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The Influence of Parameters of Interface Contact Elements of the LSB48 Blade's Computational Model on the Resonant Frequencies of the Bladed Disc

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Abstract: To perform the nonlinear vibration analysis of a bladed disc with contacts between blades, the properties of the interface contact elements of the computational model used must be set correctly. In this study, the numerical method is proposed for calculation of the resonant frequencies of the computational model of the bladed disc in case that the so-called "contact stiffnesses" are given. The method is demonstrated for the case of the bladed disc provided with LSB48 blades that comprise two integral contact elements, one in the middle and the second one on the tip of the blade. The values of contact stiffnesses are fitted to get the calculated values of resonant frequencies in accord with the measured ones. The method utilizes the cyclic symmetry properties of the bladed disc.

Introduction

In the works of Petrov and Ewins [1,2], the effective computational method is proposed for the nonlinear vibration analysis of the bladed disc. It utilizes the cyclicity of the mechanical assembly and allows for the modelling of the frictional dampers between blades via dry Coulomb friction model. One of the parameters of the friction model is the so-called "contact stiffness". This is rather a misleading naming since this parameter is not related solely to the physical properties of the contact. It also captures the numerical approximation of the frequency response function of one sector of the bladed disc with all the contact elements omitted, which is pointed out in the work [3]. In the following, we present the method of calculation of natural frequencies of the bladed disc with all contact elements stuck, i.e., in the linear situation with no macroscopic slip, using (i) the values of natural frequencies, (ii) mode shapes and (iii) the values of contact stiffnesses. All the input parameters are the outputs of the model with no contact elements employed.

Mathematical background

Let $\omega_{k,i}$ and $\boldsymbol{w}_{k,i}$ be the i-th natural frequency and the respective mode shape with k nodal diameters of the (cyclically symmetric) bladed disc provided with the N blades with no contact elements. Making use of the cylindrical coordinate system we can write $\boldsymbol{w}_{k,i} = \frac{1}{\sqrt{N}}\,[1, \mathrm{e}^{\mathrm{j}k\sigma}, \cdots, \mathrm{e}^{\mathrm{j}(N-1)k\sigma}]^T \otimes \boldsymbol{v}_{k,i}$, where $\sigma = -2\pi/N$, $\boldsymbol{v}_{k,i}$ denotes the restriction ("imprint") of the eigenvector $\boldsymbol{w}_{k,i}$ to the reference sector, i.e., to the section of the bladed disc embracing the first blade and the corresponding part of the disc. The $\otimes$ symbol stands for the Kronecker product. The quantities $\omega_{k,i}$ and $\boldsymbol{v}_{k,i}$ are obtained performing the modal analysis of the reference sector

$$-\omega^2 \boldsymbol{M}\boldsymbol{v} + \left(\boldsymbol{K}_0 + \boldsymbol{K}_1 \mathrm{e}^{\mathrm{j}k\sigma} + \boldsymbol{K}_{N-1}\mathrm{e}^{-\mathrm{j}k\sigma}\right)\boldsymbol{v} = \boldsymbol{0}\,. \tag{1}$$

Using this notation, the mass matrix of the entire bladed disc is $\boldsymbol{I}_N \otimes \boldsymbol{M}$ and its stiffness matrix is $\boldsymbol{I}_N \otimes \boldsymbol{K}_0 + \boldsymbol{P}_N \otimes \boldsymbol{K}_1 + \boldsymbol{P}_N^{\mathrm{T}} \otimes \boldsymbol{K}_{N-1}$. Here, $\boldsymbol{I}_N$ stands for the identity matrix and $\boldsymbol{P}_N = [\delta_{i,j-1}]_{i,j=1,\cdots,N}$ $(\delta_{i,0} = \delta_{i,\mathrm{N}})$, for the permutation matrix of the same order N. Note that $\boldsymbol{K}_{N-1} = \boldsymbol{K}_1^{\mathrm{T}}$. Let the contact stiffnesses in the normal and the tangential directions of the α-th contact be denoted as $k_{\mathrm{N},\alpha}$, $k_{\mathrm{T1},\alpha}$ and $k_{\mathrm{T2},\alpha}$, which are the parameters of the computational model [3]. If there is no slip in the contacts, the model is linear. In such a case, the objective is to calculate the natural frequencies $\omega_{\mathrm{S},k,r}$ and (restricted) mode shapes $\boldsymbol{v}_{\mathrm{S},k,r}$, given the corresponding quantities $\omega_{k,i}$ and $\boldsymbol{v}_{k,i}$ of the system with omitted contact elements. The mode shapes are of the form $\boldsymbol{v}_{\mathrm{S},k,r} = x_{r,1}\,\boldsymbol{v}_{k,1} + \cdots + x_{r,m}\,\boldsymbol{v}_{k,m}$, where the multipliers $x_{r,i}$ and the natural frequencies $\omega_{\mathrm{S},k,r}$ are the solution of the eigenvalue problem

$$\left(-\omega^2\boldsymbol{I}_m + \left[\omega_{k,i}^2\delta_{i,j} + h_{k,i,j}\right]_{i,j=1,\cdots,m}\right)\begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} = \boldsymbol{0} \tag{2}$$

with

$$h_{k,i,j} = \sum_{\alpha=1}^{2} \boldsymbol{v}_i^{\mathrm{H}}|_\alpha \left(\begin{bmatrix} -1 & \mathrm{e}^{-\mathrm{j}k\sigma} \\ \mathrm{e}^{\mathrm{j}k\sigma} & -1 \end{bmatrix} \otimes \mathrm{L}_\alpha^{-1} \begin{bmatrix} k_{\mathrm{N},\alpha} & 0 & 0 \\ 0 & k_{\mathrm{T1},\alpha} & 0 \\ 0 & 0 & k_{\mathrm{T2},\alpha} \end{bmatrix} \mathrm{L}_\alpha \right) \boldsymbol{v}_j|_\alpha. \tag{3}$$

Here, the matrix L_α transforms the cylindrical coordinate axes to the local coordinate system of the contact α and the $|_\alpha$ symbol indicates the restriction of the vector to the one containing the degrees of freedom of the contact element only.

Results for the LSB48 blade - example

Consider the case that the contact in the middle of the LSB48 blade (tie-boss) is stuck and the tip contact (shroud) is free. For such a configuration, the dependency of the normalized first natural frequency with $k = 0$ and $k_{\mathrm{N},2} = 5 \times 10^4$ N/mm on the value of $k_{\mathrm{T1},2} = k_{\mathrm{T2},2}$ is plotted in Fig. 1. In Fig. 2 the same is plotted for $k_{\mathrm{N},1} = 10^4$ N/mm, $k_{\mathrm{T1},2} = k_{\mathrm{T2},2} = 6.5 \times 10^4$ N/mm and both contacts stuck. Moreover, the impact of the number, m, of the mode shapes employed on the values of calculated natural frequencies is depicted in the figures.

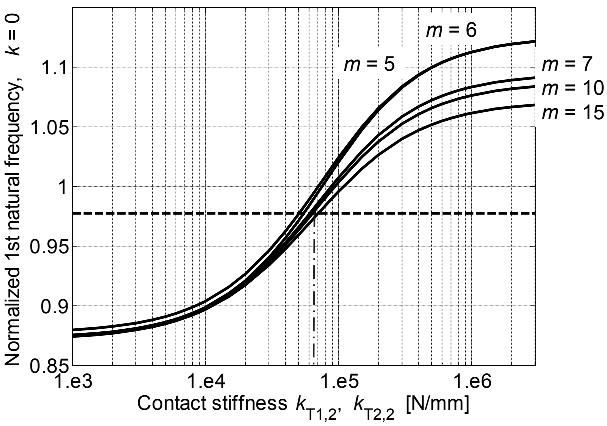

Fig. 1: Normalized first natural frequency, $k_{\mathrm{N},2} = 5 \times 10^4$ N/mm.

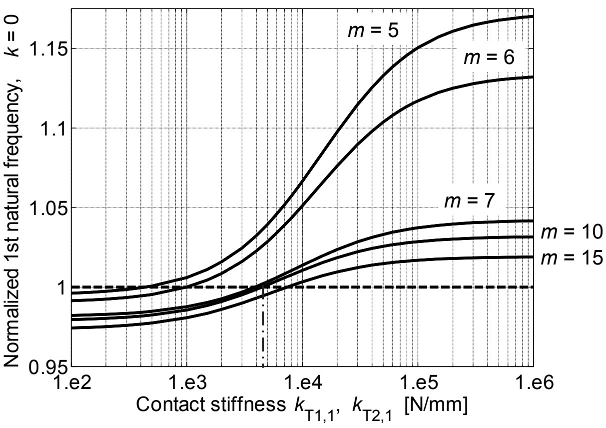

Fig. 2: Normalized first natural frequency, $k_{\mathrm{N},1} = 10^4$ N/mm, $k_{\mathrm{T1},2} = k_{\mathrm{T2},2} = 6.5 \times 10^4$ N/mm.

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