21st International Conference **ENGINEERING MECHANICS 2015**
Svratka, Czech Republic, May 11 –14, 2015
Paper **#032**, **pp. 100–101**

To the Analytical Analysis of the Internal Dynamics of Nonlinear Time Heteronymous Planetary Differential Systems

Milan Hortel[a*], Alena Škuderová[b]

Institute of Thermomechanics, AS CR, v. v. i., Dolejškova 5, 182 00 Prague 8, Czech Republic

[a]hortel@it.cas.cz, [b]skuder@it.cas.cz

Keywords: nonlinear dynamics, time heteronymous systems, damping in gear meshes

Abstract: The light high-speed transmission systems with minimum dimensions and weights may show a wide spectrum of properties extending from the regular - periodic solutions to irregular ones of chaotic character. The method of transformation of differential boundary value problems to equivalent nonlinear integro-differential equations with solving kernels of Green's type and the method of successive approximations are applied for analytical analysis of dynamic phenomena.

Introduction

For example, at a mass discretisation and with lightening holes in the cogwheel discs, the mathematical physical model of weakly and strongly nonlinear parametric, due to damping in gear time heteronymous, planetary transmission systems with spur gears is given by expression, [1]

$$\begin{aligned}\mathbf{M}\mathbf{v}'' + \left[{}_1\mathbf{K}(\beta,\delta_i,H) + {}_1\mathbf{k}_d(t)\right]\mathbf{v}' + \sum_{K_1>1}\left[{}_{K_1}\mathbf{K}(D,D_i,H) + {}_{K_1}\mathbf{k}_d(t)\right]\left|\mathbf{w}'(\mathbf{v}')\right|^{K_1}\operatorname{sgn}(\mathbf{w}'(\mathbf{v}'))\\ + {}_1\mathbf{C}(\varepsilon,\kappa,Y_n,U_n,V_n,H,t)\,\mathbf{v} + \sum_{K>1}{}_K\mathbf{C}(\varepsilon,\kappa,I_n,H,t)\,\mathbf{w}^K(\mathbf{v}) = \mathbf{F}(a_n,b_n,\overline{\varphi},H,t), \quad K_1 = 2;3\end{aligned} \tag{1}$$

where $\mathbf{v}$ means generally the n-dimensional vector of displacement of system, $\mathbf{w}^K(\mathbf{v})$ is the K-th power of vector $\mathbf{v}$ defined by expression $\mathbf{w}^K(\mathbf{v}) = \mathbf{D}(\mathbf{w}(\mathbf{v})\mathbf{w}^{K-1}(\mathbf{v}))$. $\mathbf{D}(\mathbf{w}(\mathbf{v}))$ denotes the diagonal matrix, whose elements at the main diagonal are comprised by elements of vector $\mathbf{w}(\mathbf{v}) \equiv \mathbf{v}$. $\mathbf{M}$ is the matrix of mass and inertia forces, ${}_1\mathbf{K},{}_1\mathbf{k}_d(t)$ and ${}_{K_1}\mathbf{K},{}_{K_1}\mathbf{k}_d(t)$ are matrices of linear and nonlinear constant and time variable damping forces, respectively, ${}_1\mathbf{C}$ and ${}_K\mathbf{C}$ are the matrices of quasilinear and nonlinear reversible forces, respectively, $\mathbf{F}(t)$ is the vector of nonpotential external excitation with components a_n, b_n and the phase angle $\overline{\varphi}$. H is the Heaviside's function, which allows to describe the motions (contact bounces) due to strong nonanalytical nonlinearities, for example due to technological tooth backlash $s(t)$. The corresponding *constant* linear or nonlinear coefficients of damping are denoted by β,δ_i or D, D_i, respectively. The linear parametric stiffness functions are denoted by the symbols Y_n, U_n, V_n and nonlinear parametric functions, so-called parametric nonlinearities, by the symbol I_n. ε and κ are the coefficients of the mesh duration, and the amplitude modulation of the *resulting* linear tooth stiffness function $C(t)$, t is the time.

Method of analysis

Based on the quantitative evaluation of each member of vector equation system (Eq.1) of the given planetary system with kinematic constraints, this can transform into form [1], [2]

$$\mathbf{F}(\mathbf{v}) \equiv \mathbf{M}\mathbf{v}'' + {}_1\mathbf{C}(H)_k\,\mathbf{v} = \mu\,\mathbf{V}(\mathbf{v}'',\mathbf{v}',\mathbf{v},\ldots,H,\alpha,t), \tag{2}$$

where μ is a small parameter, $\mathbf{V}$ functional operator containing all nonlinearities, parametric functions, damping and nonpotential excitation sources of vibration, α factor of frequency variation and ${}_1\mathbf{C}(H)_k$ regular matrix of linear reversible forces (both symmetrical and asymmetrical). For periodic solutions with period 2π, the boundary conditions are then

$$N_{n1} \equiv \mathbf{v}(t+0) = \mathbf{v}(t+2\pi), \qquad N_{n2} \equiv \mathbf{v}'(t+0) = \mathbf{v}'(t+2\pi). \tag{3}$$

Due to limited extent of the contribution, the *successive* approximation method to the differential boundary problem (Eq.2, Eq.3) of equivalent integrodifferential equation system will be indicated for the analytical solution of this deterministic multi-frequency excited system briefly only for the case of a *symmetric* matrix of reversible forces, by means of E.Schmidt's method of split cores.

Introducing normal coordinates $\hat{\mathbf{v}}$, further simultaneous transfer of diagonal matrix $\mathbf{M}$ on unit one $\mathbf{E}$, symmetric matrix ${}_1\mathbf{C}(H)_k$ on the diagonal one ${}_1\hat{\mathbf{C}}(H)_k$ and the introducing Green's influence function (Müller's-Breslau's method) with variables t,ξ complying with the given boundary conditions of solved differential boundary problems, then we get to the differential boundary problem Eq.2, Eq.3 equivalent integrodifferential system of equations after longer calculations and integration both for regular – nonresonant $\lambda_n(H) \neq M_k^2$, and singular - resonant $\lambda_n(H) = M_k^2$ cases, where $\lambda_n(H)$ are the eigen values of boundary problem.

$$\begin{aligned}\hat{v}_n(t) = \int_0^{2\pi} \left[\frac{\overline{v}_{n0}^2}{\lambda_n(H)} + \sum_{m=1}^{\infty} \frac{\overline{v}_{nm}(t)\,\overline{v}_{nm}(\xi) + \overline{w}_{nm}(t)\,\overline{w}_{nm}(\xi)}{\vartheta_{\lambda_n(H)}^{\lambda_{nm}(H)} \lambda_{nm}(H) - M_m^2} \right] \times \hat{V}_n(\hat{v}'', \hat{v}', \hat{v}, H, \alpha, \xi)\, d\xi \\ + \sum_{i=1}^{p} \delta_{\lambda_i(H)}^{M_k^2} \left(r_{2i-1}\overline{v}_{nM}(t) + r_{2i}\overline{w}_{nM}(t) \right),\end{aligned} \tag{4}$$

where the expression in square brackets represents solving the core - *Green's empty resolvent* in an extended sense of word. $\lambda, \overline{v}, \overline{w}$ are eigen values independent on linkage of main part of the system (Eq.2), r_{2i-1}, r_{2i} are then branching parameters resulting from branching equations[2]

$$\begin{pmatrix} r_{2i-1} \\ r_{2i} \end{pmatrix} = \int_0^{2\pi} \hat{v}_n(\xi) \begin{pmatrix} \overline{v}_{nM}(\xi) \\ \overline{w}_{nM}(\xi) \end{pmatrix} d\xi\,. \tag{5}$$

Conclusion

When matrices of reversible forces ${}_1\mathbf{C}(H)_k$ in main part of Eq.2 are asymmetrical, the solution may lead to difficulties, compared with case of a symmetric matrix with the normal coordinates. There is necessary then make the transformation of differential boundary problem and solution integrodifferential equations for linkage dependent main part of Eq.2 in the original coordinates $\mathbf{v}$. For a limited range of paper, will deal with this issue in next expanded publications.

References

[1] M. Hortel, A. Škuderová, Lineární, nelineární – kvadratické a kombinované časově heteronomní tlumení v záběrech ozubení ve vnitřní dynamice nelineárních parametrickýych soustav, Colloqium Dynamics of Machines 2014, IT AS CR, v.v.i., Prague, Ed. L. Pešek, 53-64.

[2] M. Hortel, Integrodifferentialgleichungen in der Analyse von Phänomenen in nichtlinearen parametererregten Systemen. In: Recent Advances in Mechanics of Solids and Fluids Bd. 1-3, Technische Universität Wien, 28. November 1997.

Acknowledgement: Study has been elaborated with institutional support RVO 61388998 in the Institute of Thermomechanics AS CR, v.v.i. and with TACR support, project Nr. TA04011656.