22nd International Conference

ENGINEERING MECHANICS 2016

Svratka, Czech Republic, 9 – 12 May 2016

COMPUTATIONAL HOMOGENIZATION OF FRESH CONCRETE FLOW AROUND REINFORCING BARS

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Abstract: *Motivated by casting of fresh concrete in reinforced concrete structures, we introduce a numerical model of a steady-state non-Newtonian fluid flow through a porous domain. Our approach combines homogenization techniques to represent the reinforced domain by the Darcy law with an interfacial coupling of the Stokes and Darcy flows through the Beavers-Joseph-Saffman conditions. The ensuing two-scale problem is solved by the Finite Element Method with consistent linearization and the results obtained from the homogenization approach are verified against fully resolved direct numerical simulations.*

Keywords: fresh concrete flow, porous media flow, homogenization, Stokes-Darcy coupling.

1. Introduction

This work is motivated by the computational modeling of self-compacting concrete (SCC). In contrast to the conventional concretes where the aim during its design lies in achievement of desirable compressive strength, SCCs must meet additional rheological requirements, such as higher liquidity, in order to fill in all the possible gaps in the whole form-work while keeping the risk of phase segregation at low level, see (Roussel et al. 2007). For this reason, the focus of the numerical modeling of SCC is not only on the structural, but also on the casting performance, and thus it relies on techniques of computational fluid mechanics.

The constitutive models suitable for structural-scale applications consider concrete as a homogeneous non-Newtonian fluid and the concrete flow can be then efficiently simulated using the Finite Element Method. The efficiency relies on how much details are involved in the computational model. Therefore, sub-scale phenomena can only be accounted for approximately. For example, the effect of traditional reinforcement can be accounted for by heuristic modification of constitutive parameters. This is critical especially in case of modeling of casting processes in highly-reinforced structures, which represent the major field of application for SCC.

In this paper, we propose an efficient approach which incorporates the effects of traditional reinforcement on fresh concrete flow. The tools of computational homogenization will be utilized to avoid the need to resolve flows around each reinforcing bar, which would lead to excessive simulation costs comparable to those of the particle-based models. To this purpose, the structure is decomposed into three parts: reinforcement-free zone occupied by a homogeneous non-Newtonian fluid, reinforced zone where a two-scale homogenization scheme is employed, and homogenization-induced interface separating the reinforced and reinforcement-free zones. At his stage, we restrict ourselves only to steady state flows.

In the reinforced domain, we assume that the reinforcing bars are rigid, acting as obstacles to the flow, and that their size is small compared to a characteristic size of the structure or of the concrete form-work. It now follows from the results of mathematical homogenization theory, that the flow in this region can be accurately approximated by a homogeneous Darcy law, see (Jäger et al. 2001). The relation between the macro-scale pressure gradient and the seepage velocity is defined implicitly, via a micro-

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scale boundary value problem that represents a Stokes flow in the representative volume element (RVE) of the reinforcing pattern, driven by the gradient of the macro-scale pressure. For the numerical treatment of the ensuing two-scale model, we will rely on the variationally-consistent approach developed recently in Sandström & Larsson (2013), which combines the variational multi-scale method with first-order computational homogenization, see also (Hughes et al. 1998).

As a result of the homogenization procedure, an artificial interface appears that separates the Stokes domain from the Darcy domain. In order to couple the flows in both domains, we will employ the Beavers-Joseph-Saffman interface conditions. The ensuing interface constants, relating the traction vector and the relative tangential slip in velocity, can be estimated from an auxiliary boundary value problem at the cell level.

2. Formulation of the Problem

As a point of departure, we consider a Stokes flow over a perforated domain as shown in Fig. 1. We denote, in agreement with Fig. 1, Ω_F as the reinforcement-free part of the domain and Ω_P as a part of the domain with the obstacles (further called perforated sub-domain). Boundary of the obstacles is denoted as $\partial\Omega_P$, while the outer boundary $\partial\Omega_F$ is split into two disjoint parts $\partial\Omega_F^P$ and $\partial\Omega_F^u$ corresponding to the type of applied boundary condition; Γ stands for the interface between the perforated, Ω_F, and the un-perforated, Ω_P, domains. By $\mathbf{n}$, we denote both the outer unit normal vector to $\partial\Omega_F$ and Γ , in the latter case pointed from Ω_P to Ω_F.

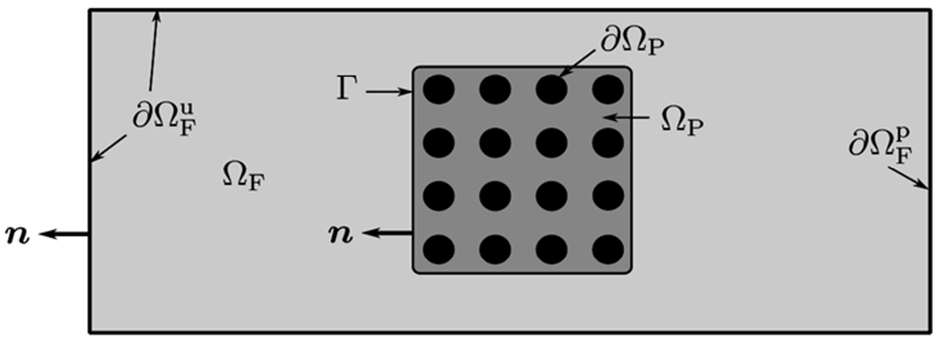

Fig. 1 Stokes flow over the bars modeled as a perforated domain.

The governing equations of the steady-state flow of an incompressible fluid in the union of domains Ω_P and Ω_F take the form

$$\begin{gathered} -\boldsymbol{\nabla}\cdot\boldsymbol{\tau}\big(\boldsymbol{D}(\boldsymbol{u})\big)+\boldsymbol{\nabla}p=\rho\boldsymbol{b} \quad \text{in } \Omega_F\cup\Omega_P \\ \boldsymbol{\nabla}\cdot\boldsymbol{u}=0 \quad \text{in } \Omega_F\cup\Omega_P \\ \boldsymbol{u}=\boldsymbol{0} \quad \text{on } \partial\Omega_P \\ (\boldsymbol{\tau}-p\boldsymbol{I})\cdot\boldsymbol{n}=-\hat{p}\boldsymbol{n} \quad \text{on } \partial\Omega_F^P \\ \boldsymbol{u}=\hat{u}_n\boldsymbol{n} \quad \text{on } \partial\Omega_F^u \end{gathered} \tag{1}$$

Our notation is standard; $\boldsymbol{\tau}$ stands for the deviatoric part of a stress tensor, the strain rate tensor $\mathbf{D}$ is obtained as the symmetrized gradient of the unknown velocity field $\mathbf{u}$, p denotes pressure, $\rho\mathbf{b}$ are body forces, $\mathbf{I}$ is a unit second order tensor and $\hat{u}_n$ and $\hat{p}$ refer to the boundary data.

In order to properly average the flow in the perforated domain Ω_P, we follow the idea of the variational multi-scale method and its application to porous media. The next step then consists in formulation of (1) in a weak sense and in introduction of a decomposition of the unknown pressure field p and its corresponding test funcion δq into the macro-scale and subscale parts as

$$p = p^M + p^S, \quad \delta q = \delta q^M + \delta q^S. \tag{2}$$

By substitution of (2) into the weak formulation of (1), we can split the equations according to the macro-scale and sub-scale part of the test function δq into the the macro-scale and sub-scale problem respectively. Following the procedure introduced in Sandstrom & Larsson (2013), which we skip here for the sake of brevity, the equations on macro-scale can be transformed using the averaging rule and employing the first order homogenization, see (Jäger et al. 2001), into the form of a conservation equation

$$\int_{\Omega_D} \delta q \, (\boldsymbol{\nabla} \cdot \bar{\boldsymbol{u}}) \mathrm{dx} = 0, \tag{3}$$

where Ω_D refers to the homogenized macro-scale domain covered by RVE's. As $\bar{\boldsymbol{u}}$, we denote so called seepage velocity, obtained from the sub-scale problem by a simple averaging rule; the bar refers to the homogenized domain. The relation (3) can be recognized as a Darcy law which governs the flow in a porous media. The flow inside the homogenized domain Ω_D is coupled with the flow in the un-perforated domain Ω_F through the interface Γ with the help of so called Beavers-Joseph-Saffman conditions. These conditions prescribe continuity of the velocities and stresses in the normal direction and relates stress and velocity in the tangential direction (with respect to the interface) through the parameter β, which can be interpreted as a friction parameter. The conditions have the following form

$$\begin{aligned} u_n - \bar{u}_n &= 0 \\ p - \bar{p} &= \boldsymbol{n} \cdot \boldsymbol{\tau} \cdot \boldsymbol{n} \\ \beta(u_t - \bar{u}_t) &= \boldsymbol{n} \cdot \boldsymbol{\tau} \cdot \boldsymbol{t}, \end{aligned} \tag{4}$$

and have to be satisfied on the interface Γ. The parameter β in the last equation of (4) can be determined, in case of a linear Newtonian fluid, from the geometry of the domain. However, up to our best knowledge, there is no way how to determine this parameter in case of non-Newtonian fluid, which is used in our work as a constitutive relation for the concrete. Therefore, the parameter β has to be set up in advance by the rule of a thumb. The sub-scale problem itself represents the Stokes flow solved over the RVE. It comes from the sub-scale part of the weak formulation of (1) with the help of localization of the test functions, see Sandström & Larsson (2013) for the details. It consists in finding $(\boldsymbol{u^S}, p^S)$ such that

$$\begin{aligned} \int_{\Omega_S} \boldsymbol{\nabla} \delta \boldsymbol{w^S} : \boldsymbol{\tau}\left(\boldsymbol{D}(\boldsymbol{u^S})\right) \mathrm{dx} - \int_{\Omega_S} (\boldsymbol{\nabla} \cdot \delta \boldsymbol{w^S}) p^S \mathrm{dx} &= \int_{\Omega_S} \delta \boldsymbol{w} \cdot (\rho \boldsymbol{b} - \bar{\boldsymbol{g}}) \mathrm{dx} \\ \int_{\Omega_S} \delta q \left(\boldsymbol{\nabla} \cdot \boldsymbol{u^S}\right) \mathrm{dx} &= 0. \end{aligned} \tag{5}$$

In above, we denote Ω_S the domain of the RVE and also the super script S refers to the sub-scale problem. The macro-scale problem (3) is coupled with the sub-scale problem (4) in the following way. The sub-scale velocity field u^S is averaged over the domain Ω_S, resulting in $\bar{\boldsymbol{u}}$, which acts on the macro-scale, while the flow on the sub-scale is drive by the macroscopic pressure gradient, in (5) denoted as $\bar{\boldsymbol{g}}$**.**

3. Numerical Examples

In this section, a benchmark test is presented to illustrate the capability and performance of the proposed method. The example illustrates the complex flow of the concrete over the reinforced area. The solution based on the homogenization technique is verified against a fully resolved solution computed by Direct Numerical Simulation (DNS). The reinforced area is located in the middle of the problem domain, so the fluid is not forced to go through the reinforcing bars and the whole situation is closer to real casting problems. The schematic setup of the situation is outlined in Fig. 2, bottom right corner. Uniform velocity is prescribed on the left side, no friction on the top and the bottom and zero "do nothing" boundary condition on the right. The radius of the obstacle in the RVE is chosen as r = 0.25. The fresh concrete is modeled as a Bingham fluid. For numerical purposes, we used modified Bingham model, which has the following form

$$\boldsymbol{\tau} = \left[\mu_0 + \frac{\tau_0}{\sqrt{J_2^D}}\left(1 - \exp(-m\sqrt{J_2^D}\right)\right] \boldsymbol{D} \tag{6}$$

It has two physical parameters, plastic viscosity μ_0 and yield stress τ_0. Additional parameter m governs how close the exponential curve of the modified Bingham model is to the original bilinear one. In the numerical example, the parameters were chosen as $\mu_0 = 20Pa.s$, $\tau_0 = 20Pa$, $m = 15$. The results obtained from the homogenized procedure are in excellent agreement with the DNS, the maximal error is less than 13%. In presented examples, computational time in both approaches is comparable but we assume that in more complex situations, the advantage of the multi-scale approach would be obvious.

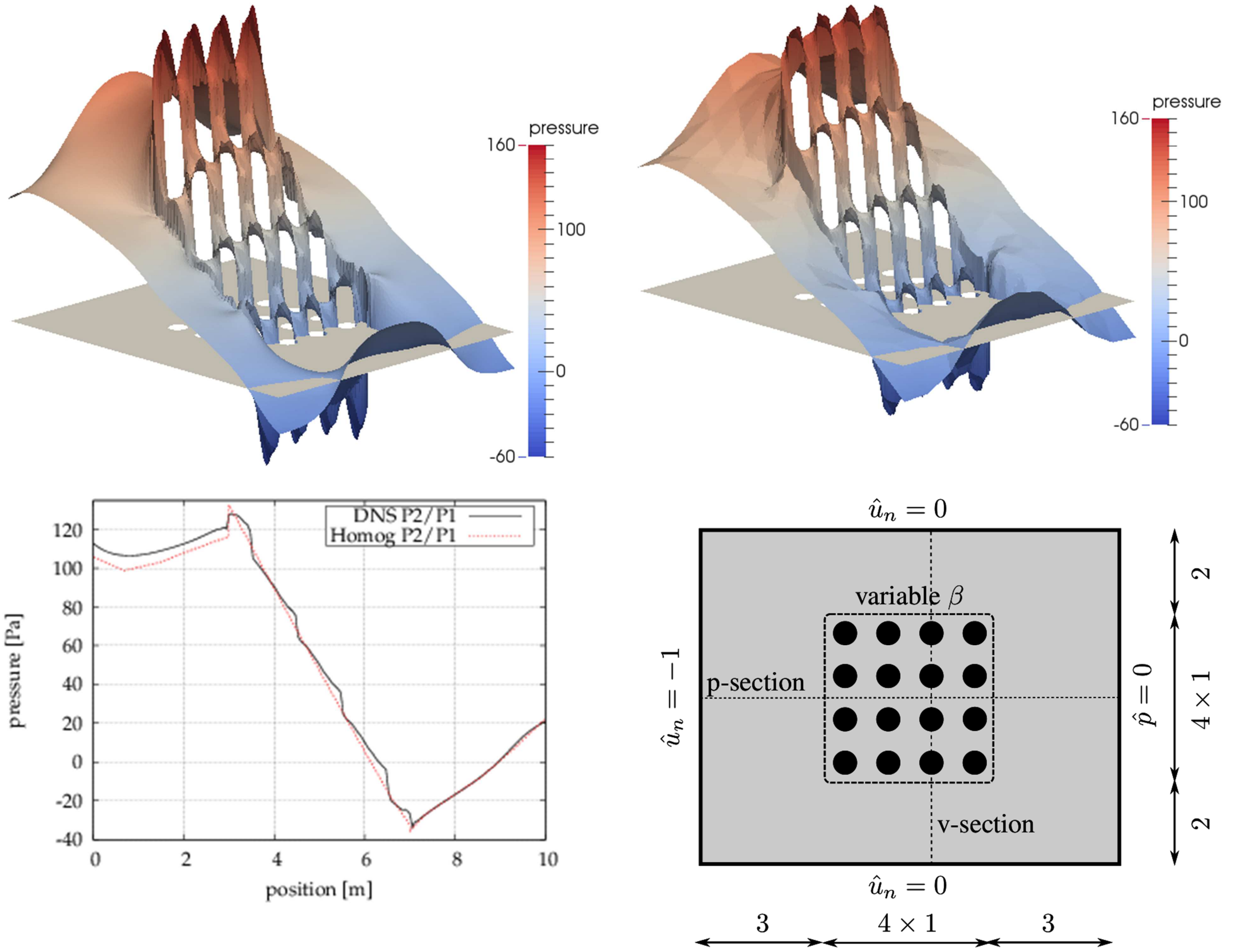

Fig. 2 Top left – fully resolved (DNS) pressure distribution. Top right – pressure distribution obtained with the homogenization approach, reconstructed from the micro-scale. Bottom left Comparison of the pressures along the horizontal section according to the scheme in the bottom left picture.

4. Conclusions

In this paper, the homogenization approach to flow of a Bingham fluid through a porous media is presented. The paper presents unified formulation of coupled Stokes and Darcy flows obtained by consistent homogenization of Stokes flow in porous sub-domain.

Acknowledgement

The research was carried out within the project SGS - Advanced algorithms for numerical analysis and modeling of structures and materials, Project No. SGS16/038/OHK1/1T/11 and the authors gratefully acknowledge the financial contribution of the MŠMT ČR.

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