was kept constant at 2 Hz. During the test, the amplitude of the sine function was kept constant and the circular frequency increased linearly with time. The dimensions of the specimen were 10 times smaller than the ones in the modelled experiment and the material properties were identical with the properties of the specimen used by Pastor, et. al. (2008) in their study. Only 1/4 of the body was modelled employing 2 planes of symmetry, which were meshed with 3D brick elements using linear shape functions. Convective and radiative heat transfer was considered through all free surfaces of the body using 273.15 K bulk temperature. At the moving end of the body, under the grip of the testing machine, the heat transfer coefficient value was increased to $h = 10^{22} \text{ W/m}^2 \cdot \text{K}$ to keep the temperature of the surfaces constant at these locations as in the test by Pastor et. al., 2008. Tab. 1 outlines the used material parameters.

Tab. 1: Material parameters of the specimen.

$E = 73100000000.0 \text{ Pa}$	$\gamma_0 = 0.002$
$E^{damp} = 73100.0 \text{ Pa}$	$\omega = 10.0$
$E^{cycl} = 73100000.0 \text{ Pa}$	$\rho_0 = 2770.0$
$\nu = \nu^{damp} = \nu^{cycl} = 0.33$	$c = 876.0 \text{ J/(kg} \cdot \text{K)}$
$\sigma_y = 345000000.0 \text{ Pa}$	$k_{xx} = k_{yy} = k_{zz} = 120.0 \text{ W/(m} \cdot \text{K)}$
$Q = 138000000.0 \text{ Pa}$	$\alpha_x = \alpha_y = \alpha_z = \alpha = 0.0000234 \text{ K}^{-1}$
$b = 3.0$	$h = 10.0 \text{ W/(m}^2 \cdot \text{K)}$
$\gamma_\infty = 0.001$	$\Psi = 1.0, \sigma_{EMS} = 11.343 \cdot 10^{-9} \text{ W/(m}^2 \cdot \text{K}^4)$

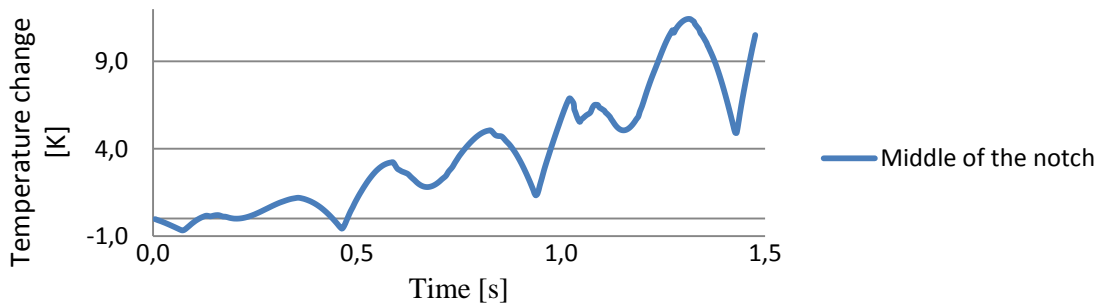


Fig. 1: Temperature change time history in the middle of the notch of the specimen.

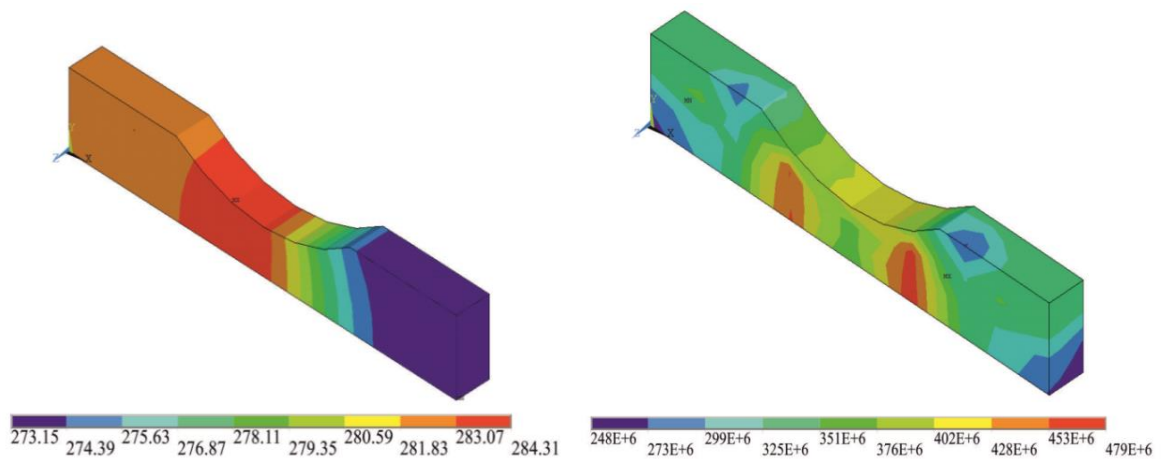


Fig. 2: Absolute temperature [K] and axial Cauchy's stress distribution [Pa].

Unfortunately at the time of writing the paper the analysis had not yet been finished, so that a few partial results only could be presented herein. They are the temperature change time history at the centre of the notch of the specimen (Fig. 1) and the absolute temperature and the axial Cauchy's stress distribution over the specimen body (Fig. 2). As can be seen in the figures, there is a significant temperature rise within less than 1.5 s of the analysis, the area of which gradually increases due to heat conduction toward the fixed end of the specimen near the origin of the coordinate system in Fig. 2 where the body is not cooled, while there is an insignificant change in temperature under the moving grips where the body is cooled. The beginning of the temperature time history agrees with the temperature time history reported by Pastor, et. al. (2008), which the authors consider to be very positive.

3. Conclusions

In this paper we have presented some recent developments in the research into fully coupled thermal structural finite element problems with convective heat transfer, radiation heat transfer, material damping within the framework of finite strain elastoplasticity. Cyclic tension of a notched specimen was studied. The analysis results are in agreement with available experiments. The authors consider the results positive and hope that the model might open new perspectives in the study and numerical simulation of metals.

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