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EFFICIENT BAYESIAN PARAMETER IDENTIFICATION

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Abstract: *Many important parameters influencing structural behaviour involve unacceptable uncertainties. An extensive development of efficient methods for stochastic modelling enabled reducing these uncertainties in input parameters. According to Bayes' rule, we obtain a more accurate description of the uncertain parameter involving an expert knowledge as well as experimental data. The aim of this contribution is to demonstrate two techniques for making the identification process more efficient and less time consuming. The first technique consists in replacement of the full numerical model by its polynomial approximation in order to reduce the computational effort. The particular approximation is based on polynomial chaos expansion constructed by linear regression based on Latin Hypercube Sampling. The obtained surrogate model is then used within Markov chain Monte Carlo sampling so as to update the uncertainty in the model inputs based on the experimental data. The second technique concerns a guided choice of the most informative experimental observation. Particularly, we apply sensitivity analysis to determine the most sensitive component of the structural response to the identified parameter. The advantages of the presented approach are demonstrated on a simple illustrative example of a frame structure.*

Keywords: Bayesian identification, Markov chain Monte Carlo, Stochastic modelling, Polynomial chaos expansion, Sensitivity analysis.

1. Introduction

Bayesian approach allows us to reduce uncertainties in parameters influencing structural reliability such as material or structural properties. The principal idea of Bayesian identification is based on a common way of thought when the resulting belief about a random event is given by a combination of all the available information (Gelman et al., 2004). An initial expert knowledge formulates prior distribution of the uncertain parameters and likelihood function contains new data obtained from experiments. The result of Bayesian identification is posterior distribution whose formulation includes the whole structural model. For this reason, the corresponding statistical moments cannot be generally computed analytically, but their computation can be done by Markov chain Monte Carlo (MCMC), see e.g. Geyer (2011). The disadvantage of this method is its high computational effort resulting from necessity of high number of model simulations. In order to accelerate the sampling procedure, a polynomial approximation of the model response can be used instead of the full numerical model.

Here, we employ polynomial chaos expansion (PCE) for the approximation of the model response in the stochastic space (Matthies, 2007; Stefanou, 2009; Marzouk et al., 2007). The efficiency of this technique depends on computational requirements of the PCE construction and its consequent accuracy. PCE can be used to approximate the response with respect to the probability distribution of the random variables. The convergence of the approximation error with the increasing number of polynomial terms is optimal in case of orthogonal polynomials of a special type corresponding to the probability distribution of the underlying variables (Xiu & Karniadakis, 2002). In particular, we employ Hermite polynomials associated with the Gaussian distribution. A comparison of several methods for the construction of the PCE-based approximation of a model response can be found in Janouchová et al. (2013). In this contribution we focus on linear regression (Blatman & Sudret, 2010), which is a very general method of

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computing the PCE coefficients based on a set of model simulations. The samples are drawn according to a stratified procedure called design of experiments (DoE), in particular well-known Latin hypercube sampling (LHS) which is able to respect the prescribed probability distributions (Janouchová & Kučerová, 2013).

Global sensitivity analysis (SA) is an important tool for investigating properties of complex systems. It is a valuable part of solution of an inverse problem such as a parameter identification. SA provides some information about the relationship between the system parameters/model inputs and the system response/model outputs. Several approaches to SA have been developed, see e.g. Saltelli et al. (2000) for an extensive review. The presented contribution is focused on widely used sampling-based approaches (Helton et al., 2006), particularly aimed at an evaluation of Spearman's rank correlation coefficient (SRCC). The aim of sensitivity analysis in the identification process is to determine the most sensitive component of the structural response to the identified parameter.

2. Bayesian Approach

Consider a stochastic problem

$$\boldsymbol{z} \approx M(\boldsymbol{m}) \tag{1}$$

with uncertain model parameters $\boldsymbol{m}$ and random observable data $\boldsymbol{z}$, which can be predicted by a function M of the parameters. In Bayesian statistics, probability represents a degree of belief about the values of the parameters. Combining the initial knowledge in the form of the prior distribution p($\boldsymbol{m}$) and the experimental data as the likelihood function p($\boldsymbol{z}$ | $\boldsymbol{m}$) according to Bayes' rule

$$\mathrm{p}(\boldsymbol{m} \mid \boldsymbol{z}) = \frac{\mathrm{p}(\boldsymbol{z} \mid \boldsymbol{m})\mathrm{p}(\boldsymbol{m})}{p(\boldsymbol{z})} = \frac{\mathrm{p}(\boldsymbol{z} \mid \boldsymbol{m})\mathrm{p}(\boldsymbol{m})}{\int_{\boldsymbol{m}} \mathrm{p}(\boldsymbol{z} \mid \boldsymbol{m})\mathrm{p}(\boldsymbol{m})\mathrm{d}\boldsymbol{m}}, \tag{2}$$

we obtain the posterior distribution of the parameters.

3. Illustrative Example

In order to demonstrate a performance of Bayesian identification on an engineering structure, we have chosen a simple frame presented in Marek et al. (2003). The geometry and load distribution of the frame are shown in Fig. 1.

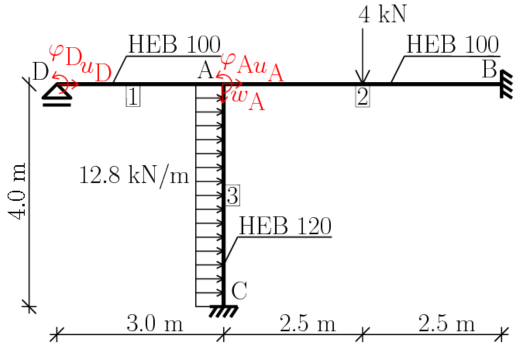


Fig. 1: Scheme of a frame structure.

Material of the frame is steel with uncertain Young's modulus E. Prior of this parameter is lognormal distribution with parameters μ_m = 210 GPa and σ_m = 110 GPa. Likelihood is based on the displacements and their normally distributed measurement errors.

3.1. Markov chain Monte Carlo with polynomial chaos expansion

MCMC is based on a creation of an ergodic Markov chain of required stationary distribution equal to the posterior. There are different algorithms for the construction of this chain (Spall, 2003). In this contribution we employ Metropolis algorithm with a symmetric proposal distribution. Suitable setting of the proposal distribution is important and can be evaluated on the basis of acceptance rate AR (Rosenthal, 2011) or autocorrelation which is required to be minimal, as you can see in Fig. 2.

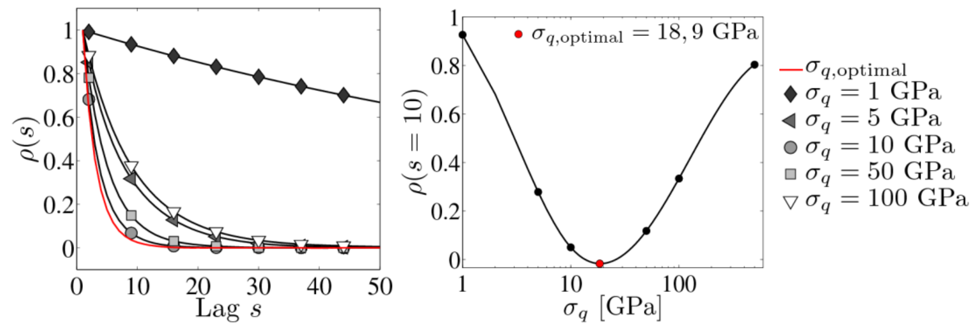
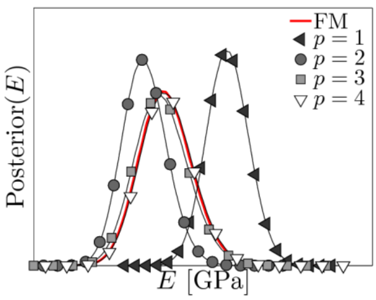


Fig. 2: A choice of the proposal standard deviation σ_q according to autocorrelation of the chain.

The computational effort of MCMC is reduced by using the polynomial approximation. A comparison of the time requirements and resulting accuracy of PCE and the full model (FM) is in Fig. 3.



MCMC	AR	Time [s]	μ_X [GPa]	σ_X [GPa]
FM	0,447	4693	221,55	7,879
LHS	0,449	755	221,92	7,941

Fig. 3: Posteriors based on PCE of a degree p and FM. Numerical results for PCE of p = 4 and FM.

3.2. Sensitivity analysis

The remaining question concerns a choice of model response component to be measured. The answer can be obtained with the help of SRCC-based sensitivity analysis due to the monotonic dependence of the outputs on *E*. The examples of obtained results are depicted in Fig. 4.

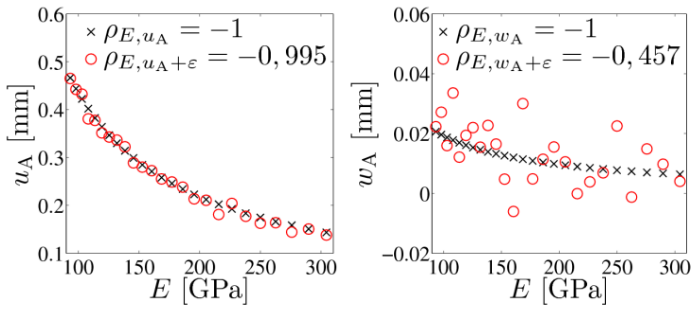
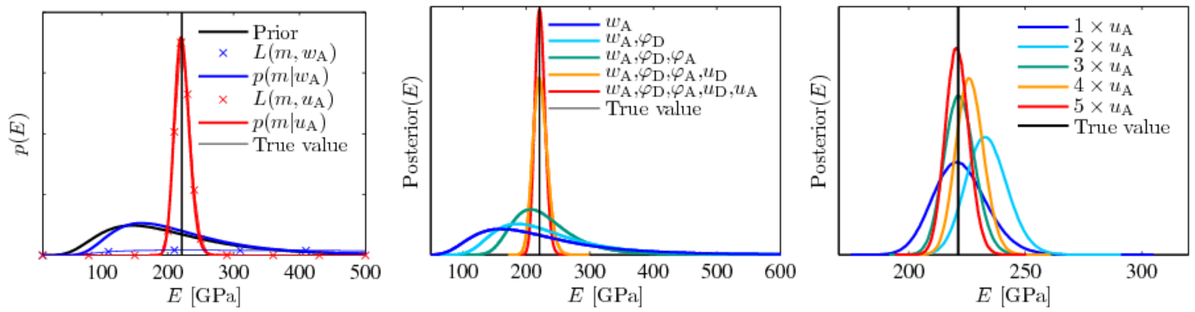


Fig. 4: Sensitivity expressed by SRCC in the case of the most and least sensitive displacements.

If we measure the exact values of the displacements, SRCC is the same in the both cases. By taking into account the measurement errors – assumed to have one value equal for all the displacements and another value equal for all the rotations – SRCC is lower in the case of the displacement with a smaller prior variance. The most sensitive structural response is then displacement u_A.

3.3. Parameter identification

Final characteristics of the posterior distribution strongly depend on the mentioned options. Fig. 5 shows results for several cases of input data for the identification process. In each graph there is a vertical black line denoting the true value of the parameter corresponding to the experiments. A significant difference in using the most or least sensitive output can be seen in Fig. 5a. The mean value of the red posterior belonging to the most informative data practically equals to the true value and the variance expresses the uncertainty which we have about this value. In the second case, the identification is not efficient and our uncertainty remains great. There are shown two variants of parameter updating in the next graphs. Fig. 5b shows a case with sequential measuring one to five of different components of the structural response while Fig. 5c presents a variant with measuring the same displacement one to five times. The both cases lead to a considerable reduction of the uncertainty in the input parameter.



a) *b)* *c)*

Fig. 5: Posteriors based on differently informative data a), one to five variant observations b) and one to five observations of the same variable c).

4. Conclusions

Bayesian identification provides a simple and effective approach for minimization of the uncertainty in the input parameters using all the available information. In order to obtain statistical moments of the posterior, Markov chain Monte Carlo is used. The computationally exhaustive process is accelerated by replacing the model response by its polynomial approximation. The coefficients of polynomial chaos are computed with the help of linear regression based on LHS. The efficiency of the identification depends on the choice of observed data. The best input data for the identification are chosen according to sensitivity analysis based on Spearman's rank correlation coefficient.

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