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NUMERICAL MODELING OF MAGNETOSTRICTIVE MATERIALS

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An operator-differential model for magnetostrictive energy harvesting proposed in the literature is used to calculate the amount of harvested energy under some uncertainty in the Preisach density function. The uncertainty is modeled through a fuzzy set approach. In galfenol, an alloy of gallium and iron, however, the hysteresis phenomenon is present only weakly. This allows to propose a simpler, but computationally faster model without hysteresis. The new, simpler model has been identified from the measured magnetic and magnetostrictive cycles. The amount of harvested energy is of the same order in both models. Again, an uncertainty in the identified model can be considered and its impact on the amount of the harvested energy calculated.

Keywords: Magnetostrictive materials, hysteresis, energy harvesting, uncertainty quantification, identification of parameters.

1. Introduction

The modern approach to the mathematical modeling of the materials with memory is based on the use of hysteresis operators. Hysteresis is a time-based dependence of a system's output on present and past inputs. Materials with memory are the subject of research and modeling in various fields, including nonlinear elasticity, moisture transport, magnetism, piezoelectricity, etc. Magnetostrictive materials are used in vibration sensors and energy harvesting devices, for instance. Energy harvesting is a technique for recovering small amounts of any kind of ambient (and otherwise wasted) energy (such as light, vibrations, heat, etc.). In our work, special attention is paid to uncertainty quantification and propagation in the models.

2. Energy harvesting device and its mathematical model

A magnetostrictive galfenol core of a coil with N loops is exposed to a known periodic uniform stress $\sigma(t)$ and produces variations in a background (bias) magnetic field h_0. As a consequence, an electric current is induced in the loops of wire and flows through a resistor (Davino et al., 2014). The phenomenon is described by the Faraday law

$$\frac{d}{dt}\left(\mu_0 f(t)u(t) + \mathcal{P}[u,\lambda_{-1}](t)\right) + \alpha\left(h(t) - h_0(t)\right) = 0, \tag{1}$$

where the unknown function $u(t) = h(t)/f(t)$ is to be found for the time interval $t\epsilon[0,T]$ and an initial value $u(0) = u_0 \epsilon \mathbb{R}$; h is an unknown total magnetic field in the core; μ_0 is the vacuum permeability; $f(t) = f(\sigma(t)) > 0$ is a known function that takes part in the magnetostrictive response to pre-stress; $\mathcal{P}[\cdot,\cdot]$ is the Preisach hysteresis operator; λ_{-1} is an initial memory state; α is a known model parameter dependent on the coil properties.

The model is free of spatial wave propagation, which may play a role for high frequency loading in the range of several tens or hundreds of kilohertz. Energy harvesting is a low frequency application. In addition, realistic spatially distributed data are not easy to get.

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The Preisach model is a multiyield model. It is particularly suitable for modeling energy harvesting due to easy parameter identification and an explicit formula for the instantaneous energy dissipation rate.

The Preisach operator $\mathcal{P}$ allows for taking hysteresis phenomena into account

$$\mathcal{P}[u,\lambda_{-1}](t) = \int_0^\infty \int_0^{\mathfrak{p}_r[u,\lambda_{-1}](t)} \psi(r,v)\mathrm{d}v\mathrm{d}r \tag{2}$$

and is determined by the Preisach density function ψ as well as the play operator $\mathfrak{p}_r\colon (u,\lambda_{-1}) \mapsto \xi_r$, where ξ_r solves a differential variational inequality defined by u_0, u, and λ_{-1}. In (2), t is fixed and $\mathfrak{p}_r[u,\lambda_{-1}](t)$ is a function of r.

It is proved in (Davino et al., 2014) that a unique solution $u\epsilon W^{1,2}(0,T), u(0) = u_0$, exists and depends continuously on initial data. If $h_0(t)$ and $f(t)$ are T_p-periodic functions, then u is T_p-periodic, too. The proof is based on an approach that can serve as a numerical method for solving the problem (1).

A Preisach density function based on galfenol measurements is identified in Davino et al. (2014), see Fig. 5 (left).

Let us recall (1) and $u(t) = h(t)/f(t)$. The energy E harvested during a time period T_p is equal to

$$E(h_0,\alpha,\gamma) = \gamma \int_{\hat{T}}^{\hat{T}+T_p} (h_0 - h(t))^2 \,\mathrm{d}t, \tag{4}$$

where γ is a known model parameter, see (Davino D. et al. (2014)).

Numerical tests indicate that the calculated harvested energy converges with the rate Cs, where s is the time-step size used in solving (1) and $C > 0$ is a constant of order 0 if the bias field h_0 is constant, but of order 2 if h_0 is nonconstant, i.e., periodic.

3. Fuzzy input data

The function ψ_{M} is approximated by a continuous piecewise bilinear function $\psi_{\mathrm{A}}{}^{\mathrm{bilin}}$ defined on a rectangular mesh. Due to the symmetry of ψ_{M}, only one half of the graph of $\psi_{\mathrm{A}}{}^{\mathrm{bilin}}$ is depicted in Fig. 1. The value of $\psi_{\mathrm{A}}{}^{\mathrm{bilin}}$ is uncertain at some mesh nodes and fixed at the others.

The uncertainty is modeled by 16 fuzzy numbers with a triangular membership function defined on $[0.9\psi_{\mathrm{M}}(r_i,v_i), 1.1\psi_{\mathrm{M}}(r_i,v_i)]$, where (r_i,v_i) are the coordinates of the nodes that bear the uncertainty. A fuzzy saturation condition is considered

$$\int_0^{r_\infty} \int_0^{v_\infty} \psi_{\mathrm{A}}{}^{\mathrm{bilin}}(r,v)\mathrm{d}r\mathrm{d}v = c_{\mathrm{fuzzy}}, \tag{5}$$

where r_∞ and v_∞ are sufficiently large and c_{fuzzy} is a fuzzy triangular number. The approximate membership function of the fuzzy harvested energy can be constructed on the basis of the worst and best case scenario problems solved on α-cuts (α-level optimization), see Chapter 5.2.2 (Möller & Beer, 2010).

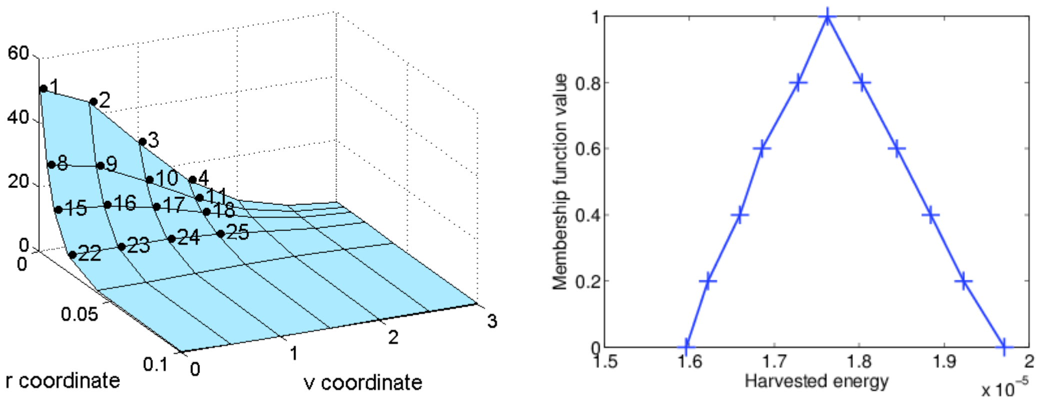

Fig. 1: Left: The graph of $\psi_A{}^{bilin}$ with 16 uncertain nodal values. Right: The calculated membership function of the harvested energy.

4. Identification of a magnetostrictive material model without hysteresis

In galfenol, an alloy of gallium and iron, the effect of the hysteresis is rather small, and this leads us to considering a simplified model where hysteresis is neglected. In this model and for a fixed stress σ, the magnetization m_σ and the strain ε_σ are assumed to depend on a function g and a value $f(\sigma)$ and to take the form

$$m_\sigma(h) = g\left(\frac{h}{f(\sigma)}\right), \qquad \varepsilon_\sigma(h) = -f'(\sigma)G\left(\frac{h}{f(\sigma)}\right) \tag{6}$$

where $h \in \mathbb{R}$ and $G(u) = \int_0^u vg'(v)\mathrm{d}v$, the prime stands for the derivative with respect to the indicated variable. In (6), ε_σ is the inelastic part of the total strain, that is, $\varepsilon_\sigma = \varepsilon_{\text{total}} - \frac{\sigma}{E}$, where E is the Young modulus.

Our goal is to identify g and $f(\sigma)$ from a set of measurements generously provided by Prof. Daniele Davino from Università degli Studi del Sannio di Benevento, see a selection of measured data in Fig. 2. or Davino et al. (2013). For this purpose, the weighted least squares optimization method was used, see Fig. 3 and Fig. 4. It has turned out that g and $f(\sigma)$, see Fig. 5 (right), cannot be uniquely identified unless a restrictive condition such as $G = \int_0^\infty vg'(v)\mathrm{d}v = 1$ is imposed. By (6), the shape of g corresponds to a magnetization curve, cf. Fig. 2 (left).

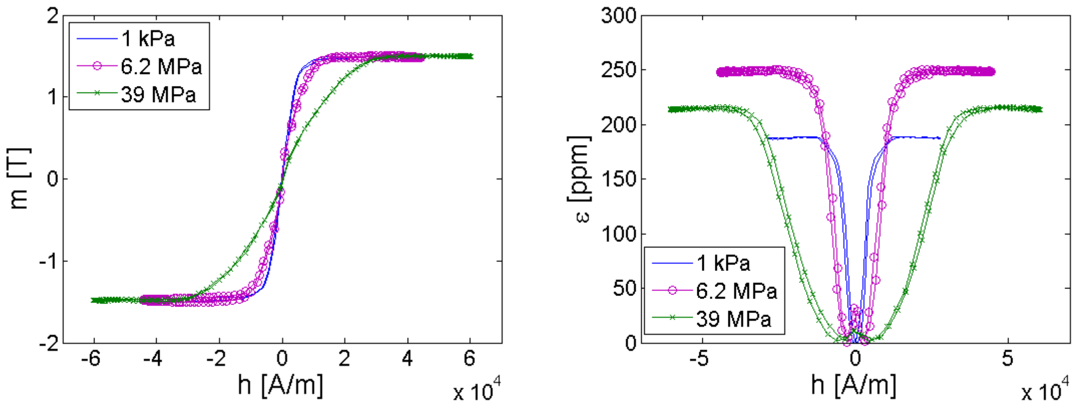

Fig. 2: Magnetic (left) and magnetostrictive (right) cycles at different constant stresses

In the hysteresis-free model, the equation (1) transforms into the following form:

$$\frac{d}{dt}\left(\mu_0 f(t)u(t) + g(u)(t)\right) + \alpha\left(h(t) - h_0(t)\right) = 0. \tag{7}$$

The energy E harvested during a time period is then determined by (4), where $h(t) = u(t) \cdot f(t)$, $u(t)$ solves (7). The difference between the energy values in the model with hysteresis ($E = 1.31061 \cdot 10^{-5}$ for a particular setting) and the model without hysteresis ($E = 2.31054 \cdot 10^{-5}$) is acceptable.

The function g can be considered uncertain and results similar to those depicted in Fig. 1 (right) can be obtained.

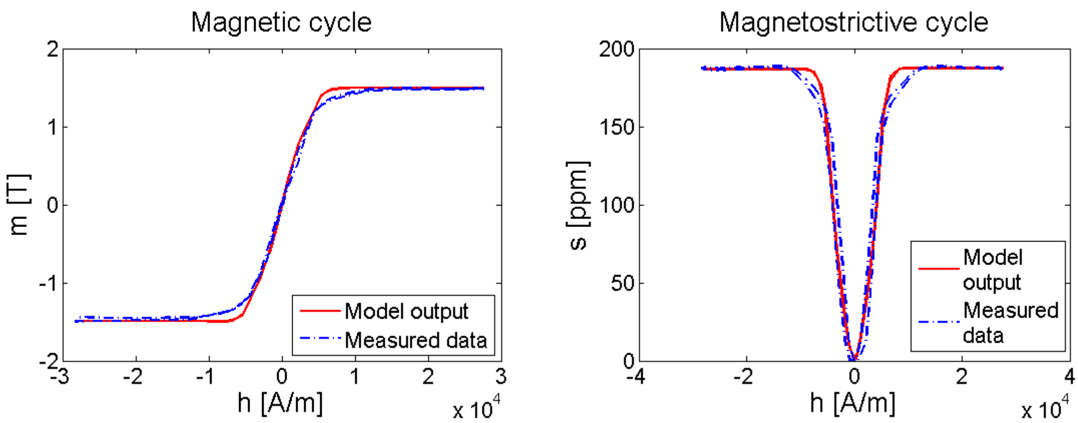

Fig. 3: Difference between the experimental data and the model output for σ_1 = 1 [kPa].

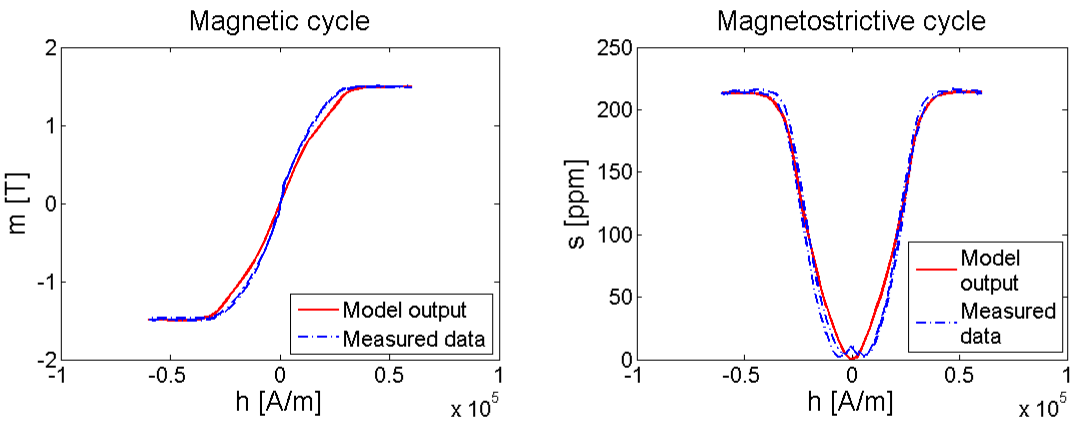

Fig. 4: Difference between the experimental data and the model output for σ_3 = 39 [MPa].

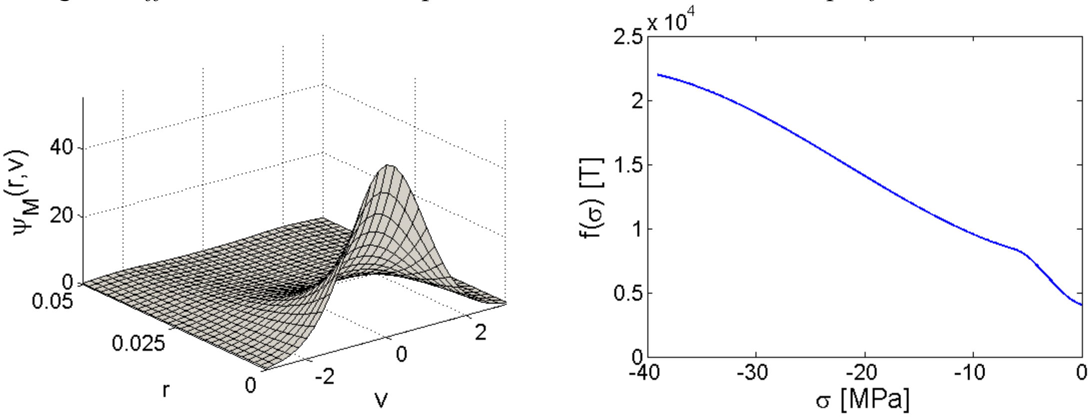

Fig. 5: Left: Identified function ψ_M. Right: Identified function $f(\sigma)$ for a galfenol rod.

5. Conclusions

In the hysteresis model, the bottleneck is in solving the differential equation containing the Preisach operator. Although the best/worst case scenarios are searched for in parallel to speed up the membership function construction, the calculations take hours in the Matlab environment. The model without hysteresis is much faster and its accuracy is on par with the model with hysteresis. As a consequence, the simplified model can be used to accelerate calculations if only a small hysteresis effect is present.

A question has arisen about the degree of the correctness of the Preisach density function published in the literature. A new, nonparametric identification would be useful for further uncertainty quantification.

It can also be observed that the accuracy of the hysteresis-free model is limited especially if the stress is large, see Fig. 4 for instance. A more advanced model with a feedback is the subject of the current research.

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