20th International Conference
ENGINEERING MECHANICS 2014
Svratka, Czech Republic, 12 – 15 May 2014

DEVELOPMENT OF MATHEMATICAL MODEL OF RAIN ZONE

T. Hyhlík*

Abstract: *The article deals with the development of one of the key part of the complex CFD model of natural draft wet-cooling tower flow namely model of rain zone. The physical phenomena occurring in the natural draft cooling tower can be described by the combined Eulerian-Lagrangian model with additional model of film type fill. The Eulerian part is the model of the flow of homogeneous mixture of dry air and water vapour. The Lagrangian part is the model of transport of liquid water droplets. The heat and mass transfer between Lagrangian phase and Eulerian phase is included via source terms in the model of homogeneous mixture of dry air and water vapour. The simplified model of rain zone based on the solution of boundary value problem for the system of ordinary differential equation is also derived.*

Keywords: **Natural draft wet-cooling tower, Eulerian-Lagrangian model, Rain zone, Evaporative cooling.**

1. Introduction

The aim of the wet-cooling tower is transfer of heat from cooled water into the moist air. The cooling of flowing water is connected with warming up of flowing moist air and with increase of its humidity. The density of warmed air is decreasing unlike of surrounding air and this density difference produce natural draft.

Cooled water is sprayed above the fill using the set of sprayers. In the channels of counterflow cooling tower fill water flows vertically down as a liquid film. Air is driven by tower draft and flows in the opposite direction. Evaporation and convective heat transfer cool down the water. The water is leaving the fill zone and falling down in the form of rain to the pond at the bottom part of the cooling tower. This work concentrates mainly on the development of the model of heat and mass transfer in the rain zone. The water droplets are considered as Lagrangian phase and moist air is considered as Eulerian phase.

2. Eulerian Model of Moist Air Flow

Moist air can be considered as homogeneous mixture of dry air and water vapour. Continuity equation for dry air can be written as

$$\frac{\partial(\rho_a)}{\partial t} + \frac{\partial(\rho_a v_k)}{\partial x_k} - \frac{\partial}{\partial x_k}\left[D_{a,v}\,\frac{\partial \rho_a}{\partial x_k}\right] = 0, \tag{1}$$

where $D_{a,v}$ is diffusion coefficient of dry air in the water vapour. We can express the continuity equation of water vapour in the form

$$\frac{\partial(\rho_v)}{\partial t} + \frac{\partial(\rho_v v_k)}{\partial x_k} - \frac{\partial}{\partial x_k}\left[D_{v,a}\,\frac{\partial \rho_v}{\partial x_k}\right] = \sigma_v(x_k,t), \tag{2}$$

where $\sigma_v(x_k,t)$ represents the density of source of vapour and diffusion coefficient of water vapour in the air is $D_{v,a} = D_{a,v}$. The system of momentum equations can be written in the form

$$\frac{\partial(\rho v_i)}{\partial t} + \frac{\partial}{\partial x_k}\left[\rho v_i v_k - \sigma_{ik}\right] = -\rho g \delta_{i3} - \zeta(x_k,t)\rho\frac{|v|^2}{2}\delta_{i3} + \sigma_h(x_k,t)\delta_{i3}, \tag{3}$$

* Ing. Tomáš Hyhlík, PhD.: Department of Fluid Dynamics and Thermodynamics, Faculty of Mechanical Engineering, Czech Technical University in Prague, Technická 4; 166 07, Prague; CZ, tomas.hyhlik@fs.cvut.cz

where $-\rho g \delta_{i3}$ represents gravitational force acting on the flowing moist air, σ_{ik} is the stress tensor in a flowing fluid, ζ represents a loss coefficient per meter of the fill zone and $\sigma_h(x_k, t)$ is momentum source. The energy equation is written in the form

$$\frac{\partial(\rho e)}{\partial t} + \frac{\partial}{\partial x_k}\left[\rho v_k e - \sigma_{kj} v_j + \dot{q_k}\right] = \sigma_q(x_k, t) - g\rho v_3, \qquad (4)$$

where $\sigma_q(x_k, t)$ is density of heat source where the only sensible part of heat source is considered because of the definition of total energy

$$\rho e = \rho\left(c_v T + \frac{v_k v_k}{2}\right) = \left(\frac{r}{\kappa - 1} T + \frac{v_k v_k}{2}\right), \qquad (5)$$

where κ is Poisson coefficient

$$\kappa = \frac{c_p}{c_v}. \qquad (6)$$

Pressure can be expressed using the equation of state as

$$p = \frac{\rho R T\left(1 + \frac{M_a}{M_v} x\right)}{M_a(1+x)} = \rho R T\left(\frac{M_v - w_v(M_v - M_a)}{M_a M_v}\right), \qquad (7)$$

where x is specific moisture and w_v is water vapour mass fraction. Detailed description of the model of the flow of homogeneous mixture of air and water vapour is in reference Hyhlík (2014).

3. Lagrangian Model of Water Droplets

There is flow of moist air with droplets in the rain zone and in the spray zone. Mean diameter of droplet falling down in the rain zone and in the spray zone is in the order of millimeters. It is stated in the reference Kroger (2004) that mean diameter in rain zone is 3.5 mm. Pierce (2007) measured mean diameter 3.22 mm and Sauter mean diameter 5.73 mm in rain zone. Viljoen (2006) measured droplet mean diameter for medium pressure spray nozzles about 2 mm and Sauter diameter was between 2.4 and 3.4 mm in spray zone. For low pressure spray nozzles Viljoen (2006) measured in spray zone mean diameter about 3.2 mm and Sauter mean diameter was about 7.4 mm.

Equation of motion of a single droplet falling down can be derived by using Newton's second law

$$m_d \frac{dv_d}{dt} = -\frac{4}{3}\pi R^3 g \rho_w + c_x(Re)\pi R^2 \rho_{ma} \frac{[v_a - v_d(z)]^2}{2}, \qquad (8)$$

where first term on the right side represents gravitational force and second term is drag force with relative velocity of air to droplet $|v_a - v_d(z)|$, m_d is droplet mass, v_d is droplet velocity, R is droplet radius, g is gravitational acceleration, ρ_w is density of water, $c_x(Re)$ is drag coefficient and ρ_{ma} is moist air density.

3.1. Droplet heat and mass transfer

In the case of dilute mixture the rate of change of mass of single droplet can be expressed as

$$\frac{dm_d}{dt} = -h_m \rho_{Ama} 4\pi R^2 \left(x^{"}\left(t_d(z)\right) - x(z)\right), \qquad (9)$$

where h_m is mass transfer coefficient, ρ_{Ama} is averaged density of moist air, $x^{"}\left(t_d(z)\right)$ is specific humidity of saturated moist air at the temperature of droplet and $x(z)$ is specific humidity of moist air flowing around the droplet. If we assume uniform drop temperature, then convection heat transfer can be expressed using Newton's law of cooling

$$\dot{Q} = \alpha 4\pi R^2\left(t_a - t_d(z)\right), \qquad (10)$$

where α is heat transfer coefficient, t_a is moist air temperature and $t_d(z)$ is droplet temperature. Rate of change of internal energy of single droplet can be then expressed as

$$\frac{d}{dt}(m_d c_w t_d) = m_d c_w \frac{dt_d}{dt} + c_w t_d \frac{dm_d}{dt} = \dot{Q} + \frac{dm_d}{dt} h_v(t_d), \qquad (11)$$

where c_w is specific heat capacity of water and $h_v(t_d)$ is enthalpy of water vapour at droplet temperature. Heat and mass transfer can be evaluated using correlations of Ranz and Marshall (1952)

$$Nu = 2 + 0.6Re^{1/2}Pr^{1/3}, \quad (12)$$

$$Sh = 2 + 0.6Re^{1/2}Sc^{1/3} \quad (13)$$

which are based on Nusselt number Nu, Reynolds number Re, Prandtl number Pr, Sherwood number Sh and Schmidt number Sc.

4. Simplified Model of Rain Zone

The definition of droplet velocity can be used to modify system of equations (8-11) to get similar equations like Fisenko et al. (2002). The equation describing the change in radius can be derived from equation (9)

$$\frac{dR}{dz} = -\frac{h_m \rho_{Ama}}{v_d(z)\rho_w}\left[x^{"}\left(t_d(z)\right) - x(z)\right]. \quad (14)$$

The equation for the velocity of falling droplet is based on equation (8)

$$\frac{dv_d}{dz} = -\frac{g}{v_d(z)} + \frac{3}{8}\frac{c_x(Re)}{R\, v_d(z)}\frac{\rho_{ma}}{\rho_w}[v_a - v_d(z)]^2. \quad (15)$$

The equation for temperature of droplet is based on equation (11)

$$\frac{dt_d}{dz} = \frac{3\left[\alpha(t_a - t_d) - h_m \rho_{Ama}\left[x^{"}\left(t_d(z)\right) - x(z)\right]\left(l_0 + t_d(c_{pv} - c_w)\right)\right]}{v_d(z) R c_w \rho_w}. \quad (16)$$

The density of water droplets per meter of the rain zone can be defined as

$$N_D(z) = \frac{\dot{m}_w}{\frac{4}{3}\pi R_i^3 \rho_w |v_d(z)|}, \quad (17)$$

where $\dot{m}_w$ is water mass flow rate at rain zone inlet and R_i is droplet radius at the inlet. Mass balance of incremental step of rain zone can be expressed from the point of view of flowing moist air as

$$\dot{m}_a \frac{dx}{dz} = -N_D(z) v_d(z) \frac{dm_d}{dz}, \quad (18)$$

where $\dot{m}_a$ is dry air mass flow rate. The change in moist air total enthalpy of incremental step of rain zone can be expressed like

$$\dot{m}_a \frac{dh_{1+x}}{dz} = -N_D(z) v_d(z)\left[\frac{dQ}{dz} + \frac{dm_d}{dz} h_v(t_d)\right], \quad (19)$$

The change in specific moisture can be expressed from equation (18) and by using equation (14) as

$$\frac{dx}{dz} = \frac{4\pi R^2 N_D(z) h_m \rho_{Ama}}{\dot{m}_a}\left[x^{"}\left(t_d(z)\right) - x(z)\right]. \quad (20)$$

The change in moist air temperature is expressed using derivation of moist air enthalpy definition

$$\frac{dt_a}{dz} = \frac{1}{c_{pa} + x c_{pv}}\left[\frac{dh_{1+x}}{dz} - \left(l_0 + c_{pv} t_a\right)\frac{dx}{dz}\right]. \quad (21)$$

After substitution of equation (19), (20), (9) and (10) we get

$$\frac{dt_a}{dz} = \frac{-4\pi R^2 N_D}{\dot{m}_a(c_{pa} + x\, c_{pv})}\left[\alpha(t_a - t_d) + h_m \rho_{Ama}\left(x^{"}(t_d) - x\right)c_{pv}(t_a - t_d)\right]. \quad (22)$$

Equations (14), (15), (16), (20) and (22) form our model of rain zone. We have boundary value problem for the system of ordinary differential equations in rain zone. We should prescribe droplet radius, droplet velocity and temperature on the upper boundary. Moist air temperature and specific humidity should be stated on the lower boundary of the rain zone.

5. Source Terms

From the point of view of Eulerian model given by equations (1), (2), (3) and (4) it is necessary to solve them iteratively together with initial value problem for the system of equations (14), (15) and (16) and prescribe source terms. The density of water vapour source is

$$\sigma_v(x_k,t) = \frac{4\pi R^2 N_D(z) h_m \rho_{Ama}}{A(z)} \left[x^{"}\left(t_d(z)\right) - x(z)\right]. \tag{23}$$

The source in the system of momentum equations can be expressed as

$$\sigma_h(x_k,t) = -c_x(Re)\pi R^2 \rho_{ma} \frac{[v_a - v_d]^2}{2} \frac{N_D(z)}{A(z)}. \tag{24}$$

The density of heat source where only sensible part of heat source is considered is

$$\sigma_{qs}(x_k,t) = \frac{-4\pi R^2 N_D(z)}{A(z)} \left(\alpha(t_a - t_d) - h_m \rho_{Ama} \left(x^{"}\left(t_d(z)\right) - x(z)\right) c_{pv} t_d\right). \tag{25}$$

Because of the definition of total energy (5) the density of heat source should by calculated as

$$\sigma_q(x_k,t) = \sigma_{qs}(x_k,t) + 273.15 c_{pv} \sigma_v(x_k,t). \tag{26}$$

6. Conclusions

The Eulerian-Lagrangian model of moist air flow with droplets falling down is developed. The key part of the proposed model is the calculation of sources of mass, momentum and energy defined by equations (23), (24), (25) and (26). The developed model is an extension of previously developed model of natural draft cooling tower by Hyhlík (2014) where heat and mass transfer in rain zone was omitted. Byproduct of the development is rain zone model given by equations (14), (15), (16), (20) and (22). Practically it is the model of counterflow rain zone. We have boundary value problem for the system of ordinary differential equations in this case. Not least should be mentioned that proposed model is ignoring droplet distribution function and the influence of droplet deformation on the heat and mass transfer. The more complex models will be probably developed in future.

Acknowledgement

This work has been supported by Technology Agency of the Czech Republic under the project Advanced Technologies for Heat and Electricity Production –TE01020036.

References

Hyhlík, T. (2014) Concept of CFD model of Natural Draft Wet-Cooling Tower Flow, submitted to European Physical Journal Web of Conferences.

Kroger, D. G. (2004) Air-Cooled Heat Exchangers and Cooling Towers, Pen Well Corporation, Tulsa.

Pierce, D. J. (2007) Evaluation and Performance of Cooling Tower Rain Zones, M.Sc. Thesis, University of Stelllenbosch.

Viljoen, D. J. (2006) Evaluation and Performance Prediction of Cooling Tower Spray Zones, M.Sc. Thesis, University of Stellenbosch.

Ranz, W. E., Marshall, W.R. (1952) Evaporation from Drops, Part I, Chemical Engineering Progress, Vol. 48, No. 3, pp. 141-146.

Fisenko, S. P., Petruchic, A. I., Solodukhin, A. D. (2002) Evaporative cooling of water in a natural draft cooling tower, International Journal of Heat and Mass Transfer, Vol. 45, pp. 4683-4694.