

## KINEMATIC MODEL OF A MECHANICAL DEVICE FOR THE SUPPORT HUMAN LOWER LIMB

M. Ostaszewski<sup>\*</sup>, F. Siemieniako<sup>\*\*</sup>

**Abstract:** The paper presents the structure of a mechanical device for the support of a human lower limb or for rehabilitation in the sagittal plane. The device has a parallel - serial mechanical structure. Three linear actuators with indirect displacements were used as executive devices. Inverse kinematics model was built for the mechanical design. The input signals for the model are angular displacements in the joints of the human lower limb. The outputs of the model are the displacements of the rods pistons actuators. The article presents the results of measurements of angular displacements using of the laboratory station for the determination of angular displacement in the joints of the lower limb. The paper also presents the displacement of the actuators in a function of angular displacements.

**Keywords:** Inverse kinematic, Rehabilitation, Support system.

### 1. Introduction

The subject of the paper is the kinematic analysis of the mechatronic device, which is designed to assist the human lower limb motion in a plane perpendicular to the axis of rotation of the knee joint of human. The mechanical structure is show in Fig. 1a. In Fig. 1b the kinematic diagram of the device is presented.

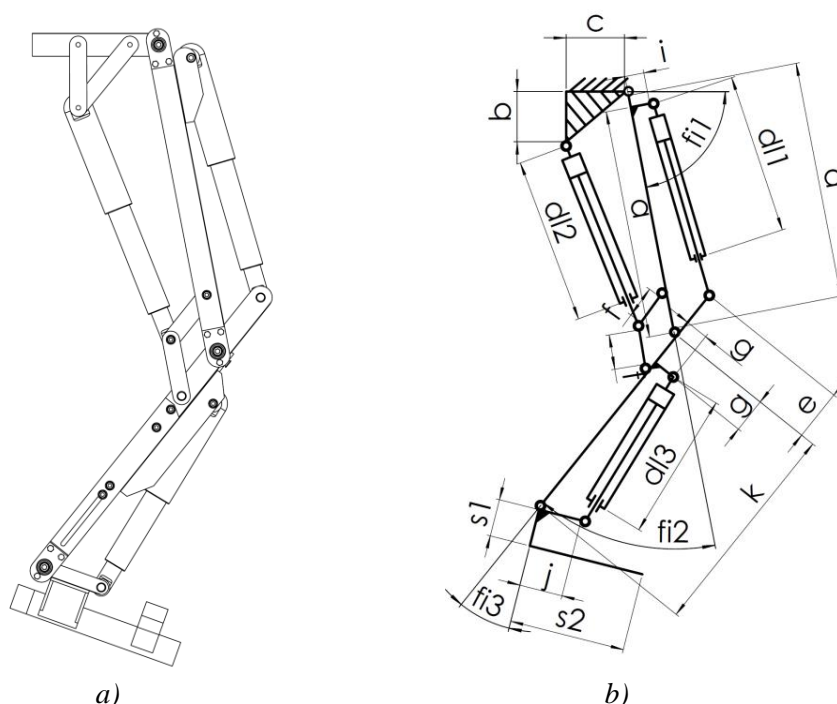


Fig. 1: The device for the rehabilitation of the human lower limb: a) mechanical structure;  
 b) Kinematic diagram of the device.

<sup>\*</sup> Michal Ostaszewski, MSc.: Department of Automatic Control and Robotics, Faculty of Mechanical Engineering, Bialystok University of Technology Wiejska 45C, 15-351 Bialystok, Poland, m.ostaszewski@pb.edu.pl

<sup>\*\*</sup> Prof Franciszek Siemieniako: Department of Automatic Control and Robotics, Faculty of Mechanical Engineering, Bialystok University of Technology Wiejska 45C, 15-351 Bialystok, Poland, f.siemieniako@pb.edu.pl

The device was designed to support the muscular system and to enforce the correct movements in the respective joints of the lower limb of a man during rehabilitation. The correct movements and trajectories are forced by the person which coordinate the rehabilitation by using the test station for angular displacements.

The device consists of components connected in joint in class V. Groups of components consist of two serial and one parallel structures. The used parallel structure increases the operating range of the device. In the mechanical design three electric linear actuators with indirect displacement are placed (Siemieniako, 2013). The input signals for the model are the angular displacement in the joints of the human lower limb. The outputs of the model are the displacements of the rods pistons actuators. The required range of motion and orientation of the executive actuator are determined experimentally using laboratory station for angular displacements in the joints of the lower extremities in the sagittal plane. (Ostaszewski, 2013).

## 2. Mathematical Model

To determine the inverse kinematic model for the mechanical structure the equations of motion of piston rod actuators as functions of angular displacements  $\varphi_1$ ,  $\varphi_2$ ,  $\varphi_3$  have been designated:

$$\begin{aligned} w_1 &= f_1(\varphi_1, \varphi_2), \\ w_2 &= f_2(\varphi_1, \varphi_2), \\ w_3 &= f_3(\varphi_3), \end{aligned} \quad (1)$$

where:  $w_1$ ,  $w_2$ ,  $w_3$  - displacements of piston rods of the actuators 1, 2, 3.

In order to determine the equations of inverse kinematics the simplified model equations of angular displacements have been developed, as a function of the displacements relative to axes OX, OY, and the orientations of feet relative to the initial base:

$$\begin{aligned} \varphi_1 &= f_4(x, y, \varphi), \\ \varphi_2 &= f_5(x, y, \varphi), \\ \varphi_3 &= f_6(x, y, \varphi). \end{aligned} \quad (2)$$

The final form of the equations describing the inverse kinematics of the support mechanical device have been written as follows:

$$\begin{aligned} \varphi_1 &= \arctan 2(y + \sin(\sigma_6)\sqrt{s_1^2 + s_2^2}, x - \cos(\sigma_5)\sqrt{s_1^2 + s_2^2}) - \arctan 2(b\sqrt{\frac{\sigma_1^2}{4d^2k^2} - 1}, d + \frac{\sigma_1}{2d}), \\ \varphi_2 &= \arctan 2(b\sqrt{\frac{\sigma_1^2}{4d^2k^2} - 1}, d + \frac{\sigma_1}{2d}), \\ \varphi_3 &= -(\varphi - \varphi_1 - \varphi_2), \end{aligned} \quad (3)$$

where

$$\begin{aligned} \sigma_1 &= (x - \cos(\arctan\left(\frac{s_2}{s_1}\right) - \varphi)\sqrt{s_1^2 + s_2^2})^2 + (y - \sin(\arctan\left(\frac{s_2}{s_1}\right) - \varphi)\sqrt{s_1^2 + s_2^2})^2 - d^2 - k^2, \\ \sigma_6 &= \arctan\left(\frac{s_2}{s_1}\right) - \varphi. \end{aligned}$$

$$\begin{aligned}
w_1 &= \sqrt{a^2 + e^2 + i^2 - 2e \cdot \cos(\arcsin\left(\frac{i}{a}\right) - \varphi_2)} \sqrt{a^2 + i^2} - dl_1, \\
w_3 &= \sqrt{j^2 + l^2 + m^2 - 2j \cdot \cos(\arcsin\left(\frac{m}{l}\right) + \varphi_3 - \frac{\pi}{2})} \sqrt{l^2 + i^2} - dl_3, \\
w_2 &= \sqrt{b^2 + c^2 - \sigma_3 + f^2 + (d-h)^2 + 2\cos(\arccos\left(\frac{2(d-h)^2 - \sigma_3}{2(d-h)\sigma_2}\right) + \varphi_1 + \sigma_5)} \sqrt{b^2 + c^2} \sigma_2 - dl_2,
\end{aligned} \tag{4}$$

where

$$\begin{aligned}
\sigma_2 &= \sqrt{\sigma_3 + f^2 + (d-h)^2}, \\
\sigma_3 &= 2f \cos(\arccos\left(\frac{h+g \cos(\sigma_2)}{\sigma_4}\right) + \arccos\left(\frac{\sigma_4}{2f}\right))(d-h), \\
\sigma_4 &= \sqrt{g^2 + 2\cos(\varphi_2)g h + h^2}, \\
\sigma_5 &= \arccos\left(\frac{b}{c}\right) - \frac{\pi}{2}.
\end{aligned}$$

### 3. Range of Motion of the Device

In order to verify the device for the support of the lower limb the simulations of the required range have been performed. Schematic structure of the simulation was shown in Fig. 2.

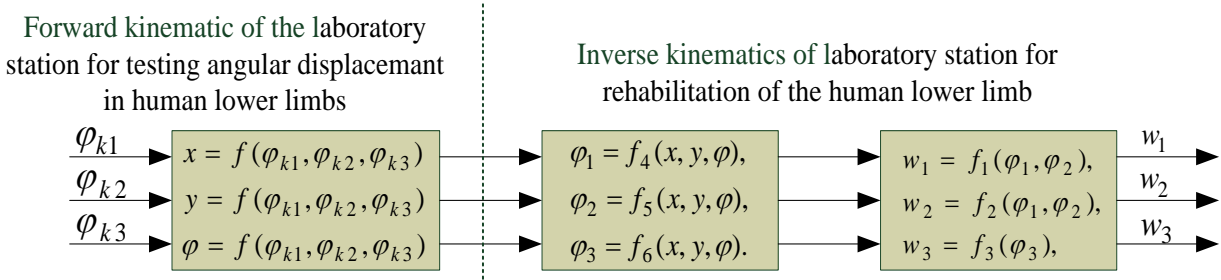
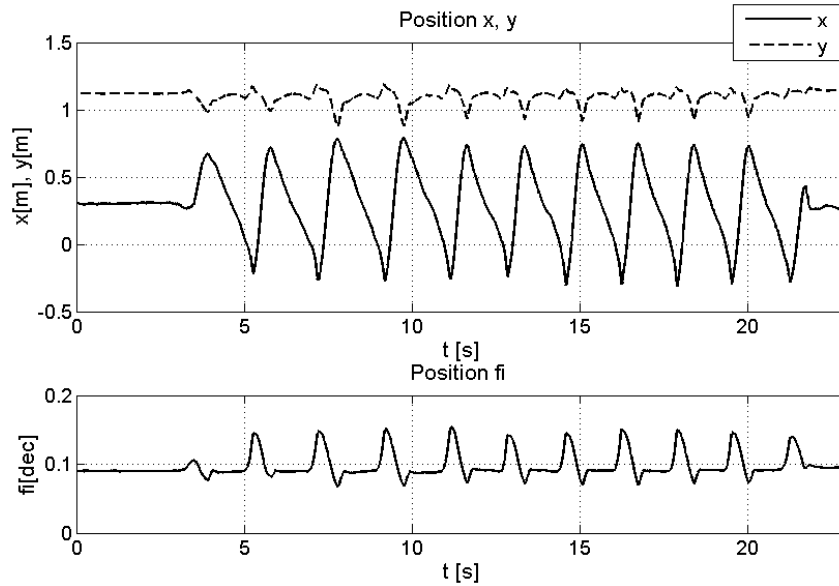


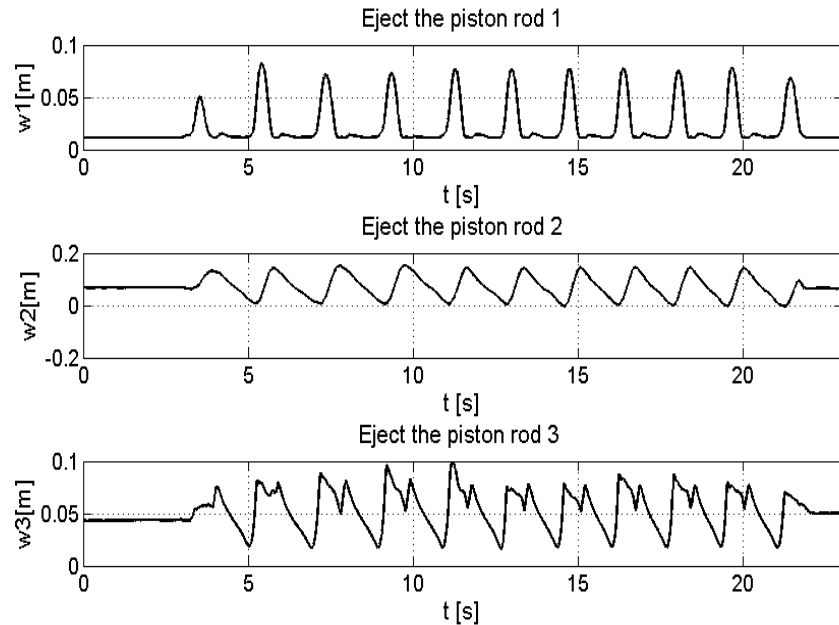
Fig. 2: Schematic structure of the simulation model.

The input signals to the system are displacement in the joints for the human lower limb. Signals have been collected using the laboratory station to measure the position of the angular displacement. The process of measurement and the laboratory station is described in (Ostaszewski, 2013). Using the station measurements of angular displacements are realized by six 12 bit hall effect absolute encoder. The encoder is installed in the joints of measurement station. The joints directly map the movements of individual joints (hip, knee, ankle) lower limb during the test. Sampling frequency was equals was 1 kHz. The maximum height of persons who have been subjected to the tests is 1.95 meter. In the procedure angular displacement which was collected on the measuring station were converted to global coordinates of movement and orientation of the human feet. Then by using the equitation inverse kinematics the signal were transformed into angles and then the displacements of rod cylinders mounted on the devise.

Fig. 3 presents measurements of the position and orientation of the human foot in XOY plane for the left leg in normal walking phase at a speed of 0.15 m/s. The measurement was for 1.95 meter high person. The distance between the hip joint and the knee joint was 0.47 meter and between the knee joint and the ankle was 0.5 meter. Required displacements for the actuators are shown the Fig. 4.



*Fig. 3: Time charts of angular displacements during normal walk.*



*Fig. 4: Time charts of the movements of the actuators as answer to the angular displacement.*

#### 4. Conclusions

The rehabilitation device for the lower limb is equipped with actuators of the maximum rod output stroke equal, respectively for actuator 1 - 0.15 m, actuator 2 - 0.15 m, actuator 3 - 0.10 m based on the designed inverse kinematics model. We can observe that the designed structure and the maximum displacements for actuators are in allowable ranges. A further step will be design of a dynamic model of device for rehabilitation. The dynamic model will be used to investigate the regularity of selected cylinders due to the generated power.

#### References

- Ostaszewski, M., Siemieniako, F. (2013) Laboratory Station for Research on Angular Translocation in Human Lower Limbs, Solid State Phenomena, Trans-Tech Publications, Durnten-Zurich, Vol. 199, pp. 326-331.
- Siemieniako, F., Ostaszewski, M. (2013) Identification of in-line electric actuator, In: Proc. 14<sup>th</sup> International Carpathian Control Conference ICC'2013, Rytro, pp. 280-283, DOI 10.1109/CarpathianCC.2013.6560553.