20th International Conference
ENGINEERING MECHANICS 2014
Svratka, Czech Republic, 12 – 15 May 2014

INFLUENCE OF REDUCER DISK BALANCE ON FLEXURAL VIBRATION OF PROPELLER SHAFT

M. Hać, W. Ostapski*

Abstract: *In the paper vibration analysis of a propeller shaft of a light airplane piston engine is considered. The bending vibration of the shaft occurs due to unbalanced assembly: reducer disk – shaft as a consequence of machining tolerance. The finite element method is used for determination of the shaft deflection and the geometric non-linear model is considered. The dynamic equations for rotating shafts are developed and the results for bending vibration of the shaft are presented.*

Keywords: Bending vibration, FEM, Propeller shaft, Airplane piston engine.

1. Introduction

The object considered in the present paper is a propeller shaft of a light airplane. The drive unit consists of new-generation, multifuelled airplane piston engine, planetary reducer, and propeller shaft (Ostapski, 2012). The design of the reducer involves the use of a planetary gear, thus a disk with the ring gear is embedded on the propeller shaft (see Fig. 1). For safety reasons it is very important to properly balance this assembly since the shaft speed during operation is high (approx. 2600 rpm). Moreover, during operation the roller bearings develop additional clearance due to wear, which also influences the shaft balance. Therefore it is important to analyze the bending vibration of the shaft due to unbalanced elements.

In order to estimate the possible imbalance of the reducer the machining tolerance of the splined connection of the shaft and the disk is taken into account. The line of deflection of the shaft is determined using the finite element method. The axis of symmetry for the shaft pin and the disk cannot be ideally coaxial (within the area of machining tolerance) and this is the reason for vibrations appearing during rotation of the shaft. These vibrations cause additional deflection of the rotating shaft and influence the gear wheel meshing. This problem was carefully addressed in (Hać, 2005) and the method of calculation of the load distribution corrections along gear width was also presented.

The propeller shaft is designed for torque transmission from engine to the propeller and dominant strains are those from conveying torque. However, the presence of heavy elements – such as a reducer disk – causes the possibility of bending vibration to develop, which is highly dangerous when not controlled.

The model of the propeller shaft presented in Fig. 1 for further consideration is simplified to the form presented in Fig. 2

2. Modelling of Shafts by FEM

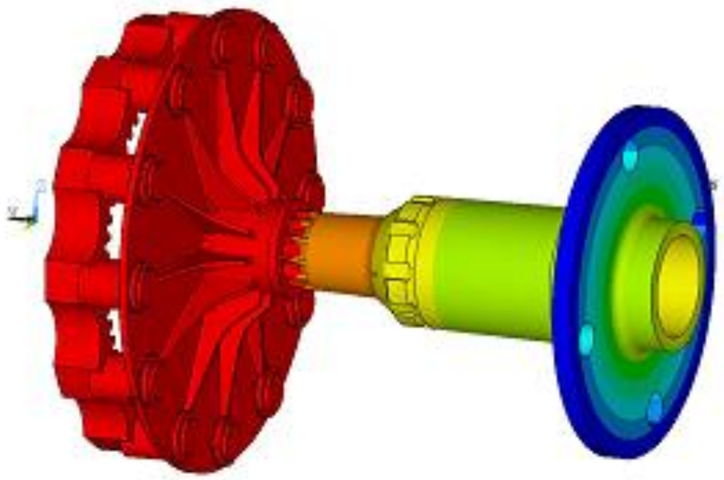
The finite element method is used in order to analyze elastic transverse deformation of the propeller shaft. The planar Bernoulli-Euler beam finite elements are used and the nodal displacement vector consists of both transverse and longitudinal displacements of nodes and nodal angular deformations.

2.1. Modelling of articulated joint (hinge)

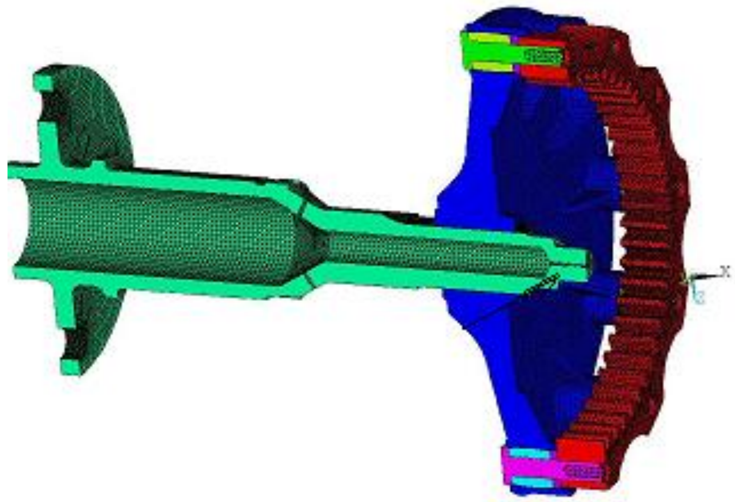
Usually for modelling a hinge joint an additional very short finite element (compared to other elements used in construction) is used. In this way the deformation angle at the left side of the hinge is different

* Dr. hab. Michał Hać, PhD., Prof. Wiesław Ostapski, PhD.: Institute of Machine Design Fundamentals, Warsaw University of Technology, Narbutta 84, 02-524 Warszaw, Poland, mha@simr.pw.edu.pl, wos@simr.pw.edu.pl

than at the right side. In order to avoid introducing additional elements, the node (number 2) at the articulated joint (hinge) is modelled by assuming two independent rotational degrees of freedom (left and right) - see Fig. 3.



a)



b)

Fig. 1: Propeller shaft of airplane engine: a) 3D view; b) Its cross-sectional view.

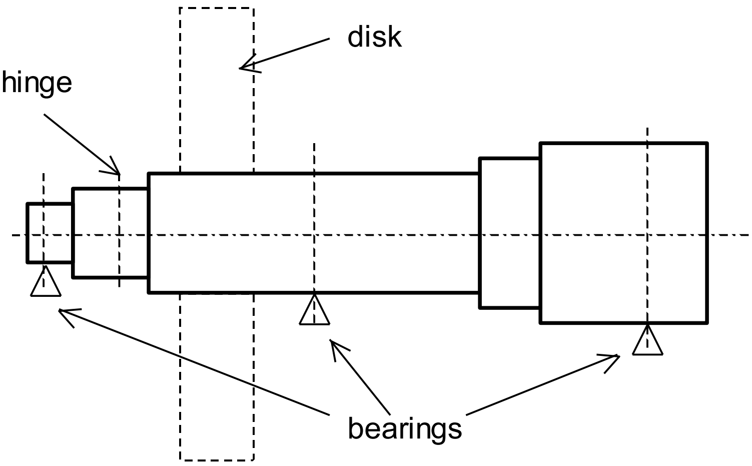


Fig. 2: Model of propeller shaft for finite element analysis.

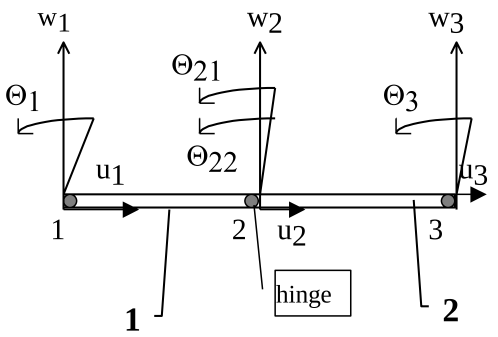


Fig. 3: Displacements of nodes of two finite elements connected by the hinge.

Nodal displacement vectors for elements 1 and 2 connected in the hinge are as follows

$$\{\delta_1\}^T = [u_1, w_1, \Theta_1, u_2, w_2, \Theta_{21}] \tag{1}$$

$$\{\delta_2\}^T = [u_2, w_2, \Theta_{22}, u_3, w_3, \Theta_3] \tag{2}$$

where: Θ_{21}, Θ_{22} are the left and the right angular deformations of the hinge node, respectively.

In this way the articulated joint is modelled in a precise way (no artificial elements in the hinge are used). Moreover, the size of the problem is reduced because the global stiffness matrix compared to the problem without hinge joint is increased by only 1 (i.e. additional deformation angle in the hinge). In the case when additional element is used the size of the problem increases more (co-ordinates of additional node). In most cases longitudinal displacements of nodes of finite element neighboring with the hinge joint are equal – the hinge does not transfer longitudinal force and the neighboring finite elements are not longitudinally loaded. In such case it can be assumed: $u_1 = u_2 = u_3$.

3. Equations of Motion of Rotating Shaft

The equations of motion of rotating shafts can be obtained by any method used in derivation of motion of dynamic systems such as the principle of virtual work, the Gibbs-Appel equations of motion, or Lagrangian equations of motion. The equations of motion of rotating shaft in the global coordinate system can be expressed as follows (Brown and Shabana, 1997):

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [C_g]\{\dot{x}\} + ([K] + [K_c])\{x\} = \{F\} \tag{3}$$

where coefficient matrices are global matrices obtained from appropriate element matrices: $[M_e]$ is the element inertia matrix, $[C]$ is the global damping matrix, $[C_g]$ is the gyroscopic matrix, $[K]$ is the global stiffness matrix, $[K_c]$ is the centrifugal matrix, $\{F\}$ represents generalized forces, and $\{\ddot{x}\}$, $\{\dot{x}\}$and $\{x\}$ represent acceleration, velocity, and displacement vectors (in nodal points). Matrices $[C_g]$ and $[K_c]$ are obtained from element matrices $[C_{ge}]$ and $[K_{ce}]$ defined as follows:

$$[C_{ge}] = 2\omega\rho A\int_0^L [N_e]^T[\Omega][N_e]\mathrm{d}\varsigma , \quad [K_{ce}] = -\omega^2\rho A\int_0^L [N_e]^T[N_e]\mathrm{d}\varsigma \tag{4}$$

where ω is the angular velocity of the shaft, ρ is mass density, A is the cross-sectional area, $[N_e]$ is the beam finite element shape function, $0 \le \varsigma \le L$, L is the length of the beam finite element, and $[\Omega] = [0, -1; 1, 0]$ is the Boolean type operator matrix.

The damping matrix can be presented as a linear combination of the mass and stiffness matrices

$$[C] = \alpha[M] + \beta[K] \tag{5}$$

where α and β are constants to be determined from first few natural frequencies of the system. Usually damping proportional to stiffness is considered (i.e. “structural” damping), and in that case $\alpha = 0$, and β is obtained based on natural frequencies and assuming material damping coefficient for steel in the range of 0.02÷0.03.

In order to calculate the natural frequencies of the system the homogeneous version of equation of (3) is considered. In the further analysis the structural damping (proportional to stiffness) is considered and the damping matrix is formulated as $[C] = \beta[K]$.

4. Numerical Results and Conclusions

Dynamic behavior can significantly influence the deflection of the shaft. In order to conduct the vibration analysis the damping coefficient β for determination of the damping matrix [C] (see Eqn (5)) should be calculated. The coefficient β is calculated based on the first two natural frequencies of the system. For the given data of the propeller shaft the natural frequencies are as follows [Hz]: 39; 99.

In our case of steel shaft the damping ratio ξ_β is assumed to be 0.02 for the first two modes. The coefficient β is calculated from the formula (Rakowski & Kacprzyk, 1993):

$$\beta = 2\xi_\beta /(\omega_1 + \omega_2) \tag{6}$$

For the obtained natural frequencies this yields $\beta = 2.897 \cdot 10^{-4}$.

In the dynamic analysis it is assumed that the imbalance of the shaft-disk assembly results from the shaft and the disk axes not being coaxial (within the assumed tolerance of the joint). Thus additional centrifugal forces act on the shaft and influence its deflection. The deflection graph of the shaft obtained for the highest inertia force possible is presented in Fig. 4.

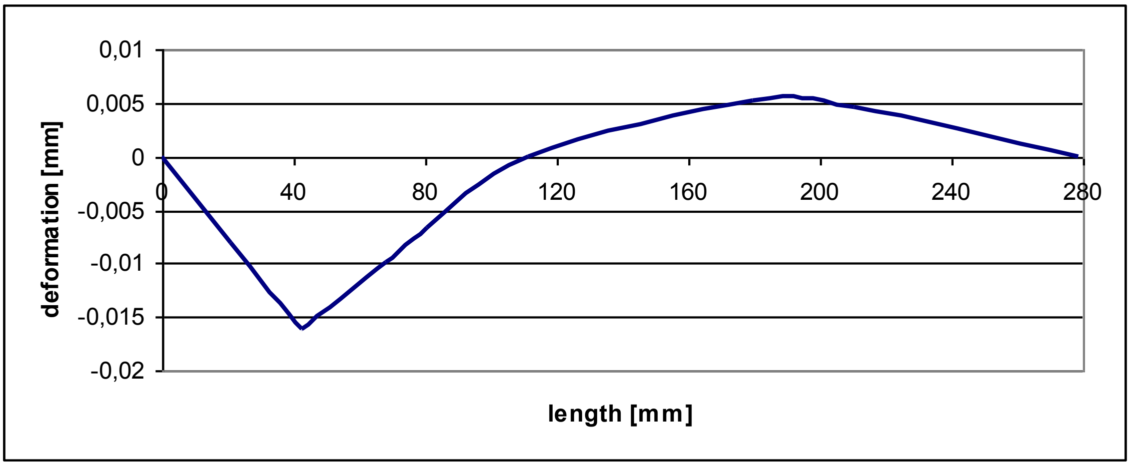


Fig. 4: Shaft deflection.

The results show that, within the given tolerance of shaft – disk connection, significant centrifugal forces (approximately 60 times the weight of the disk) may occur, which cause additional deflection of the shaft. It should be noted that the presented analysis does not contain additional dynamic response due to clearances in bearings. The problem is very important since the bending vibration of shafts is often the reason for destruction of the structure in both piston and jet airplane engines.

References

Brown, M.A., Shabana, A.A. (1997) Application of multibody methodology to rotating shaft problems, Journal of Sound and Vibration, 204, 3, pp. 439-457.

Hać, M. (2005) Influence of static and dynamic deflection of shaft pins on strength parameters of toothed gear. Machine Dynamic Problems, Vol. 29, 1, pp. 65-76.

Ostapski, W. (2012) Report of the target project nr 6ZR62008C/07057: “Developing and implementing eco-friendly, multifuelled new generation airplane piston engine of power N = 200 kW”, Warszaw, (in Polish).

Rakowski, G., Kacprzyk, Z. (1993) Finite element method in structural mechanics, WPW, Warszawa (in Polish).