20th International Conference
ENGINEERING MECHANICS 2014
Svratka, Czech Republic, 12 – 15 May 2014

ON AN ESTIMATION OF THE EXPONENT OF THE STRESS SINGULARITY: THREE DIMENSIONAL PROBLEMS AND EFFECT OF RESIDUAL STRESSES ON A CRACK ARRESTED ON THE INTERFACE

B. Máša*, L. Náhlík, P. Hutař*****

Abstract: *The main aim of this paper is an investigation of the crack behavior in the ceramics laminates. Especially, the problem of the estimation of the stress singularity exponent in such a material using different approaches is closely described. Since analytical approach is a suitable tool for two dimensional problems, introducing residual stresses, which can be result of used procedures during composite production, may influence value of the stress singularity exponent. Unfortunately, there is formally no analytical tool available to introduce residual stresses. Moreover, when the three dimensional geometry is investigated, effects of complicated stress distribution in front of the crack tip are of crucial importance for crack behavior. Hence using numerically obtained stress distribution appears to be the only way, how to capture the effect of residual stresses and three dimensional geometry of the crack front. The stress singularity exponent can be directly determined from stress components in front of the crack tip and/or from displacements at faces of the crack. Both can provide good results and give us a solution, which cannot be obtained analytically. In following text the procedure will be described in more detail and shown results obtained on ceramic laminate.*

Keywords: **Stress singularity exponent, Residual stress, Singular stress concentrator, Material interface, Ceramic composites.**

1. Introduction

One of the most important material parameters, in terms of fracture mechanics, is fracture toughness. For example usage of ceramics is fairly limited for its brittleness. However, using special technologies and composite design it is possible to prepare ceramic materials with fracture toughness increased three times compared to single ceramic layer without special treatment (e.g. Bermejo et al., 2007). The main idea is to introduce residual stresses, which are closing the crack tip and retard further crack propagation. The interfaces are also barriers for crack propagation – the crack could extend across the interface at some angle; it could extend along the interface; reflect back into the originating material, or arrest. Therefore, the knowledge of the crack behavior in the vicinity or at the interface is of crucial importance.

The stress singularity exponent *p* of the crack propagating in isotropic, elastic and homogeneous material is $p = 0.5$ (e.g. Williams, 1957). Nevertheless, when the crack terminates the material interface, the classical square root singular field changes its value within the interval $0 < p < 1$. Both, residual stresses and material mismatch may influence the exponent of the stress singularity. This problem occurs also in the place, where the crack front extends to the free surface of the body (e.g Hutař et al., 2010). Thus, this effect should be included in further considerations about crack behavior near the free surface. Therefore, there is the need to have a suitable tool to evaluate all these effects, either analytically or numerically.

2. Analytical Approach

According to the literature (Knésl et al., 2003; Náhlík et al., 2009), the stress singularity exponent $p = 1\text{-}\lambda$ is given by the solution of the characteristic equation following from the boundary conditions:

* Ing. Bohuslav Máša: Brno University of Technology, Technická 2896/2, 616 69, Brno, CZ, masa@ipm.cz

** Assoc. Prof. Ing. Luboš Náhlík, PhD.: Institute of Physics of Materials, AS CR, Žižkova 513/22, 616 62, Brno, CZ, nahlik@ipm.cz

*** Assoc. Prof. Ing. Pavel Hutař, PhD.: Institute of Physics of Materials, AS CR, Žižkova 513/22, 616 62, Brno, CZ, hutar@ipm.cz

$$\lambda^2(-4\alpha^2+4\alpha\beta)+2\alpha^2-2\alpha\beta+2\alpha-\beta+1+(-2\alpha^2+2\alpha\beta-2\alpha+2\beta)\cos(\lambda\pi)=0\,, \quad (1)$$

where α and β are the composite parameters (Meguid et al., 1995) given as follows:

$$\alpha=\frac{\frac{E_1}{E_2}(1+\nu_2)-(1+\nu_1)}{4}\,,\ \beta=\frac{E_1}{E_2} \qquad \text{for plane stress conditions,} \quad (2)$$

$$\alpha=\frac{\frac{E_1}{E_2}\frac{1+\nu_1}{1+\nu_2}-1}{4(1-\nu_1)}\,,\ \beta=\frac{E_1}{E_2}\frac{1-\nu_1^2}{1-\nu_2^2} \qquad \text{for plane strain conditions,} \quad (3)$$

E_1, E_2 are Young moduli, and ν_1, ν_2 are Poisson's ratios of materials, which create the interface.

2.1. Numerical approach

The stress distribution around a crack tip of a crack perpendicular to the interface can be written in the form:

$$\sigma_{ij}=\frac{H_I}{\sqrt{2\pi}}r^{-p}f_{ij}\left(p,\theta\right), \quad (4)$$

where H_I [MPa.m^p] is a generalized stress intensity factor and r, θ are polar coordinates with origin at the crack tip. Thus, one of the possible approaches is to estimate the stress singularity exponent directly from numerically obtained stress distribution in front of the crack tip. For given geometry, boundary conditions and polar angle θ the stress components correspond to:

$$\sigma_{ij}\approx r^{-p}\,. \quad (5)$$

The stress singularity exponent corresponds directly to the tangent of the stress distribution (5) expressed in log-log coordinates. Similar approach can be used for the stress singularity exponent determination from numerically obtained displacements on the crack faces. The relation between displacement components and the stress singularity exponent is:

$$u_{ij}\approx r^{1-p}\,. \quad (6)$$

Let us note that estimation of the stress singularity exponent using these approaches is quite accurate and is suitable for problems, where analytical solution is not defined, e.g. for 3D problems, where the complicated stress distribution is expected. The disadvantage is dependence on the mesh density – numerical model has to contain fine mesh around the crack tip and this will cause important increase in time for model preparation, high hardware requirements and the calculation is time consuming.

3. Numerical Modeling

For estimation of the stress singularity exponent a ceramic composite was chosen in order to capture an effect of the 2D and 3D crack front geometry and how the residual stresses can influence its value. The composite design was a laminate with A-B-A architecture, which consists of 5 ATZ layers (alumina with tetragonal zirconia) t_{ATZ} = 0.52 mm and 4 AMZ layers (alumina with monoclinic zirconia) t_{AMZ} = 0.10 mm (Fig. 1). Material properties (see Tab. 1) of the laminate layers were taken from the works (Bermejo et al., 2007; Náhlík et al., 2009).

Tab. 1: Material properties of the studied laminate (Bermejo 2007, Náhlík 2009).

PROPERTY	**UNITS**	**ATZ**	**AMZ**
Young's modulus **E**	GPa	390	280
Poisson's ratio **ν**	-	0.22	0.22
Coefficient of thermal expansion $\boldsymbol{\alpha_t}$	$10^{-6}K^{-1}$	9.82	8.02

For the stress distribution around the crack tip numerical 2D and 3D models were created in commercial FEM software Ansys 13.0. However, two dimensional models represent the crack passing through the entire layer, which is not in agreement with experimental observations. The crack propagating in a plate exhibits approximately a semi-elliptical crack front shape. Thus, parametrically controlled 3D models with semi-elliptical crack front were developed. On the crack front numerous locations were selected and the stress intensity factor K_I was computed using the direct method (extrapolation of stress component to the crack tip). Under assumption of the constant stress intensity factor along the crack front, the real crack front shape was iteratively found (Hutař et al., 2010; Ševčík et al., 2012), see Fig. 2.

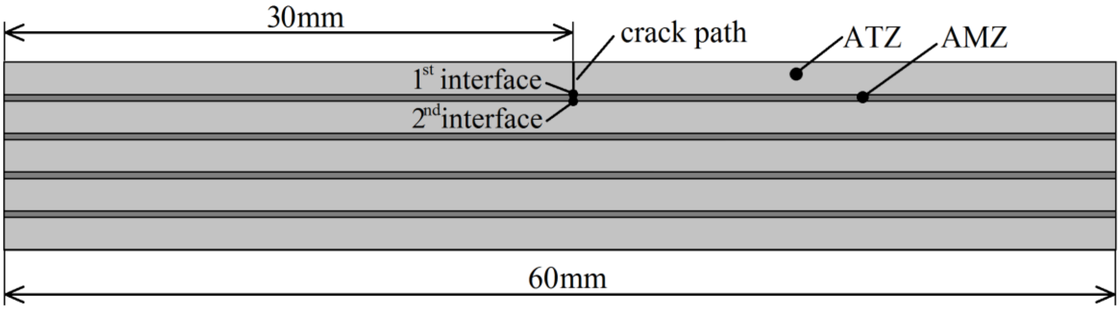

Fig. 1: Design scheme of the laminate with predefined crack path and evaluated interfaces.

Let us note that the residual stresses were prescribed to the numerical model using different thermal coefficients for ATZ and AMZ layer. The specimen is during preparation cooled down from the stress free temperature 1250°C to the room temperature. After this procedure strong residual stresses are developed in the laminate layers (for given case ATZ = +110 MPa, AMZ = -715 MPa), which are able to open/close the crack tip and influence the crack behavior, see Fig. 3.

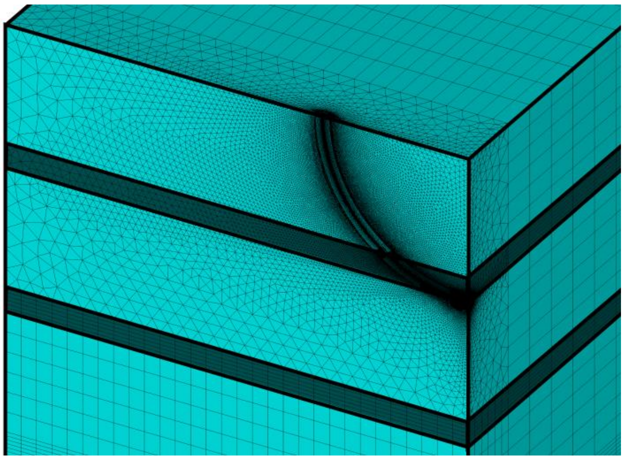

Fig. 2: The real crack front shape numerically estimated for the 3D geometry.

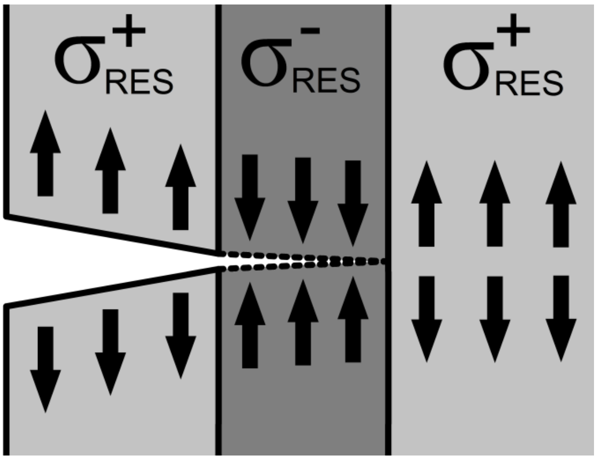

Fig. 3: Effect of the residual stresses distributed in ATZ and AMZ layers.

4. Results

Following previous methodology, analytical solution was used for the 2D geometry where the first 2 interfaces were considered. Also, numerical modeling was performed for the 2D and 3D configurations for the same interfaces, where following loading of the laminate was chosen: a) external bending loading only, b) internal residual stresses only, c) combination of the external and internal loading. The results for the first ATZ/AMZ interface are summarized in the Tab. 2. It is evident that the 2D numerical solution differs slightly from the analytical solution and error is less than 2% for all three loadings considered. When 3D models were evaluated, differences are about 10%. This is due to different stress distribution caused by semi-elliptical shape of the crack front touching the interface.

For the AMZ/ATZ interface analytical solution is in a good agreement with 2D model with external loading only. This situation represents a state, for which is the analytical approach derived and match of these two results was expected. Three-dimensional numerical model shows again a slight difference from the analytical solution. Nevertheless, the strong compressive stresses of 715 MPa cause that no crack opening is obtained and exponent of the stress singularity could not be evaluated clearly (or with a significant error) using extrapolation of the opening stress to the crack tip. Moreover, determination from numerically obtained displacements on the crack faces fails completely, because of negative values of displacement were computed. This, again, physically means that no crack opening occurs.

Tab. 2: Estimation of the stress singularity exponent for the crack on ATZ/AMZ and AMZ/ATZ interfaces.

APPLIED LOADING		**SOLUTION FOR ATZ/AMZ INTERFACE**		
External loading	**Residual stresses**	**Analytical**	**2D numerical**	**3D numerical**
Yes	no	0.54008	0.54826	0.60185
No	yes	0.54008	0.54956	0.60461
Yes	yes	0.54008	0.54874	0.60265
External loading	**Residual stresses**	**SOLUTION FOR AMZ/ATZ INTERFACE**		
Yes	no	0.46451	0.47240	0.54650
No	yes	0.46451	X	X
Yes	yes	0.46451	X	X

5. Conclusions

This paper deals with an estimation of the stress singularity exponent. Analytical approach was presented and used for the exponent evaluation. This procedure is able to capture the effect of material mismatch on the composite interface accurately. Nevertheless, does not correspond to the 3D problems. Moreover, when the effect of residual stresses needs to be captured, this method has no option to incorporate this effect to the solution. When the 3D problem with consideration of residual stresses is investigated, the numerical approach for the stress singularity exponent evaluation is needed. This approach takes into account all effects which may influence a stress field around the crack tip - this leads to the more accurate value of the exponent. Disadvantage of this approach is requirement for the fine mesh in the vicinity of the crack, which increase the computation time. Nevertheless, based on the results, no significant difference on the value of the stress singularity exponent was observed between analytical and 2D numerical solution for all considered types of loading. On the other hand, when 3D crack is investigated, the exponent should be evaluated numerically.

Acknowledgement

This work was supported through the Specific academic research grant No. FSI-S-11-11/1190 provided to Brno University of Technology, Faculty of Mechanical Engineering and grant No. CZ.1.07/2.3.00/30.0063 of the Ministry of Education, Youth and Sports of the Czech Republic.

References

Williams, M.L. (1957) On the Stress Distribution at the Base of a Stationary Crack. Journal of Applied Mechanics, 24, pp. 109-114.

Knésl, Z., Náhlík, L., Radon, J.C. (2003) Influence of interface on fatigue threshold values in elastic bimaterials. Computational Materials Science, 28, pp. 620-627.

Náhlík, L., Šestáková, L., Hutař, P. (2009) Estimation of apparent fracture toughness of ceramic laminates. Computational Materials Science, 46, pp. 614-620.

Meguid, S.A., Tan, M., Zhu, Z.H. (1995) Analysis of cracks perpendicular to biomaterial interfaces using a novel finite element. International Journal of Fracture, 73, pp. 1-23.

Hutař, P., Náhlík, L., Knésl, Z. (2010) The effect of a free surface on fatigue crack behavior, International Journal of Fatigue, 32, pp. 1265-1269.

Bermejo, R., Torres, Y., Baudín, C., Sánchez-Herencia, A.J., Pascual, J., Anglada, M., Llanes, L. (2007) Threshold strength evaluation on an Al_2O_3–ZrO_2 multilayered system. Journal of the Europian Ceramic Society, 27. pp. 1443-1448.

Ševčík, M., Hutař, P., Knésl, Z., Náhlík, L., Zouhar, M. (2012) Estimation of the critical configuration of a crack arrested at the interface between two materials. Computational Materials Science, 64, pp. 225-228.