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COMPARATIVE ANALYSIS OF BURZYŃSKI-TORRE STRENGTH HYPOTHESIS FOR DENTINE AND ENAMEL

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Abstract: *The paper presents the analysis of Burzyński-Torre strength hypothesis application for hard tissues of teeth, i.e. dentine and enamel. Comparative analysis was done with regards to the well-known strength hypotheses, as Huber-von Mises, Tresca-Guest and de Saint-Venant theories. Numerical simulations as well as the finite element modeling were done by means of ANSYS® program. The calculations have been done for the features of the normal occlusal loadings respectively for anterior and lateral teeth. The numerical stress field analyses in dental and enamel were compared with the relevant experimental and clinical data. The effort estimation according to the Burzyński-Torre hypothesis produces the relatively wide spread out of the obtained results, both in the dentine and enamel. As the result of the large negative values of the first invariant of the stress tensor for the proper occlusal loadings the obtained reduced stress values are relatively low when comparing with Huber-von Mises and Tresca-Guest assessments. For the cases of strong bending effects for anterior teeth and for mastication loadings in lateral teeth the respective values of Burzyński-Torre reduced stress rapidly increase. Those effect also seem to be a result of the strong asymmetry of the dentine, and especially enamel, at tension and compression.*

Keywords: Burzyński-Torre strength hypothesis, Dental, Enamel.

1. Introduction

The problem of application of strength hypotheses for the tooth structures seems to be very rare in the dental biomechanical literature. However dentine and enamel are very similar to the bone tissues teeth characterize with the endodermal genesis, as for instance nails or hair. With reference to the bone structures, both compact and trabecular, hard tissues of teeth are highly mineralised. In the case of enamel the rate of the mineral phase reaches even more than ninety percent. Such a structure of dentine and enamel results in treating them also as highly isotropic materials (Craig and Peyton, 1958, Currey, 1995, Powers and Sakaguchi, 2008). Tab. 1 presents a set of strength properties for hard tissues of tooth.

Tab. 1: Strength properties of dentine and enamel.

Tooth structure	Dentine*	Enamel
Modulus of elasticity E [GPa]	18.6	84.1
Poisson's ratio ν	0.31	0.33
Tensile strength σ_r [MPa]	105.5	10.3
Compressive strength σ_c [MPa]	297	382
Tangential strength σ_s [MPa]	138	90.2

* for demineralized dentine: E [GPa] = 0.26; σ_r [MPa] = 29.6

The aim of the paper was to analyze the stress distributions for the chosen strength hypotheses as well as to compare them with the experimental and clinical data in order to select the best fitting of theoretical effort estimation in dental and enamel to the relevant experimental and clinical data. A special attention was paid to the Burzyński-Torre strength hypothesis, which is considered to be the best and the most effective hypothesis for both ductile and brittle materials (Życzkowski, 1999).

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2. Methods

In general the hypothesis of Burzyński-Torre describes the material effort as a function of three stress invariants (Życzkowski, 1999)

$$s = \frac{1}{3} s_I = \frac{1}{3}\left(\sigma_x + \sigma_y + \sigma_z\right) \tag{1}$$

$$t = \frac{\sqrt{2}}{3}\sqrt{s_I^2 - 3s_{II}} = \frac{\sqrt{2}}{3}\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_x\sigma_y - \sigma_y\sigma_z - \sigma_z\sigma_x + 3\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)} \tag{2}$$

$$u = \sqrt[3]{s_{III}} = \sqrt{\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{xy}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2} \tag{3}$$

As the influence of the third invariant *u* could be neglected, the reduced stress with the sufficient accuracy is formulated as a function of two invariants: mean stress *s* and deviatoric stress *t*. In practise two approximations, linear and parabolic, are being used:

$$\sigma_{B-T} = \frac{1}{2\kappa_c}\left\{3\left(\kappa_c - 1\right)\sigma_m + \sqrt{\left[9\left(\kappa_c + 1\right)^2 - 12\frac{4\kappa_c^2}{3\kappa_s^2}\right]\sigma_m^2 + \frac{4\kappa_c^2}{3\kappa_s^2}\sigma_e^2}\right\} \tag{4}$$

where $\kappa_c = \sigma_c / \sigma_r$ and $\kappa_s = \sigma_s / \sigma_r$ are the relevant strength parameters.

Comparative analyses of Burzyński-Torre reduced stress distributions in hard tissues of tooth were done with respect to the well-known Huber-von Mises ($\sigma_{H\text{-}vM}$), Tresca-Guest ($\sigma_{T\text{-}G}$) and de Saint-Venant (ε_1) theories. Numerical simulations as well as the finite element modelling were done by means of ANSYS® programe. The calculations have been done for the features of the normal occlusal loadings respectively for anterior and lateral teeth. The same types of occlusions were applied for the strength tests carried out for the removed teeth by means of INSTRON 4465 strength machine. The numerical models of incisor and premolar were taken after Milewski, 2002. A separate code done in Ansys Parametric Design Language for Burzyński-Torre strength hypothesis was joint to the postprocessor part of ANSYS program.

The detailed stress and strain analysis were done in the areas of the teeth crowns structures which characterize with the maximal effort. Tab. 2 presents a comparison of the maximal values of the reduced stress in the incisor dentine and enamel for the considered strength hypotheses. The comparison was done with the reference to the variable occlusion angle φ in the characteristic areas: occlusal contact zone (A), base of a tooth crown near the gingival line: lingual site (B), buccal site (C) and dentine close to the apical region of pulp chamber (D). Calculations were done for the total occlusion 500 N.

Tab. 2: Comparison of the maximal values of effort in the incisor dentine and enamel for Burzyński-Torre reduced stress and other typical strength hypotheses.

φ [°]	Area	Tooth structure	$\sigma_{B\text{-}T}$ [MPa]	$\sigma_{H\text{-}vM}$ [MPa]	$\sigma_{T\text{-}G}$ [MPa]	ε_1 [x10^{-4}]	σ_m [MPa]
0	A	Enamel	13.2	160.0	174.3	8.4	-105.5
	B	Dentine	5.5	15.5	15.9	3.6	-2.3
20	A	Enamel	5.7	168.2	192.2	10.8	-77.0
	D	Dentine	16.7	41.6	45.2	8.3	-20.9
25	A	Enamel	5.2	158.8	182.5	10.8	-69.8
	D	Dentine	19.2	47.6	52.0	9.9	-23.7
30	A	Enamel	4.8	149.8	172.7	12.6	-63.4
	D	Dentine	20.6	51.2	56.2	10.9	-25.4
45	B	Enamel	313.6	224.1	256.4	26.1	104.8
	D	Dentine	24.2	60.5	66.9	18.0(B)	-29.4
90	B	Enamel	624.6	450.9	514.0	52.9	208.6
	B	Dentine	45.5	80.8	86.9	30.3	8.0

The examples of Huber-von Mises and Burzyński-Torre reduced stress distributions in the incisor dentine and enamel for chosen occlusal angles are given respectively in Fig. 1.

The results of the numerical stress analysis in the hard tissues of teeth were compared with the experimental strength tests carried out for the removed teeth. Fig. 1 shows typical two ways of crown fractures for anterior teeth for various ways of occlusal loadings. The first case, the oblique fracture line (Fig. 1a), is characteristic for the proper occlusions (for φ up to 30°), while the second one, fracture of the crown base (Fig. 1b), appears more often for higher occlusal angle φ, where bending effects dominate.

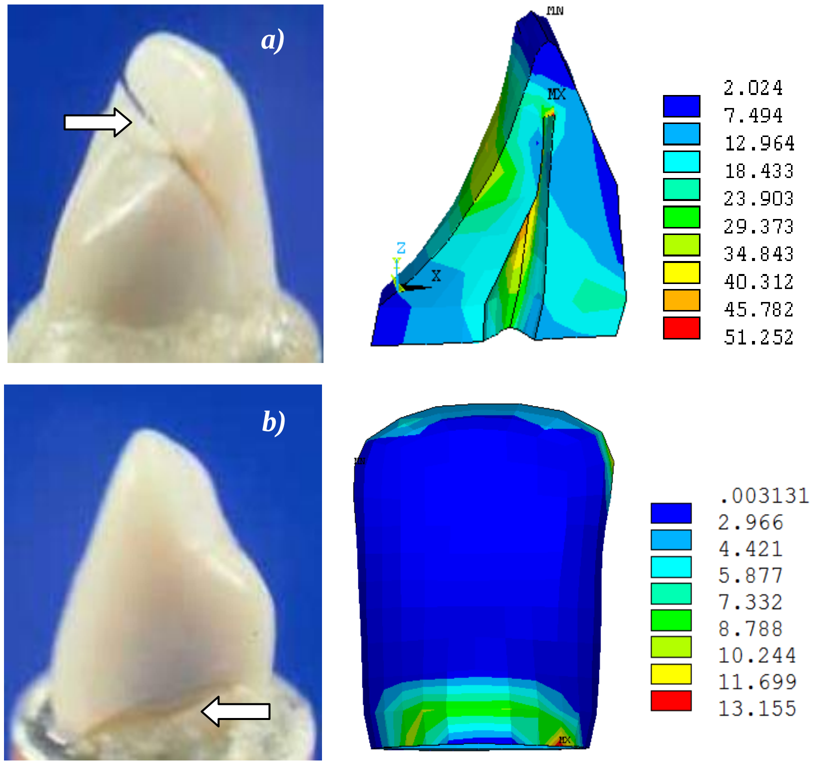

Fig. 1: Typical anterior teeth crowns fractures: a) Oblique line; b) Tooth crown base at gingival base.

For the lateral teeth (premolars and molars) a comparison of the considered strength hypotheses is presented in Tab. 3. With the reference to the described above localisation of the effort areas (A to D), additionally E stands for the dentine areas in the upper part of the crown, close to the mastication surface.

Tab. 3: Burzyński-Torre reduced stress and other strength hypotheses comparison for the maximal effort in premolar dentine and enamel for proper occlusion 500 N.

Tooth structure; Area	$\sigma_{B\text{-}T}$ [MPa]	$\sigma_{H\text{-}vM}$ [MPa]	$\sigma_{T\text{-}G}$ [MPa]	ε_1 [x 10^{-4}]	σ_m [MPa]
Enamel; A	1.4	45.9	48.1	2.0	-18.5
Enamel; B	0.6	25.3	25.9	1.1	-7.3
Dentine; E	5.9	16.8	17.7	3.5	-5.5
Dentine; D	6.1	16.6	18.3	3.9	-6.5
Dentine; B	9.3	15.9	16.7	1.4	-14.3

On the other hand the examples of the accompanying strength tests for the removed lateral teeth are shown in Fig. 2. The total teeth crowns destructions chracteristic for the normal occlusions (Figs. 2a, 2b) and for the masticatory movements (destructions of the seperate cusps - Fig. 2c) were in majority.

Further analysis of application of Burzyński-Torre approach was done with the regards to the influence of the strength hypothesis parameters on the values of the reduced stress estimation. The calculations were done for the enamel for the case of incisor at proper occlusion (φ = 30°). The results of the numerical calculations are presented in Tab. 4.

The analysis of Burzyński-Torre hypothesis sensitivity for variable values of κ_c and κ_s parameters shows that κ_c changes has almost no influence on the values of the reduced stresses while for the coefficient κ_s that influence is noticable for relatively low values of mutual ratio of tangential to tensile strength.

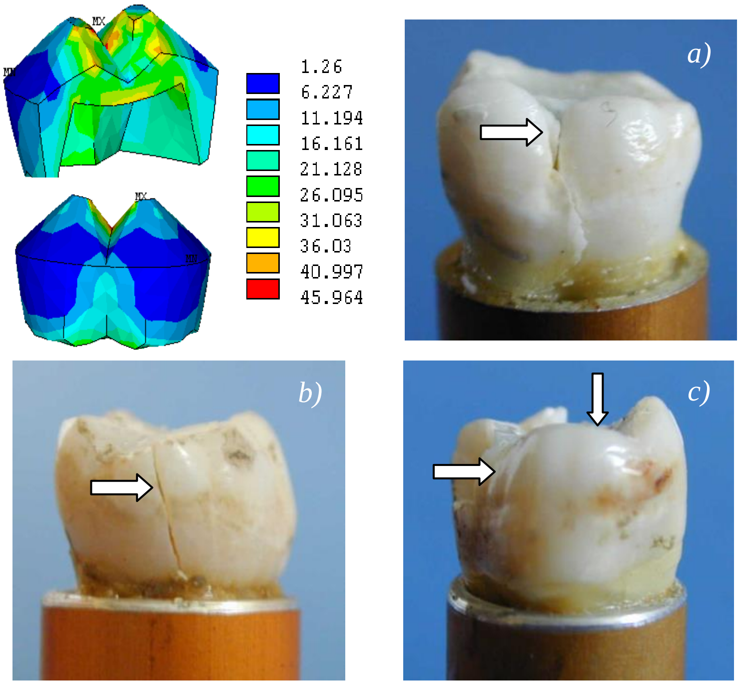

Fig. 2: Characteristic lateral teeth crowns fractures: a) In buccal-lingual plane; b) In distal-mesial plane; c) Separate cusps fractures; total occlusion in numerical calculations 500 N.

Tab. 4: Burzyński-Torre strength hypothesis sensitivity for variable values of κ_c and κ_s parameters.

κ_c	$\sigma_{B\text{-}T}$ [MPa] for $\kappa_s = 8.72$	κ_s	$\sigma_{B\text{-}T}$ [MPa] for $\kappa_c = 37.28$
37.28	101.448	8.72	101.448
30	101.450	8	101.403
25	101.451	6	101.190
20	101.453	4	100.577
10	101.463	2	97.130
8.72	101.466	1.5	93.271

3. Conclusions

Numerical simulations and experimental strength tests prove that, especially for dentine, the areas of maximal effort correspond to the distributions of the maximal tangential stresses. It implies the correctness of application of both the von Mises and Tresca-Guest strength hypotheses. Initiations of cracks in the enamel in the occlusal contact zone as well as in the area of the crown base point out the usefulness of the hypothesis of the maximal principal strain ε_1. It is also worth underlining a relatively wide spread out of the effort estimation according to the Burzyński-Torre hypothesis, both in the dentine and enamel. As the result of the large negative values of the I invariant of the stress tensor (mean value) for the proper occlusal loadings the obtained $\sigma_{\mathbf{B\text{-}T}}$ reduced stress values are relatively low when comparing with $\sigma_{H\text{-}vM}$ and $\sigma_{T\text{-}G}$ values. For the cases of strong bending effects for anterior teeth and for mastication loadings in lateral teeth the respective values of $\sigma_{B\text{-}T}$ rapidly increase. Those effect also seem to be a result of the strong asymmetry of the dentine, and especially enamel, at tension, compression and shearing.

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