

## THE INFLUENCE OF MESH MORPHOLOGY ON THE SCF IN 2D FEM ANALYSIS OF FLAT BARS WITH OPPOSITE V-NOTCH UNDER TENSION

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**Abstract:** *The paper covers an issue of accuracy of numerical determination of stress concentration factor values for flat specimens with opposite V-notch. Own calculations were performed with finite element method with application of both meshes types, free mesh and mapped mesh. Dependency of  $K_t$  on finite element characteristic size was determined with application of the taken into consideration meshes. Based upon the obtained results, the mesh morphology influence on the accuracy of  $K_t$  calculations for the discussed notches was specified. The presented considerations allow for determination of optimum mesh morphology and size for obtaining the precise  $K_t$  value at minimum DOF number.*

**Keywords:** *Notch, Stress concentration factor, Finite element method, Mesh morphology.*

### 1. Introduction

Analytical solution to the issue of stress concentration in the flat specimens with opposite V-notch under tension was described in the paper (Nisitani & Noda, 1986). With approximating formulae (Noda et al., 1995) one can estimate  $K_t$  values for any notch with the accuracy of up to 1%. Higher accuracy of  $K_t$  determination can be achieved by the approach presented in the paper (Noda & Takase, 2002) which proposes a division of notches into six groups, distinguished by depth and radius with separate dependencies in each group. Approximate values of  $K_t$  can be estimated with simplified method described in the specific handbooks (Pilkey & Pilkey, 2008). Numerical verification of handbook values of  $K_t$  values performed by FEM in the paper (Shin et al., 1994) for flat specimens notch under tension revealed discrepancy of  $K_t$  values of up to 8%.

The paper (Karolczuk, 2013) shows that oversized mesh size in the notch may cause up to 8% error in the calculated fatigue life, and undersized mesh may even double the duration of analyses made by Finite Element Method FEM. The problem of the influence of finite element order on calculation accuracy was addressed in the paper (Cichański, 2011). Based upon the conducted analyses it was determined that second order elements are characterized by good accuracy relation to the problem size. The issue of mesh morphology impact on the accuracy of FEM analyses was discussed in the paper (Cichański, 2015). It was stated that the mapped mesh with sub-area in the shape of a circle ring segment guarantees calculation accuracy control in the whole range of dimension variability of the finite element size. The issue of specimen geometry shaping with respect to the mesh was discussed in the paper (Soltysiak & Boronski, 2015). In geometrical model of notch specimen, apart from sub-areas specified for modelling of various material properties in weld zones, sub-areas for control of mesh structure with respect to accuracy were introduced. In the paper (Blacha, 2013) in the notch area, subsequent sub-areas were distinguished, allowing for the control of finite elements size increase along with distancing from the notch root.

The paper discusses the issue of the mesh morphology influence in the notch area on  $K_t$  value determined by FEM for flat specimens under tension. During the analyses, comparison of the accuracy of free meshes with triangular and quadrature elements with quadrilateral mapped meshes was performed. The calculations estimated the mesh quality parameters and degree of freedom DOF number change along with the change of finite element characteristic size.

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## 2. The object and test conditions

The tests were conducted in plane stress condition for flat specimens of  $W$  width with opposite V-notch (Fig. 1). The analyses were performed for the specimen of  $W=100\text{mm}$  width and  $L=200\text{mm}$  length, corresponding to  $L=2W$ . The notch of  $\rho=1.5\text{mm}$  radius was adopted for considerations, corresponding to  $2\rho/W = 0.03$ . For the adopted notch, it was analytically determined (Nisitani, 1986) that the extreme  $K_t=5.58$  value was present for the depth of  $t=15\text{mm}$ , corresponding to  $2t/W=0.3$ .

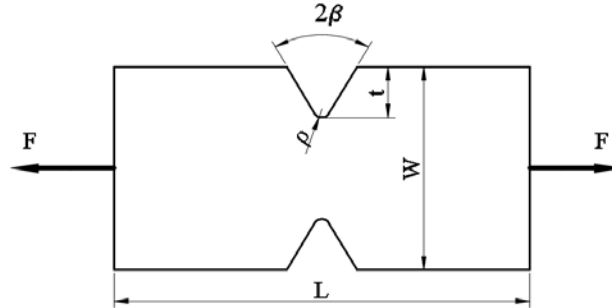


Fig. 1: Specimen adopted for tests.

Linear numerical analyses were conducted with finite element method in ANSYS software environment. Due to the symmetry of geometrical shape and boundary conditions a quarter of the specimen was adopted for the analysis. The analyses were performed with second order finite elements PLANE183 (Cichański, 2011). For free mesh and mapped mesh each, two different morphology variants were adopted (Fig. 2).

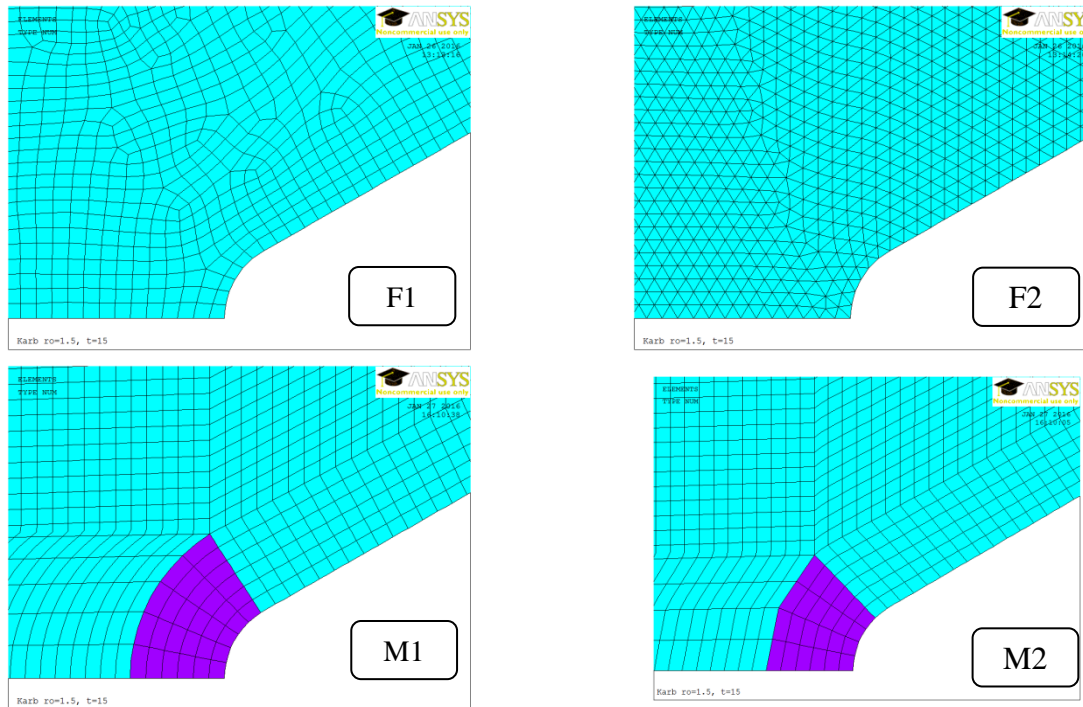


Fig. 2. Mesh morphology in the notch area: F1) free mesh with quadrangle elements; F2) free mesh with triangle elements; M1) mapped mesh with circular sub-area; M2) mapped mesh with polygonal sub-area.

For free meshes the whole specimen surface was modelled with a single area. The tests were performed with quadrangle elements (Fig. 2, F1) and triangular elements (Fig. 2, F2). In this case the shape of each element was determined by meshing algorithm. For the purpose of the notch root element shape control mapped meshes with application of two different sub-areas specified in the notch area were used. In the first variant, the sub-area is of a circular ring segment shape (Fig. 2, M1). The second variant assumed determination of the sub-area in the notch area in such manner that its two delimiting lines are the chords of the concentric circle with the notch arch (Fig. 2, M2).

### 3. Test results

For the discussed methods of mesh modelling (Fig. 2) a number of calculations with application of meshes with various characteristic element sizes was performed. For each mesh the relative error  $\delta K_I$  was calculated based upon analytical value of  $K_I$  for the tested notch (Nisitani & Noda, 1986). Along the decrease of the mesh characteristic size, the analysis accuracy increased which was reflected in decrease of  $\delta K_I$  relative error (Fig. 3a). The increase of calculation accuracy was connected with the increase of finite elements number, necessary for specimen area meshing which was reflected in the increase of degree of freedom DOF number (Fig. 3b).

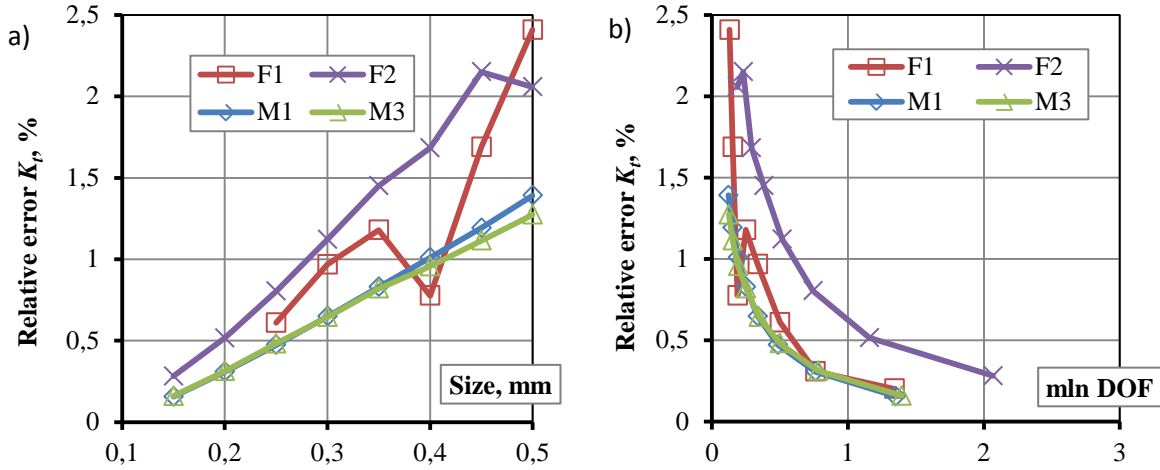


Fig. 3: Dependency of  $\delta K_I$  relative error on: a) element size; b) DOF number

During analyses was collected the mesh quality factors for the notch root element. The course of aspect ratio being a measure of its distortion is presented on Fig. 4a. The improvement of shape along with the decrease of the element size was observed for both mapped meshes. For free meshes, low correlation between the size and the element quality factors was observed. The course of the error of stress discontinuity described by absolute value of the maximum variation of any nodal stress component SDSG is presented on Fig. 4b. Stress discontinuities in triangular elements are much greater than those which were set for both meshing methods with use of quadrate elements. F1 mesh for element size 0.4mm exhibit deviation of  $\delta K_I$  error course (Fig. 3a) as a result of the finite element distortion which is described by the aspect ratio increase (Fig. 4a). For this particular case the notch root element belongs to the free mesh distortion which involves sharing one node by five elements.

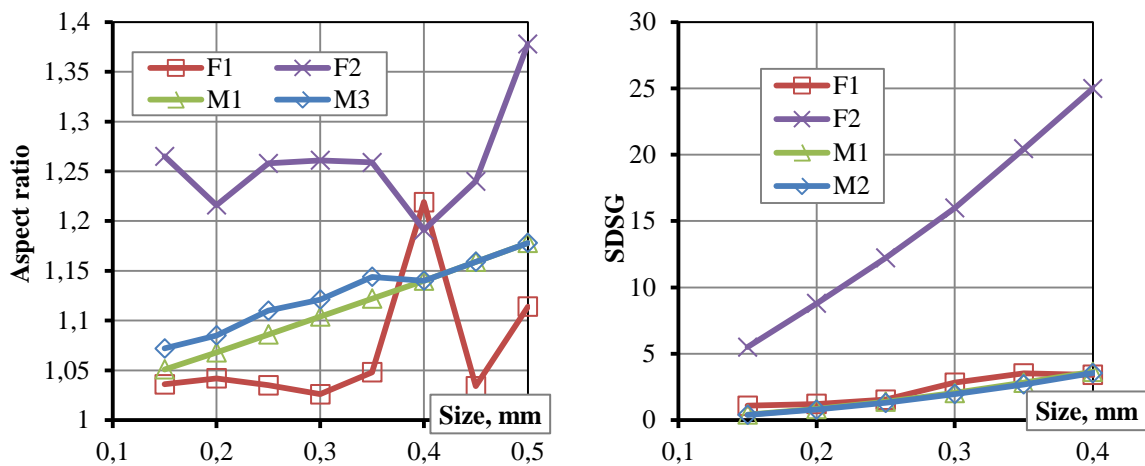


Fig. 4: Mesh quality factors at notch root: a) aspect ratio; b) stress discontinuity error.

The analysis of graphs on Fig. 3b shows that, depending on the mesh preparation method, the assumed calculation accuracy can be obtained for various model sizes. Additional estimation was conducted to establish the problem size for which the assumed error level could be obtained with each model. The estimated problem sizes are presented in Tab. 1 with DOF number and in DOF relative values determined for individual meshes corresponding to DOF for M1 mesh.

Tab. 1: Estimated problem size for obtaining of the assumed  $\delta K_t$  relative error

$\delta K_t$ , %	F1	F2	M1	M2	F1	F2	M1	M2
	mln DOF				DOF / (DOF for M1)			
2	0.144	0.251	0.076	0.055	1.90	3.31	1.00	0.73
1	0.324	0.604	0.195	0.179	1.66	3.11	1.00	0.92
0.2	1.354	2.385	1.182	1.197	1.14	2.02	1.00	1.01

The quality of free mesh is lower than the quality of mapped meshes for which we can observe the linear influence of the element size on aspect ratio. Triangular elements, compared to quadrate elements, require three times as much as DOF quantity in order to reach the error over 1% and double the number of DOF for more precise analyses in which models are of the size above 1 mln of DOF. The shape of the notch sub-area borders has an influence on the calculation accuracy for mapped models of the dimension below 0.1 mln of DOF.

#### 4. Conclusions

The accuracy of FEM analyses for flat specimens with opposite V-notch under tension is not only influenced by the size of the finite element, but also by mesh morphology. Quadrate elements allow for more precise estimation of high gradient stresses which are present at the notch root. Introduction of mapping to the mesh significantly improves the calculation accuracy.

For the mapped meshes, preparation of the notch root surrounding sub-area, in the manner that the generated mesh consists of non-deflected elements, gains crucial importance. The geometrical form of this sub-area border has an impact on the accuracy of analyses for relatively large characteristic sizes of finite elements.

In order to guarantee the clarity of our discussion, the paper assumes a homogeneous value of the finite elements characteristic size in the whole specimen area, also in its gripping part. It allows for separation of effect of the mesh morphology influence on calculation accuracy for each mesh variant. In practice, it is often used the approach which assumes variation of the elements size and the mesh morphology, between the notch area and the rest of the specimen, by refining the mesh at locations of stress gradients. Also, in such cases, the tendencies of relations of estimation accuracy of  $K_t$  to DOF number remain unchanged in the terms of quality. It should be expected that quantity measures shall be subject to change.

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