

COMPUTATION OF AERODYNAMIC DAMPING IN AEROELASTIC SYSTEMS BASED ON ANALYTICAL AND NUMERICAL APPROACH

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Abstract: *The paper describes computation of aerodynamic damping and natural frequencies of aeroelastic systems. The damping is a critical parameter for the stability analysis of aeroelastic systems. Structural damping of the system is important for very low fluid flow velocities, however by increasing the flow velocity, the aerodynamic damping dominates in the instability search. The damping can be evaluated in time or in frequency domain. The presented computation of aerodynamic damping consists of two analytical and one numerical approach. The analytical approaches are represented by the well-known pk method and the unsteady panel method. The pk method is based on Theodorsen unsteady aerodynamics and on the computation of complex eigenvalues of the system as functions of the flow velocity. The unsteady panel method enables the computation of the interaction between aeroelastic system and fluid flow. The aerodynamic damping is evaluated in time domain from the system response to given initial conditions. The numerical approach is based on the finite volume method (FVM) modelling the complete fluid-structure interaction (FSI) coupled problem. The aerodynamic damping is also computed from the system response to a given initial condition. The results of the mentioned methods are compared for the profile NACA 0012 with two degrees of freedom (2-DOF) for plunge and pitch motion around an elastic axis.*

Keywords: aeroelastic instability, aerodynamic damping and eigenfrequency analysis, fluid structure interaction

1. Introduction

Aeroelastic systems are represented by wide range of systems where vibrating structure interacts with fluid. The fluid flow affects the structure and the structure affects the flow field. It yields the coupled problem where governing equations of both structure and fluid have to be solved together. One of the basic important problems is the computation of stability boundaries of the coupled system. These boundaries are usually calculated for critical fluid flow velocities. If a certain value is achieved, the system becomes unstable. Two basic types of instability are usually defined – divergence and flutter. Divergence is defined as instability with negative damping and zero frequency of the motion of the structure. Flutter is defined as instability with negative damping and positive frequency of the motion. Calculation of the damping as a function of the fluid flow velocity represents a classical procedure for the stability investigation. For zero and very low velocities the damping of the structure is usually positive and consists mainly of the structural damping. With increasing the flow velocity the aerodynamic damping becomes more significant. At certain velocity the aerodynamic damping starts to decrease. When the total damping crosses the zero value, the instability occurs (Dowell et al., 1995).

2. Methods

Three different computational methods are introduced. They differ in the form of FSI calculation. The pk method only solves an eigenvalue problem of the FSI system, see e.g. Dowell et al. (1995). The unsteady panel method (Basu & Hancock, 1978) and the FVM (Rodden et al., 1979) enable the FSI solution in time domain. These methods are based on the evaluation of aerodynamic damping and natural frequencies. In all cases considered here, we suppose that the fluid flow is inviscid and incompressible in a 2D computational domain.

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2.1. Pk method

The pk method represents a technique for computation of aeroelastic stability solution where p indicates the eigenvalue and k the reduced frequency. This method is based on the computation of complex eigenvalues of the equation of motion

$$M\ddot{x}(t) + B\dot{x}(t) + Kx(t) = Q(k)x(t), \quad (1)$$

in which for the considered 2-DOF system M, B, K and Q stand for/are mass, damping, stiffness and aerodynamic forces matrices of dimension 2×2 respectively. The matrix of aerodynamic forces is derived for inviscid and incompressible fluid using the Theodorsen unsteady aerodynamics and it is a function of reduced frequency $k = \frac{\omega b}{U_\infty}$ where ω is the natural frequency, b is the reference dimension and U_∞ is the fluid flow velocity. Aerodynamic matrix $Q(k)$ for aeroelastic system with 2 degrees of freedom (DOF) can be found in classic aeroelastic literature, see e.g. Dowell et al., (1995). The eigenvalues are computed in the form of complex conjugate numbers

$$p_{j_{1,2}} = -b_{rj}\Omega_{0j} \pm i\Omega_{0j}\sqrt{1 - b_{rj}^2}. \quad (2)$$

The solution using the pk method is iterative and the convergence criterion is the difference of frequency ω and imaginary part of eigenvalue $|\Omega_{0j}\sqrt{1 - b_{rj}^2} - \omega| < \varepsilon$. Once the convergent solution is computed, the damping ratio is evaluated as

$$b_{rj} = -\frac{\text{Re}(p_j)}{|p_j|}, \quad (3)$$

and the eigenfrequency is evaluated as the positive imaginary part of (2). Results of the pk method are valid in accordance with its derivation only for zero fluid flow velocity, or for the velocity, where the flutter occurs. Nevertheless, the benefits of pk method are reflected in its implementation into commercial software NASTRAN for aeroelastic calculations (Rodden et al., 1979).

2.2. Unsteady panel method

Unsteady panel method was derived for 2D inviscid and incompressible fluid flow as a tool for fluid structure interaction calculation. The calculation of the flow field is based on the solution of the Laplace equation for the total velocity potential $\Delta\phi = 0$. The solution for the total velocity potential is considered as a sum of free stream, source and sink, and vortex components. The geometry of the structure has to be defined and the computational domain is considered to be infinite. Once the complete flow field is computed, the aerodynamic forces acting on the structure can be evaluated. The motion of the structure is described by the equation of motion

$$M\ddot{x}(t) + B\dot{x}(t) + Kx(t) = V(t), \quad (4)$$

where $V(t)$ is the vector of aerodynamic forces. The structure motion and the fluid flow have to be solved for every time step. The response of the structure can be computed for arbitrary initial displacement of the structure at defined flow field velocity. The damping ratio and natural frequencies can be evaluated based on the structure response in time domain, see e.g. Chládek et al., (2016).

2.3. Finite volume method

The inviscid incompressible 2D fluid flow field is described by the Euler equation in the conservative form

$$D \cdot W_t + F_x + G_y = 0, \quad (5)$$

where $W = \{p, u, v\}^T$ is the vector of conservative variables, $F = \{u, u^2 + p, uv\}^T$, $G = \{v, uv, v^2 + p\}^T$ represent inviscid physical fluxes and $D = \text{diag}(0,1,1)$ represents diagonal matrix. The solution in the time domain was calculated by the finite volume method in discrete points of the computational domain where the proper initial and boundary conditions were defined. Computational domain is shown with the boundary conditions in the left-hand side of Fig. 1. On Γ_i , which represents inlet of the channel, there was the Dirichlet boundary condition prescribed in the form $[u, v] = (U_\infty, 0)$. On Γ_o , which represents outlet of the channel, a value of pressure p was prescribed. Γ_w denotes boundary condition of type wall and it was prescribed on the structure and on the walls of the channel. It defined the slip boundary condition in

the case of the inviscid fluid. For given initial conditions of the structure position was the coupled FSI problem solved. In every time step the complete flow field was calculated, the aerodynamic forces were evaluated and the equation of motion (4) was solved. Based on the structure response in time domain, the damping ratios and the natural frequencies were evaluated similarly as for the unsteady panel method.

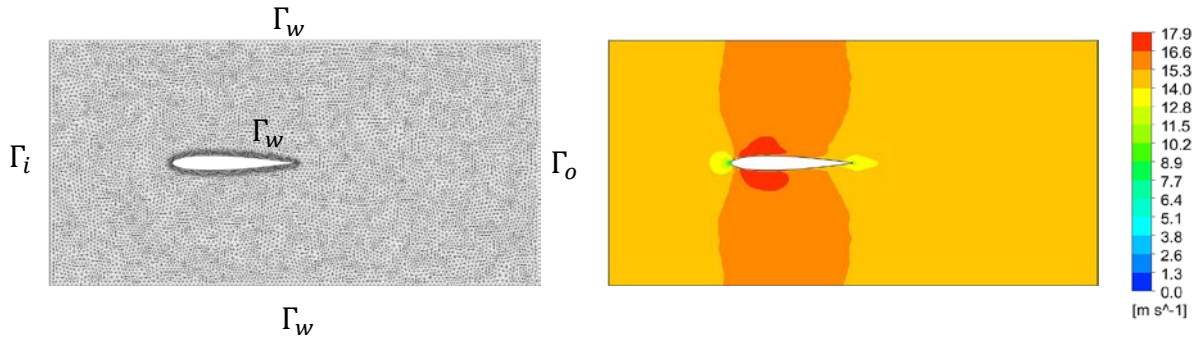


Fig. 1: Computational domain for FVM (left) and example of velocity flow field at $U_\infty = 15 \text{ m.s}^{-1}$ (right).

3. Results and discussion

Algorithms for the pk method and the unsteady panel method were programmed in Matlab. The FVM solution was implemented in the commercial software Ansys/Fluent and the Runge-Kutta method was used for numerical solution of the equation of motion (4) as in the case of the panel method. Structural properties of the profile NACA 0012 were taken from Sváček et al. (2012). Only the static moment was multiplied by -1 to be positive. The profile response was computed for the far field velocities $U_\infty = \{10, 15, 20, 26, 30\} \text{ m.s}^{-1}$. The initial position of the profile was set to $h(0) = 2 \cdot 10^{-3} \text{ m}$ for the vertical translation and $\alpha(0) = 2.86^\circ$ for the rotation. The example of the FVM solution is shown in the right-hand side of Fig. 1 in the form of velocity flow field. The resulting natural frequencies are shown as a function of the flow velocity in Fig. 2 and the evaluated damping ratios are presented in Fig. 3.

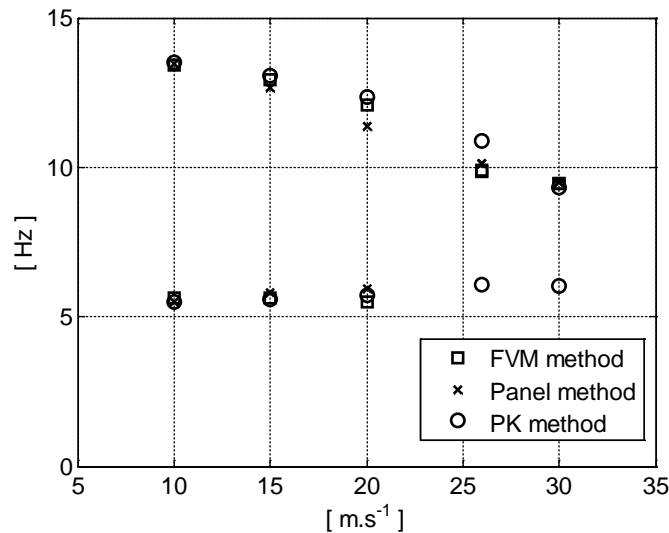


Fig. 2: Comparison of computed natural frequencies as a function of flow velocity.

Considering the data in Fig. 2 it can be concluded that for the flow velocities lower than $U_\infty = 26 \text{ m.s}^{-1}$ here is a good agreement of all three methods, however, for the higher velocities two natural frequencies were computed only by the pk method, while by the panel and FVM methods we computed only the higher natural frequency. Nevertheless, these frequencies are close to the results obtained by the pk method. The results for the damping ratio are not clear. There are some flow field velocities where the results are in good agreement but it is difficult to make some general conclusions. The results of the stability analysis are summarized in Tab. 1. The lowest stability limit was computed by the panel method when the flutter occurs at far field velocity $U_\infty = 26 \text{ m.s}^{-1}$. Using FVM method the flutter velocity was computed at $U_\infty = 28 \text{ m.s}^{-1}$. Finally, the pk method estimated the flutter instability at $U_\infty = 30 \text{ m.s}^{-1}$.

Tab. 1: Aeroelastic stability analysis based on three different computational methods.

Velocity $U [m.s^{-1}]$	Stable or type on instability		
	PK method	Panel method	FVM method
5	stable	stable	stable
10	stable	stable	stable
15	stable	stable	stable
20	stable	stable	stable
26	stable	FLUTTER	stable
28	stable	FLUTTER	FLUTTER
30	FLUTTER	FLUTTER	FLUTTER

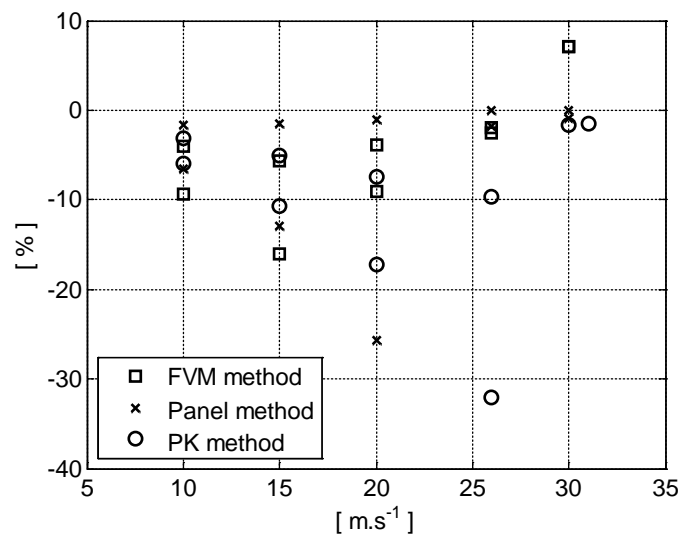


Fig. 3: Comparison of evaluated damping ratios as functions of the air flow velocity.

4. Conclusions

Three different methods for the airfoil stability calculation were compared. The results show a good agreement in case of natural frequency evaluation. Comparison of the damping ratios appeared more complicated and only for some values of the fluid flow velocity a good agreement was obtained. A more detailed study is needed for clarification of such discrepancies.

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