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INFLUENCE OF UNBALANCED BENDING MOMENTS ON PUNCHING RESISTANCE OF FLAT SLABS

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Abstract: *Unbalanced bending moments influence punching resistance of flat slabs depending on position of columns in structure. Paper presents methods of calculation of this phenomenon by different approaches with different levels of accuracy. The contribution begins with an analysis of forces distribution depending on position of columns and continues with an analysis of stresses round the column and also brings explanation and simplification of Eurocode approaches.*

Keywords: Flat slab, Punching, Shear resistance, Unbalanced bending moment, Eurocodes.

1. Introduction

There are two possible ways of structural failure due to punching. The first one is strut diagonal failure (crushing of concrete) at control perimeter u_0 of the column (Fig. 1a). The second one is the failure in shear crack surrounded by control perimeters u_i, which are analysed in distances 2*d* from face of column (Fig. 1b).

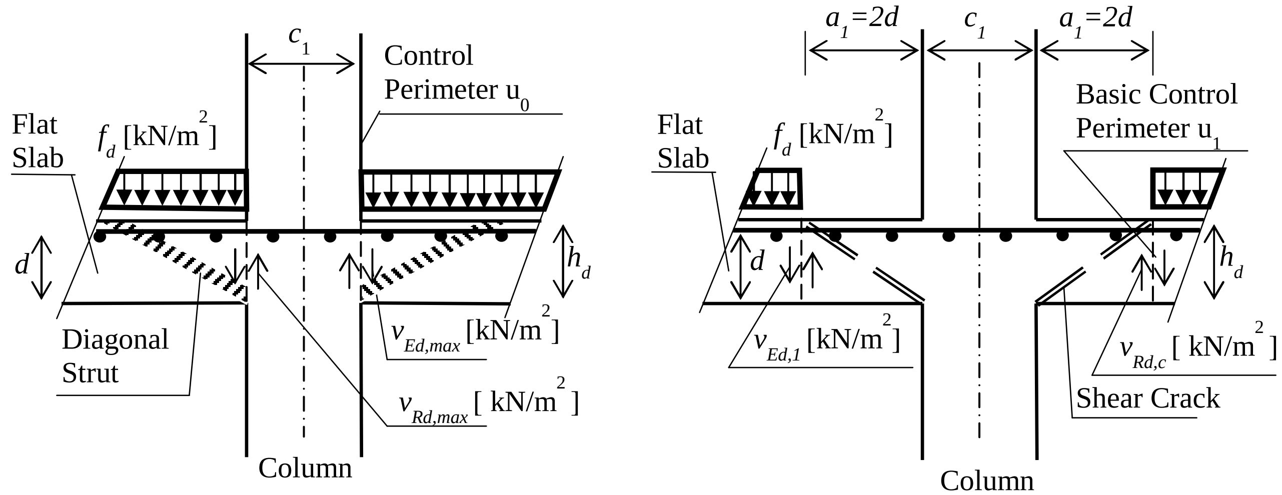

Fig. 1: a) Strut diagonal failure model (left), b) model of failure in shear crack (right)

Crushing of the struts at column perimeter is controlled by reduced compressive strength of concrete (EN 1992-1-1:2004/AC (2010)) (1).

$$v_{Ed,max} = \frac{\beta V_{Ed,max}}{u_0 d} \leq v_{Rd,max} = 0.4\, \nu\, f_{cd} \tag{1}$$

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The failure in shear crack is limited by shear resistance of concrete without shear reinforcement (2). The maximum shear resistance with shear reinforcement in basic control perimeter is limited by $k_{max}\, v_{Rd,c}$ where k_{max} (1.4 - 1.9) depends on form of shear reinforcement (EN 1992-1-1:2004/AC (2010)) (3).

$$v_{Ed,1} = \frac{\beta V_{Ed,1}}{u_1 d} \leq v_{Rd,c} = \frac{0.18}{\gamma_C} k_h \left(100 \rho_l f_{ck}\right)^{1/3} \tag{2}$$

$$v_{Ed,1} \leq v_{Rd,cs} = 0.75 v_{Rd,c} + \left(\frac{1.5d}{s_r}\right) \frac{A_{sw} f_{ywd,ef}}{u_1 d} \leq k_{max} v_{Rd,c} \tag{3}$$

2. Unbalanced Bending Moment

Punching failure also depends on a position of a column in a plan of a building – corner, edge and internal position of the column. This position is very important because of unbalanced bending moment coefficient β calculation. There are several levels of calculation accuracy by Eurocodes. For the structure up to 25% adjacent span difference is allowed to use recommended values of coefficient β (corner column – β = 1.5, edge column – β = 1.4, internal column – β = 1.15). It is also possible to use simplified equations for the calculation of coefficient β for the complete perimeter of internal column (4) and reduced perimeters for corner (5) and edge column (6).

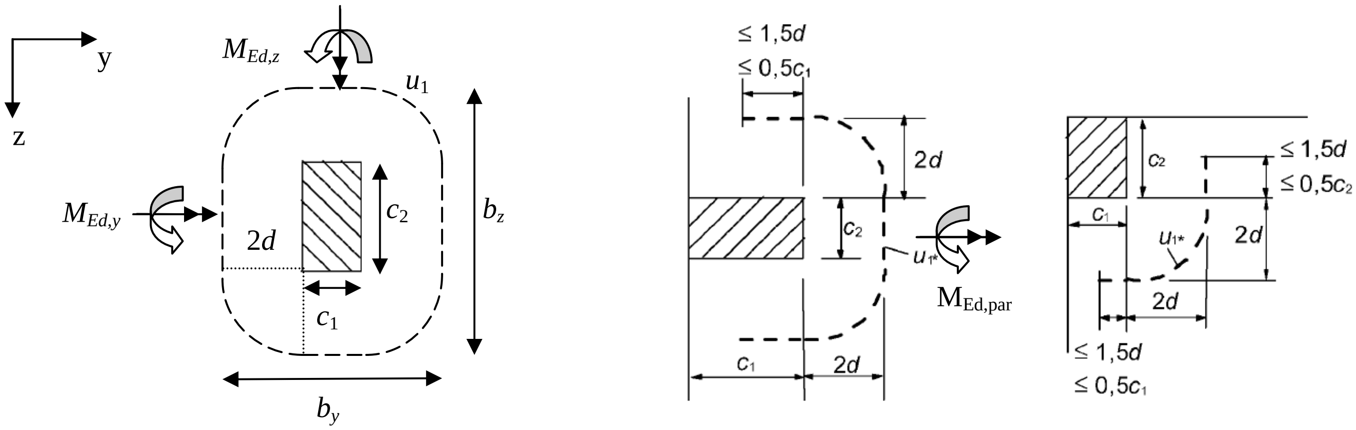

Fig. 2: Perimeter for internal column position (left), reduced perimeters for edge (middle) and corner (right) column position

$$\beta = 1 + 1.8 \sqrt{\left(\frac{M_{Ed,z}}{b_z V_{Ed,1}}\right)^2 + \left(\frac{M_{Ed,y}}{b_y V_{Ed,1}}\right)^2} \tag{4}$$

$$\beta = \frac{u_1}{u_1^*} \tag{5}$$

$$\beta = \frac{u_1}{u_1^*} + k \frac{M_{Ed,par}}{V_{Ed,1}} \cdot \frac{u_1}{W_1} \tag{6}$$

All cases of column positions can be calculated by general equation.

$$\beta = 1 + k \frac{M_{Ed}}{V_{Ed}} \cdot \frac{u_1}{W_1} \tag{7}$$

Where: u_1 is the length of the basic control perimeter; u_1^* is the length of the reduced basic control perimeter; M_{Ed} – unbalanced bending moment; k is a coefficient dependent on the ratio of the column dimensions c_1 and c_2, this value represents proportion of the unbalanced bending moment transferred by shear into column (EN 1992-1-1:2004/AC (2010)).

Tab. 1: Coefficient k - ratio of the column dimensions c_1 *and* c_2

c_1/c_2	≤ 0.5	1.0	2.0	≥ 3.0
k	0.45	0.60	0.70	0.80

W_1 corresponds to a distribution of shear stresses as illustrated in Fig. 3 and it is a function of the basic control perimeter u_1. The parameter W_1 is determined by the formula (8) and also by simplified equations (Fingerloos et al., 2012).

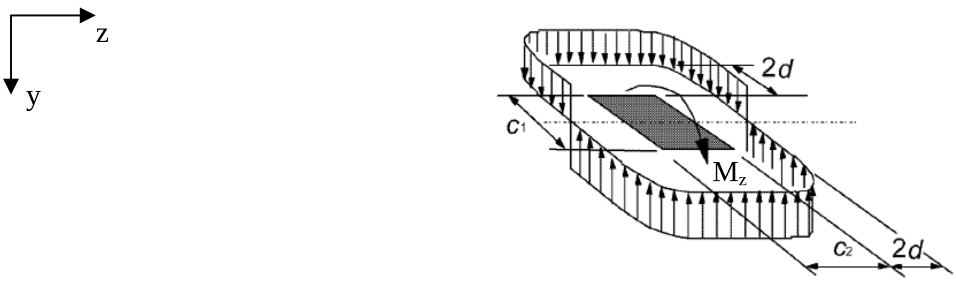

Fig. 3: Shear stresses distribution due to an unbalanced moment at a slab-column connection

$$W_1 = \int_0^{u_i} |\, e \,|\, dl \qquad (8)$$

Where: *dl* – the length increment of the perimeter; *e* - the distance of *dl* from the axis about which the moment M_{Ed} acts.

3. Analysis of coefficient *β*

The analysis was done for following data: four floor apartment building, with height of one floor 3200 mm (the second floor was chosen for analysis), h_d = 200 mm (260 mm for slab overhang 300 mm), column span in "x axis" 7000 mm, column spans in "y axis" 7000 x 5000 x 7000 mm, slab overhang 1750 mm (300 mm alternative), column dimensions 400 x 400 mm, characteristic permanent load g_k = 2 kN/m^2 (without self-weight) and variable load q_k = 2.5 kN/m^2.

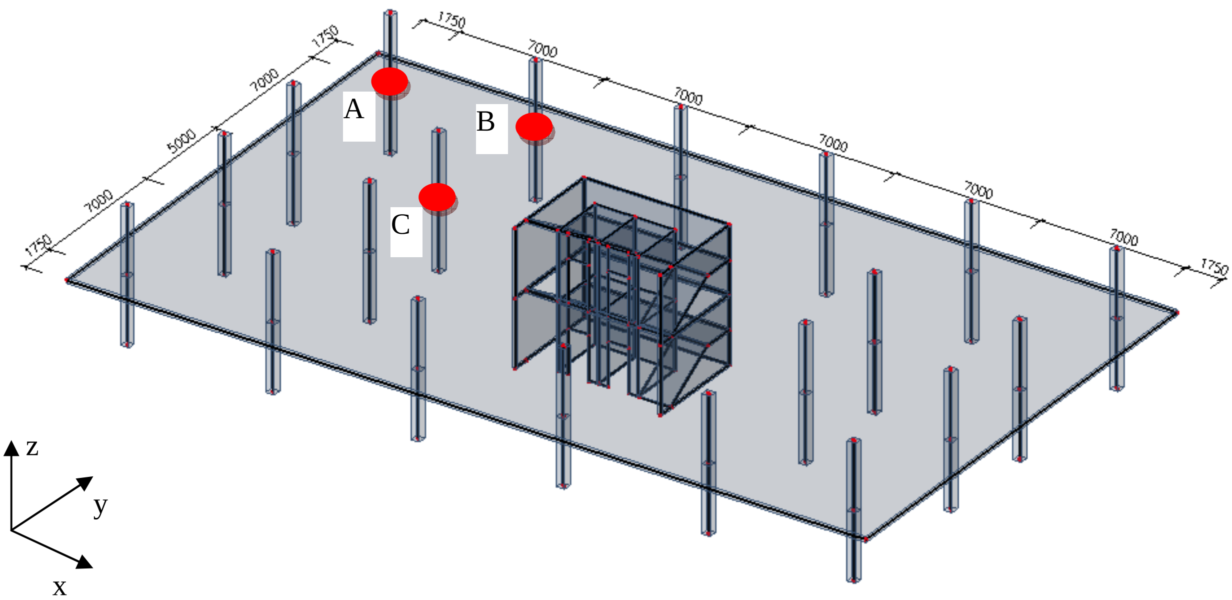

Fig. 4: Analysed plan of flat slab

Tab. 2: Comparison of the coefficient β for columns

	Columns				
	Corner – A (overhang 1750 mm)	Corner - A (overhang 300 mm)	Edge – B (overhang 1750 mm)	Edge - B (overhang 300 mm)	Internal - C
V_{Ed} [kN]	362	232	505	435	590
M_{yEd} [kNm]	113	130	136	208	90
M_{zEd} [kNm]	106	130	9	11	16
W_{1y} [m^2]	1.182	1.039	1.182	1.495	1.182
W_{1z} [m^2]	1.182	1.039	1.182	0.993	1.182
β Simplified calculation	1.81 (4)	1.96 (5)	1.51 (4)	1.52(6)	1.29 (4)
β General calculation (7)	1.77	1.52	1.48	1.56	1.28
β Recommended	1.5		1.4		1.15

4. Conclusions

In the paper is presented a contribution to the problem of flat slab punching. Two possible ways of structural failure due to punching are introduced on Fig.1. Punching failure also depends on a position of the column in a plan of a building – corner, edge and internal. This position is very important because of a coefficient *β* calculation (4)-(7), which depends on unbalanced bending moments. Paper presents methods of calculation of this phenomenon by different approaches with different levels of accuracy. Tab. 2 brings results of the coefficient *β* comparison for the different calculation methods and two possible overhangs of the edge slab cantilever (Fig.4). It's obvious that simplified calculation brings values on the safe side, but the Eurocode recommended values of the coefficient β are on the unsafe side.

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