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REDUCTION FOR HIGH-RISE BUILDINGS SEISMIC ANALYSIS

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Abstract: *This paper presents reduction as a special case of static condensation. The reduction was designed for checking seismicity of high-rise buildings. A reference point is assigned to each floor. Mass matrix and the stiffness matrix of the original structure are reduced to these reference points. The presented procedure is based on the requirements of design codes and engineering practice.*

Keywords: Modal analysis, Dynamic reduction, Seismicity, High-rise buildings, Sensitivity analysis.

1. Introduction

The seismic design of buildings is a part of the standard design procedures in many countries nowadays, even those with low or moderate seismic risk. Design codes (Eurocode EC-EN 1998, 2004) are increasingly demanding about it and efficient analysis tools are therefore necessary. The increasing complexity of structures requires the use of 3D finite element modelling to provide realistic distribution of design internal forces in complex shear-resisting systems. Most design codes recommend – or, in some cases, even request – the use of the response spectrum method, which implies a modal analysis of the structure.

Using a full 3D modelling of a building for dynamic analysis tends to be computationally heavy. Moreover, the FE discretization of all structural members introduces many DoF in the modelling which correspond to local eigenmodes and are uninteresting regarding the overall seismic behavior of the structure. Those local modes mostly correspond to very small parts of the modal mass of the structure, but their important number makes it difficult to reach the typical 90% of cumulated modal mass within a reasonable number of modes. The computation of a very high number of modes might thus be required in order to ensure that all relevant modes are indeed taken into account in the modal superposition, thus making the process unaffordable.

Another typical issue is the so-called accidental eccentricity. Design codes require that some mass eccentricity is introduced in the analysis to account for an irregular distribution of the non-structural masses (dead and live loads). As classical modal analysis uses a diagonal mass matrix to reduce the computation effort, that eccentricity cannot be taken into account directly in a full 3D FE analysis.

The Improved Reduced System technique presented in this paper addresses directly the issues above. The reduced system is directly generated from a classical 3D FE mesh, which means that no dedicated input is necessary for it, apart from the actual definition of the reference points for each storey. Not only the floor slabs, but also all supporting members are reduced at once and mapped to the closest reference point, leading to a reduced model with only 6 DoF per storey. This extremely compact analysis model automatically leads to a solution including only relevant eigenmodes for the seismic behaviour. After resolution, the end results (displacements, stresses) can be expanded back to the original FE mesh, providing detailed output in the entire structure. Additionally, the use of a full mass matrix for the reduced model allows for implicit mass eccentricity without modifying the geometry of the system. Finally, because of its ideal topology, the reduced system can provide directly results at the mass center of each storey, such as displacements, accelerations and inter-storey drift. Those are required for the seismic assessment of a building.

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2. Method

The proposed dynamic reduction should meet the following conditions. The resulting eigenmodes shall approximate significant eigen modes of the original building. With the classical reduction to the point there is a not realistic concentration of stresses (Guyan, 1965, O'Callahan et al., 1989). This is very inconvenient for the following checks seismicity. Other methods are more accurate in loading from seismicity (O'Callahan et al., 1989, Paz, M. 1984, and their modification). The resulting reduced system should also respect the mass distribution along the structure reduced to points at each storey. It is very suitable for the following sensitivity analysis and close to engineering practice. However, the mentioned methods partially conceal it. The chosen process is illustrated in Fig. 1. Most of the original node structure is allocated to the floor. The method does not require the identification of all nodes especially between floors. A reference point is chosen for each storey. This point represents the movement of the storey as a whole.

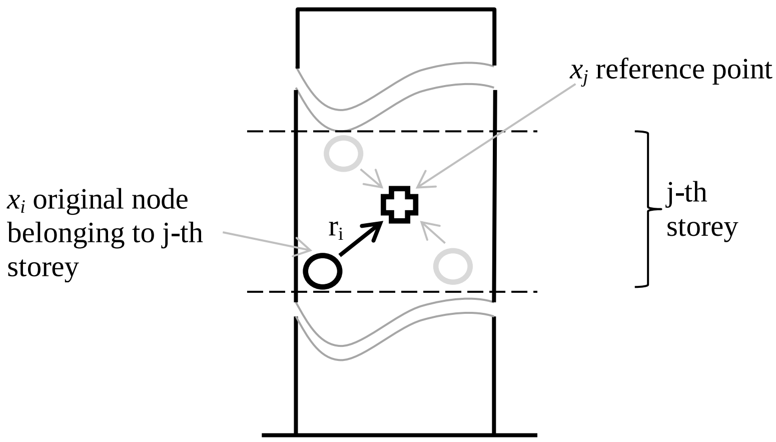

Fig. 1: Schema of method.

2.1. Main reduction

Inputs are symmetric sparse positive definite stiffness matrix **K** and sparse symmetric mass matrix **M**, which are determinated by classic FEM model over original construction. Third input is sparse matrix **H**. Matrix H defines linear relationships between forces in reduced DOF $\mathbf{F_r}$ and forces in original DOF $\mathbf{F_o}$ by following equation:

$$\mathbf{F_O} = \mathbf{H.F_r} \tag{1}$$

Using the matrix **H** does not imply any artificial reinforcement structure. Moreover, it may even better describe the structure load caused by its own mass without any concentration. If we use Lagrange multiplier, we can write the following system of equations as a special case of static condensation. Where $\mathbf{u_r}$ is deformation in reference node and $\mathbf{u_o}$ is deformation on original construction.

$$\begin{bmatrix} \mathbf{K} & -\mathbf{H} \\ -\mathbf{H}^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u_o} \\ \mathbf{F_r} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{u_r} \end{bmatrix} \tag{2}$$

$$\mathbf{F_r} = \left(\mathbf{H.K}^{-1}\mathbf{.H}\right)^{-1} = \mathbf{K_r.u_r} \tag{3}$$

$$\mathbf{u_0} = \mathbf{K}^{-1}\mathbf{.H.K_r.u_r} = \mathbf{T.u_r} \tag{4}$$

Using the transformation matrix **T,** reduced mass matrix can be obtained.

$$\mathbf{M}_r = \mathbf{T}^T\mathbf{.M.T} \tag{5}$$

Booth reduced matrix $\mathbf{M_r}$, $\mathbf{K_r}$ are full and small. Now it is possible to solve reduced eigenvalue problem over reduced matrixes.

$$\{\mathbf{M_r},\mathbf{K_r}\} \Rightarrow \{\mathbf{\Lambda},\mathbf{V_r}\} \tag{6}$$

And eigen vectors **V** of original construction can be approximated by transformation matrix **T** and eigen vectors of reduced problem $\mathbf{V_r}$ at final.

$$\mathbf{V} = \mathbf{T}.\mathbf{V_r} \tag{7}$$

2.2. Relation matrix H

Creating the matrix H is based on the following simplifying assumption. We reduce the original nodes x_i belonging to that storeys to the storey reference point (see Fig. 1.). The acceleration of each relevant original node is dependent on the acceleration of the reference node as in rigid body. This can be written classically by a kinematic relationship (where $\hat{\mathbf{r}}$ is antisymmetric matrix representing vector $\boldsymbol{r}_i$. at Fig. 1).

$$\ddot{\mathbf{x}}_i = \begin{bmatrix} \mathbf{E} & \hat{\mathbf{r}} \\ \mathbf{0} & \mathbf{E} \end{bmatrix} \ddot{\mathbf{x}}_j = \mathbf{R_i}\ddot{\mathbf{x}}_j \tag{8}$$

For inertia force in the original node x_i caused by acceleration in the reference node can be written

$$\mathbf{F_i} = \mathbf{M_i}.\mathbf{R_i}.\ddot{\mathbf{x}}_j \tag{9}$$

Force in reference node caused by acceleration in reference node is given by summation of forces in original node.

$$\mathbf{F}_j = \sum_i \mathbf{R_i}^{\mathbf{T}}.\mathbf{F_i} = \left(\sum_i \mathbf{R_i}^{T} \mathbf{M_i}.\mathbf{R_i} \right) .\ddot{\mathbf{x}}_j = \mathbf{J_J}.\ddot{\mathbf{x}}_j \tag{10}$$

Exclude $\ddot{\mathbf{x}}_\mathbf{j}$ from (10) and substitute to (9).

$$\mathbf{F_i} = \mathbf{M_i}.\mathbf{R_i}.\mathbf{J}_j^{-1}.\mathbf{F_j} = \mathbf{H_{ij}}.\mathbf{F_j} \tag{11}$$

This is founded part of matrix **H**, which represents relationship between j-th reference node and i-th reduced node and respects mass distribution over construction.

2.3. Some remarks about sensitivity analysis

Each floor can have a variable center of gravity from operational mass. The resulting eccentricity up can lead to significant torsional load of the building. It is reflected also in building design codes. To check the structural element it is necessary to select the most adverse mass distribution. In practice this leads to the evaluation of the many mass combinations. It is possible to use the reduced stiffness matrix $\mathbf{M_r}$ here. It is not difficult to find a sensitivity of the reduced mass matrix to change the center of gravity Δx_{tj} of each floor. Linear estimation can be applied for the modification.

$$\tilde{\mathbf{M}}_\mathbf{r} = \mathbf{M_r} + \Delta x_{tj} \frac{\partial \mathbf{M_r}}{\partial x_{tj}} \tag{12}$$

Now we can use a sensitivity analysis (for example in Choi and Kim, 2005).

$$\left\{ \mathbf{M_r}, \frac{\partial \mathbf{Mr}}{\partial x_{tj}} \mathbf{K_r}, \mathbf{\Lambda}, \mathbf{V}_r \right\} \Rightarrow \left\{ \frac{\partial \mathbf{\Lambda}}{\partial x_{tj}}, \frac{\partial \mathbf{V}_r}{\partial x_{tj}} \right\} \tag{13}$$

The resulting sensitivity of eigenvalues can be used to quickly find the extreme load of the part of structure.

3. Example

This 7-storey building is a part of the training centre of the ACPC in Fribourg (Switzerland). The modal behaviour of the original, full mesh model and that of the reduced system have been compared. The frequencies and modal masses of the most relevant eigenmodes are listed in the table below. The corresponding eigenshapes (not represented here) show excellent concordance for low order modes (see modes 1-2-3 in the table). For higher order modes, the full mesh analysis tends to exhibit local vibrations and spread modal mass across several modes, whence the reduced model tends to more smooth the response and group similar behaviours and corresponding modal mass. Internal forces in the main shear walls after modal superposition exhibit no significant differences. As it is a real building with an important structural eccentricity, all relevant modes have combined translational and torsional behaviour.

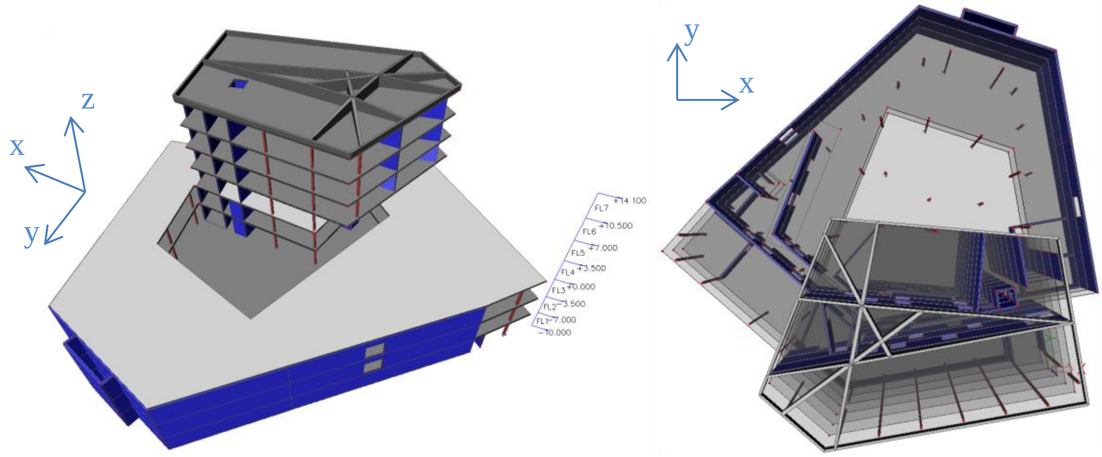

Fig. 2: Example of building.

Eigenvalue	Original model				Reduced model			
	Freq [Hz]	***Modal mass [%]***			***Freq [Hz]***	***Modal mass [%]***		
		X	***Y***	***Rz***		***X***	***Y***	***Rz***
Translation Y	***1.71 (1)***	***0.3***	***17.9***	***5.1***	***1.71 (1)***	***0.3***	***17.9***	***5.1***
Translation Y	***2.44 (2)***	***5.1***	***11.6***	***1.8***	***2.44 (2)***	***5.1***	***11.6***	***1.8***
Translation X	***3.09 (3)***	***30.0***	***2.4***	***10.7***	***3.10 (3)***	***30.7***	***2.6***	***10.9***
Translation X	***7.12 (21)***	***11.2***	***2.5***	***1.1***	***8.06 (15)***	***11.5***	***11.6***	***0.2***
Torsion Rz	***12.83 (92)***	***0.2***	***0.1***	***10.2***	***12.85 (30)***	***9.9***	***3.4***	***29.9***

4. Conclusions

The method corresponds to engineering practice of dividing the building into individual floors. It reflects the requirements of design codes. The reduced model can directly obtain mass characteristics, the relative motion and acceleration of the floors. In addition, modification of the position of center of gravity of the floor in the calculation model is easy. The reduced model can be directly used for seismic analysis and next sensitivity analysis. More accurate methods of reduction are known. However, numerical simulations confirmed its sufficient robustness for real buildings with low or moderate seismic risk. The presented method has been implemented in the CAE software SCIA Engineer for building applications.

Acknowledgement

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