

## DROPWISE CONDENSATION MODELING

P. Beňovský\*, H. Rusche\*\*, F. Ridzoň\*\*\*, M. Popovac\*, Ch. Reichl\*, T. Fleckl\*

**Abstract:** *The present paper describes an approach to the numerical modeling of the condensation on the cooled surfaces of the air driven heat pump heat exchangers. The approach is based upon the classical nucleation theory and on a continuous growth law which are the necessary inputs for the simplified population balance equation solved by the quadrature method of moments. An example result is presented where one of the models' parameters: the driving force, is varied. The influence of the driving force on the condensate rates of a simple condensing heat exchanger is shown.*

**Keywords:** Condensation, Droplet nucleation, Growth, CFD, QMOM.

### 1. Introduction

To date different methods have been developed to calculate the amount of the precipitating condensate on a cooled surface. These methods (VDI, 2002) are based usually upon the Nusselt theory, Nusselt (1916). The main advantage of the calculations using the Nusselt theory is, that it gives a relatively fast indication of the performance of a condensing heat exchanger at steady state and usually nominal load. The drawback of the method, however, is that it is 1-D model with certain model assumptions like presumed geometry, steady flow, negligible vapor velocity, etc. Therefore, if one is interested in the transient behavior of the flow or wants to take into account the full geometrical features of the modeled domain, the Nusselt theory based calculation may not be appropriate anymore. Moreover, reviewed literature Song et al. (2009) and the experiments conducted at AIT, Pauschenwein et al. (2010) indicate that the initial stage of the condensation begins by the nucleation of the initial droplets and subsequent growth, all of which happens in the dropwise regime. Therefore, we have decided to adopt a different approach, namely to develop a numerical model coupled with a CFD code based upon sound physical modeling. For its implementation we have chosen the open source CFD library OpenFOAM® due to the code transparency and performance.

### 2. Methods and Results

In order to get a more detailed insight into the condensation kinetics during its initial stages we developed a numerical model containing both initial phases of droplet nucleation and growth. The choice to investigate the initial stages of the condensation is not arbitrary – the places where the condensation starts may indicate spots of enhanced heat transfer, change of the surface quality or similar effects. Further, if we think of a situation where yet another phase transition – droplet freezing – takes place, then these spots may very well be the ones where frosting leads to either narrowing of the flow pathway between the fins, decreasing of the heat transfer coefficient or both.

#### 2.1. Description of the nucleation model

The nucleation is a process where the condensing species builds clusters of molecules from the available monomers in the system (Vehkamäki, 2006), e.g. water molecules in humid air condensing on a cold plate. There exists a discrete value of the droplet critical radius  $r^*$

---

\* Peter Beňovský, Mirza Popovac, Christoph Reichl, Thomas Fleckl: Austrian Institute of Technology, Giefinggasse 2, 1210 Vienna, Austria, peter.benovsky@ait.ac.at

\*\* Dr. Henrik Rusche: Wikki GmbH, Görgestraße 24, 38118 Braunschweig, Germany, h.rusche@wikki-gmbh.de

\*\*\* Assoc. Prof. František Ridzoň, PhD.: Institute of Thermal Power Engineering, Faculty of Mechanical Engineering Slovak University of Technology, Náms. slobody 17, 812 31 Bratislava; Slovakia, frantisek.ridzon@stuba.sk

$$r^* = \frac{2\sigma m}{\rho RT \ln S}. \quad (1)$$

In Eq. (1) utilized symbols stand for:  $\sigma$  surface tension [N/m],  $m$  molar mass of water [kg.mol<sup>-1</sup>],  $\rho$  density [kg/m<sup>3</sup>],  $R$  universal gas constant [J/mol.K],  $T$  temperature [K],  $S$  saturation ratio [-]

To form a droplet of this critical size (a droplet which is stable and will not evaporate), enough formation free energy needs to be available. The calculation of the formation free energy can be done according to the classical theory of Becker & Döring (1935) or its more recent updates, which address the inherent inconsistency of the theory (Girshick & Chiu, 1990), and attempt to improve the accuracy of the theory (Reiss et al., 1997). The expression for the formation free energy  $\Delta G_{BD}^*$  during homogeneous nucleation according to Becker & Döring is then

$$\Delta G_{BD}^* = \frac{16\pi}{3} \frac{v_m^2 \sigma^3}{(kT \ln S)^2}. \quad (2)$$

Here  $v_m$  is the molecular volume [m<sup>3</sup>/mol], and  $k$  the Boltzmann constant [J/K].

Finally, the nucleation rate  $J$  is calculated

$$J_{BD} = \sqrt{\frac{2\sigma}{\pi m}} v_m \left( \frac{p_v}{kT} \right)^2 e^{-\frac{\Delta G_{BD}^*}{kT}}. \quad (3)$$

Where  $p_v$  is the vapour pressure [Pa].

The expressions for the improved nucleation rates (Reiss, Kegel & Katz, 1997) and (Girshick & Chiu, 1990) use the Becker & Döring nucleation rate Eq. (3), which is multiplied by the respective factors proposed by the authors. It is important to note that the expressions listed above hold for homogeneous nucleation, while heterogeneous nucleation prevails in the engineering practice. To amend this discrepancy, the formation free energy (Eq. (2)), is multiplied by a geometric factor. Furthermore, the volumetric nucleation rate has to be transformed into an areal one. To achieve this, one can use the monolayer approach which assumes, that the surface is completely wetted with a monolayer of water molecules. The final expressions for the heterogeneous nucleation can be found in literature Vehkamäki (2006). Nevertheless, the Eqs. (2) and (3) are always present in some form.

## 2.2. Description of the growth model

The growth of condensing droplets can proceed by two different mechanisms: the continuous growth of the droplets driven through diffusion processes and the coalescence of the droplets, the latter being a discontinuous change of the size. Because we are looking at the initial stage of the droplet formation, we will limit our focus to the continuous droplet growth. In literature different growth law formulations can be found e.g.: Heidenreich & Ebert (1995); Krischer & Grigull (1971); Vemuri & Kim (2006). For brevity only the growth equation formulated by Krischer & Grigull (1971) is reported

$$G(r) = \frac{\ln(1 + 1.75Bi)}{1.75Bi} \frac{\alpha_{Kin} \Delta T}{\rho \Delta h}. \quad (4)$$

Here  $Bi$  is the Biot number [-],  $\alpha_{Kin}$  kinetic heat transfer coefficient [W/m<sup>2</sup>.K],  $\Delta T$  wall to bulk temperature difference [K],  $\Delta h$  latent heat [J/kg.K]

## 2.3. Mathematical model

Two physical models describing the nucleation and continuous growth were discussed in the previous paragraphs. The nucleation and the growth are essentially a variation of one droplet parameter out of many – the size. Thus, an equation is needed to track the variation of the number density of droplets with a certain quality (size) in physical space, time and the internal coordinate space. Such criteria are met by the population balance equation (PBE), Ramkrishna (2000). Restricting our attention to the nucleation and continuous growth of the non-moving droplets the PBE reads as

$$\frac{\partial n(r, x, t)}{\partial t} + \frac{\partial}{\partial r}(G(r)n(r, x, t)) = H(r, x, t). \quad (5)$$

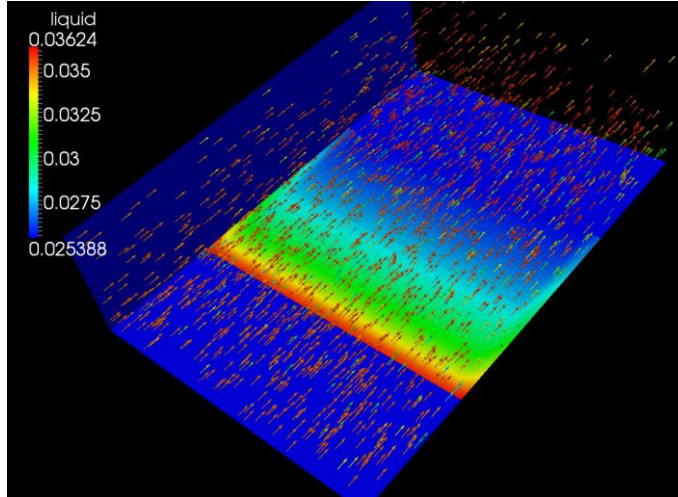
In the equation  $n(r, x_i, t)$  is the number density function which can change in the physical time and space  $(x_i, t)$  as well as in the internal coordinate space  $(r)$ . The right hand side of the equation  $H(r, x_i, t)$  accounts for the discontinuous events: nucleation, droplet coalescence and breakage. There are several ways to solve the equation. Our choice was the quadrature method of moments (QMOM) McGraw (1997). For the application of QMOM it is necessary to transform Eq. (5) into a moment equation. The  $k^{\text{th}}$  order moment  $\mu_k$  of a number density function is defined by

$$\mu_k = \int_0^\infty n(r, x, t) r^k dr. \quad (6)$$

The transformed PBE for the  $k^{\text{th}}$  order moment reads

$$\frac{d\mu_k}{dt} = J\delta(r - r^*) r^{*k} + k \cdot \sum_{i=0}^N \mu_{k-1} G(r_i). \quad (7)$$

Which is a set of ordinary differential equations (ODE) for the moments. These depend on the previously defined functions of the nucleation rate  $J$  (Eq. (3)), critical radius  $r^*$  (Eq. (1)), growth law  $G(r)$  (Eq. (4)) and  $\delta$  stands for the Dirac delta function. The mentioned models were implemented into an OpenFOAM® solver for the turbulent heat transfer. For the evaluation of the performance of the numerical apparatus several simulations of the flow over a cooled plate were done. The representative results are shown on Fig. 1.



*Fig. 1: Cooled wall in a rectangular channel (Condensing flow).*

On Fig. 2 results of five simulations are reported. The domain is a simple rectangular channel. Humid warm air of  $46^\circ\text{C}$  enters the inlet and passes a cooled wall losing humidity due to the nucleation and the continuous droplet growth.

### 3. Conclusions

A method for the calculation of the condensation rates using the classical nucleation theory and population balance equation was briefly introduced. The methods' benefit is that it uses fewer empirical correlations than the conventional methods, and allows for physical modeling of the condensation process based on first principles. The comparison with the experimental results (not reported here) reveals that the features of the process are truly represented. For instance the curves on Fig. 2 have the same ordering, which means that the influence of the driving force  $\Delta T$  is represented correctly. This paper set out to sketch an approach to the numerical modeling of the condensation within air heat exchangers. We can state that the presented procedure captures the initial stage of the condensation and the first part of our work is thus completed. The next phase will include the validation and rigorous testing of the model. Furthermore, the inclusion of the coalescence term in the PBE, model deployment in a two – phase solver, Rusche (2002), or the implementation of a second droplet internal coordinate: the droplet temperature (Marchisio & Fox, 2013) will be incorporated.

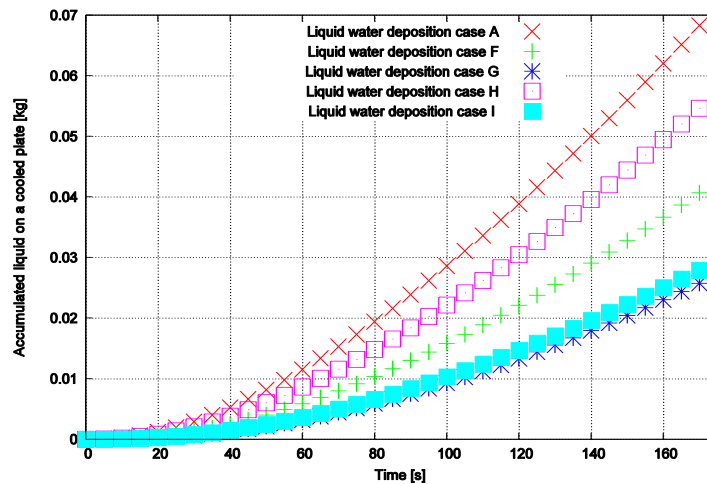


Fig. 2: Condensation rates: Case A, F, G flowrate =  $375 \text{ m}^3/\text{h}$  cooled plate temperature A:  $14^\circ\text{C}$ , F:  $17.5^\circ\text{C}$ , G:  $21.5^\circ\text{C}$ . Case H, I flowrate  $310 \text{ m}^3/\text{h}$  cooled plate temperature H:  $17.5^\circ\text{C}$ , I:  $21.5^\circ\text{C}$ .

## Acknowledgement

The authors gratefully acknowledge the funding provided by Klima- und Energiefonds for the project 829964 in the framework of the “NEUE ENERGIE 2020” programme.

## References

- Becker, R. & Döring, W. (1935) Kinetische Behandlung der Keimbildung in übersättigten Dämpfen. *Ann. Phys.*, 416, pp. 719-752.
- Girshick, S. L. & Chiu, C.-P. (1990) Kinetic nucleation theory: A new expression for the rate of homogeneous nucleation from an ideal supersaturated vapor. *J. Chem. Phys.*, 93, pp. 1273.
- Heidenreich, S. & Ebert, F. (1995) Condensational droplet growth as a preconditioning technique for the separation of submicron particles from gases. *Chemical Engineering and Processing: Process Intensification*, 34, pp. 235-244.
- Krischer, S. & Grigull, U. (1971) Mikroskopische Untersuchung der Tropfenkondensation. *Wärme - und Stoffübertragung*, 4, pp. 48-59.
- Marchisio, D. L. & Fox, R. O. (2013) *Computational Models for Polydisperse Particulate and Multiphase Systems*. Cambridge Books Online.
- McGraw, R. (1997) Description of Aerosol Dynamics by the Quadrature Method of Moments. *Aerosol Science and Technology*, 27, pp. 255-265.
- Nusselt, W. (1916) Die Oberflächenkondensation des Wasserdampfes. *VDI-Z.*, 60, pp. 541-546 and 569-575.
- OpenFOAM (2013). The open source CFD toolbox: <http://www.openfoam.com>.
- Pauschenwein, G. J.; Reichl, C.; Windholz, B.; Moretti, I. & Monsberger, M. (2010) CFD simulations for solar collectors including condensation and ice accretion., *Eurosun 2010*.
- Ramkrishna, D. (2000) *Population balances: Theory and applications to particulate systems in engineering*. Access Online via Elsevier.
- Reiss, H.; Kegel, W. K. & Katz, J. L. (1997) Resolution of the Problems of Replacement Free Energy,  $1/S$ , and Internal Consistency in Nucleation Theory by Consideration of the Length Scale for Mixing Entropy. *Phys. Rev. Lett.*, 78, pp. 4506-4509.
- Rusche, H. (2002). *Computational fluid dynamics of dispersed two-phase flows at high phase fractions*, PhD Thesis, Imperial College, London.
- Song, T.; Lan, Z.; Ma, X. & Bai, T. (2009) Molecular clustering physical model of steam condensation and the experimental study on the initial droplet size distribution. *International Journal of Thermal Sciences*, 48, pp. 2228-2236.
- Vehkamäki, H. (2006) *Classical Nucleation Theory in Multicomponent Systems*. Springer, Berlin Heidelberg.
- Vemuri, S. & Kim, K. (2006) An experimental and theoretical study on the concept of dropwise condensation. *International Journal of Heat and Mass Transfer*, 49, pp. 649-657.
- VDI-Wärmeatlas (2002) *Berechnungsblätter für den Wärmeübergang*. Springer-Verlag, Berlin Heidelberg New York.