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PERFORMANCE COMPARISON OF LOCALIZATION LIMITERS

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Abstract: *It is well known, that simulation of crack propagation using the finite element method is dependent on mesh discretization. The contribution compares two approaches that are designed to reduce the mesh influence: (I) the crack band model and (II) the nonlocal model. These localization limiters are applied to simulate three-point-bent beam with and without notch. The model of the beam is made with several variants of mesh discretization differing in finite element size and inclination. Performance of both localization limiters is discussed.*

Keywords: Quasi-brittle materials, Strain softening, Localization, Crack band model, Nonlocal model.

1. Introduction

Behaviour of quasi-brittle materials is usually represented in FEM software by material models with strain-softening. When using such a model, after reaching the tensile strength, strain continues to increase while stress decreases. In consequence, crack localizes into a band of width of one finite element and computed results become dependent on mesh discretization. There are several approaches that try to avoid such an unwanted behaviour. The most known are the Crack band model (Bažant & Oh, 1983) and Nonlocal model (Jirásek, 1998). Both approaches should theoretically ensure correct energy dissipation during the crack propagation and therefore also similar load-deflection response of FEM models irrespectively of chosen discretization.

Desired independency is unfortunately hard to achieve in reality, especially when the crack is inclined from the mesh direction. This contribution shows performance of both localization limiters on simulation of three-point-bending test with and without central notch using several variant of mesh discretization.

2. Model of the Beam

The approaches are compared on a model of the three-point-bent beam. The beam dimensions are following: span s = 400 mm, total length $l = 1.1\times s$ = 440 mm, depth and thickness are both $D = t$ = 100 mm. If notch is present, its depth is 1/3 of beam depth D.

Three element meshes differing in density of the discretization were generated for each beam variant (with and without the notch). The element size was chosen as 10, 5 and 2.5 mm, respectively. Another three element meshes for each beam variant were made to study effect of inclined mesh, and were inclined by angle of 30, 45 and 60 degrees. The element size for the inclined meshes was 5 mm. Some of the meshes are shown in Fig. 1.

The loading was done via prescribed deformation at two nodes above the beam center. The two nodes were necessary to ensure symmetric boundary condition. The total loading force F was measured as sum of forces at the loaded nodes, deflection d was taken as an average of vertical movement of two nodes at the bottom surface at the midspan.

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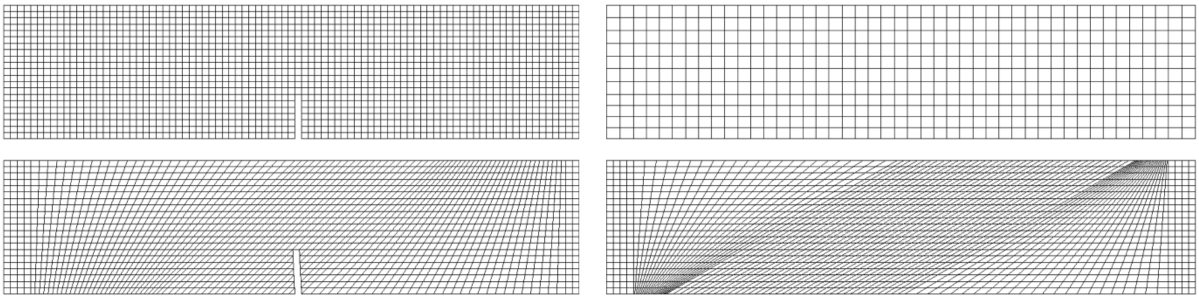


Fig. 1: Examples of straight and inclined meshes with and without notch.

Material parameters were chosen to represent behaviour of concrete in tension: Young's modulus E = 30 GPa, tensile strength f_t = 2.5 MPa, Poisson`s ratio ν = 0.18, linear strain-softening represented by isotropic damage variable. Self weight was omitted in this study. Equivalent stress was defined according to Mazars (1984)

$$\bar{\varepsilon} = \sqrt{\sum_{I=1}^{3} \langle \varepsilon_I \rangle^2} \quad (1)$$

where ε_I are principle strains and brackets $\langle \cdot \rangle$ returns positive part of the argument inside. FEM analyses were computed in open-source program Oofem (Patzák & Bittnar, 2001).

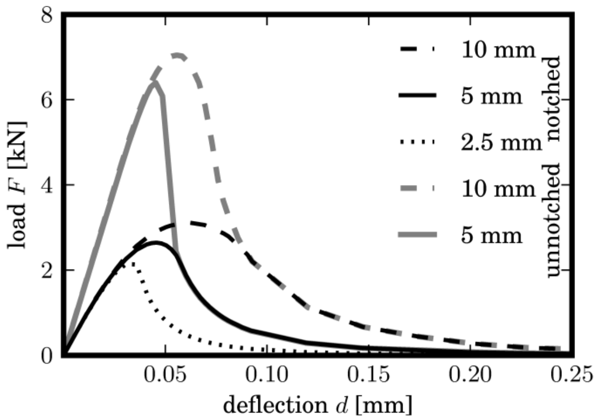
3. Demonstration of Discretization Density Effect

To demonstrate the influence of localization, the first study was performed using local constitutive law with constant value of the final strain ε_f = 0.004, which is the strain that corresponds to fully opened crack. Fig. 2 you can see that the response is different for the different finite element discretization density. Load-deflection diagrams obtained from unnotched (notched) beam simulations are shown in gray (black) color, respectively. Both peak loads and descending parts of the diagrams are different for different mesh densities. Demand for more efficient model is obvious.

4. Crack Band Model

The first tested remedy is the *crack band* model developed by Bažant & Oh (1983). The crack band model does not actually eliminate the localization; it just helps us get rid of the localization influence on the results. The crack still propagates through the thin band of one layer of finite elements. The main idea is to ensure the constant value of energy dissipated in the unit of the area. This constant value is called fracture energy G_f [N/m^2] and it is understood as a material parameter. The energy dissipated in one finite element must be equal to the fracture energy multiplied by the area of the finite element. The final strain in the constitutive law is not constant anymore and it is dependent on the fracture energy and the element width. For the linear softening, it can be calculated as

$$\bar{\varepsilon}_f = \frac{2\,G_f}{h_b f_t}, \quad (2)$$



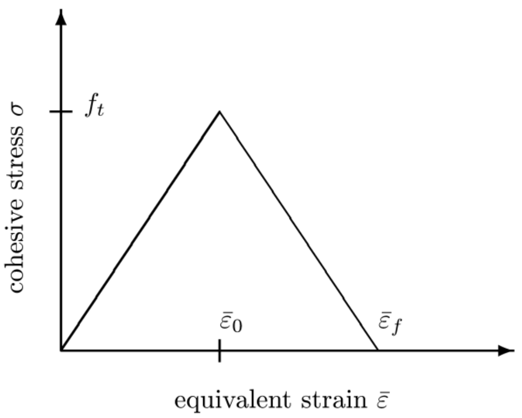


Fig. 2: Dependency of local material model with constant final strain.

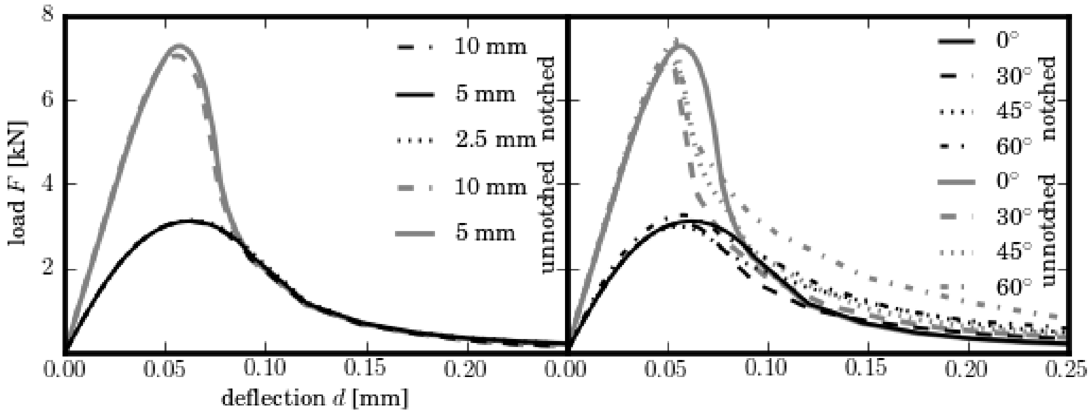


Fig. 3: Crack band model applied on meshes differing in element size and inclination.

where h_b is the band width. When crack is aligned with mesh direction, the width can be taken as size of the finite elements. But the crack does not always propagate in the same direction as the finite elements are aligned. In such cases, the band width must be artificially estimated (Jirásek & Bauer, 2012).

The *crack band* model was applied on the same three point bended beam with the same set of mesh geometries. In addition, the performance of the crack band model was verified on set of inclined mesh geometries. Fracture energy was chosen to correspond to constitutive law of the 10 mm element from Sec. 2; its value was 50 N/m^2. The results are shown in Fig. 3, the left hand side shows load-deflection diagrams for various mesh densities, whereas the right hand side displays results on meshes with different inclination angle.

Performance of the *crack band* model on aligned meshes is excellent; curves for different discretization densities almost coincide. Poorer results are obtained for inclined meshes, where calculation of the band width is not that simple. Peak load is not affected by the inclination at all.

5. Nonlocal Model

The nonlocal model does not allow the crack to localize into band of one element width. It enforces the crack to propagate through zone of constant width irrespectively of meshing. The constitutive law is not modified, it has constant final strain ε_f. However, the nonlocal equivalent strain is used and it is computed as an average of equivalent strains using some weight function α_0. The weight function can be any function that decreases with increasing distance. Besides different type of weight functions, there are also nonlocal models that average other variable instead of equivalent strain, such as damage, stress, etc.

In this contribution, nonlocal equivalent strain is considered; weight function was chosen bell shaped function according to

$$\alpha_0(s) = (1 - s^2/R^2)^2, \tag{3}$$

where s is distance and R is range of the weight function. For any point outside of the range ($s > R$), the value of the weight function is considered as zero. The function needs to be normalized to ensure, that the sum of its values is equal to 1. The nonlocal equivalent strain is then calculated from

$$\tilde{\varepsilon}(x) = \sum_V \alpha\,(s_i)\bar{\varepsilon}(\gamma_i) \tag{4}$$

where x is coordinate of the examined point and γ_i is coordinate of any point in the range of weight function

The range R was chosen as 10 mm. The final strain in constitutive law in $\tilde{\varepsilon}$f = 0.002 was chosen to provide the response comparable with the crack band model results. We applied the nonlocal model on the same geometry as was done with the crack band model. The performance on the aligned meshes (left part of the Fig. 4) is worse than what we got with the crack band localization limiter. For inclined meshes, the results look more or less the same as those provided by the crack band model. For 60 degrees inclination angle, quite poor agreement is found.

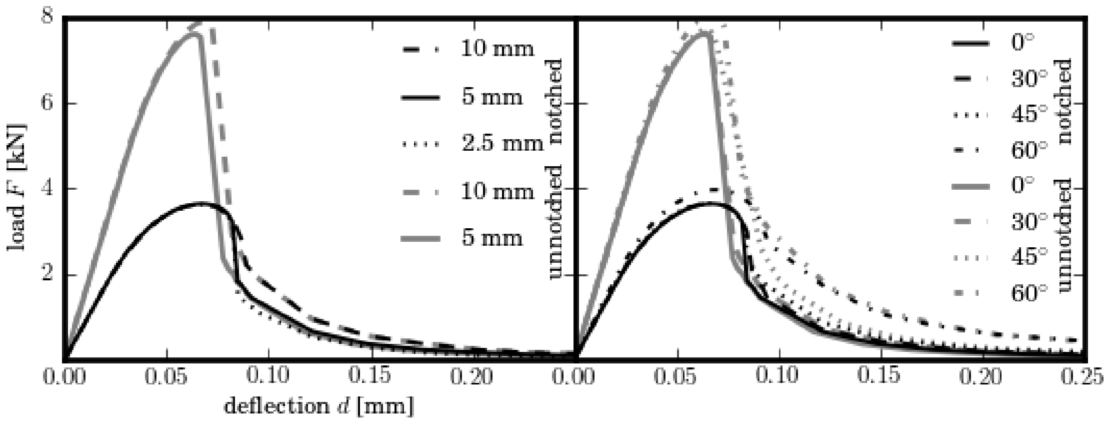


Fig. 4: Nonlocal model applied on meshes differing in element size and inclination.

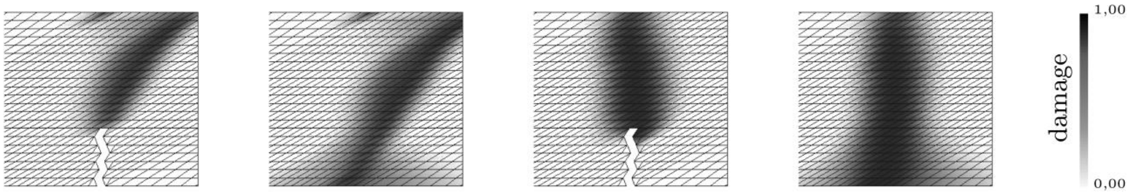


Fig. 5: Crack patterns for Crack band (left) and Nonlocal model (right) for notched and unnotched beam with mesh inclined by 60 degrees.

Besides the load-deflection curves, the crack path was compared as well. Fig. 5 displays damage variable at the end of the simulation. There are only shown central parts of the beams, the depth is not trimmed. Two figures in the right were computed using the nonlocal model, other two figures at the left by crack band model. Crack band allows the localization and the damaged band goes through only a few elements. The nonlocal model avoids such localization. In case of the crack band model and inclination angle 60 degrees, the crack pattern is spuriously skewed along the mesh orientation.

6. Conclusion

Both approaches provide reduction of dependency of the results on the finite element size. When inclined mesh is used, independency of the response seems to be more efficiently provided by the *nonlocal* formulation, especially when considering the crack pattern. On the mesh with inclination angle of 60 degrees, the crack spuriously propagates along the mesh orientation when using the *crack band* model.

Acknowledgement

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