

PROBABILISTIC CALCULATION USING PARALLEL COMPUTING

M. Krejsa^{*}, J. Brozovsky^{**}, P. Janas^{***}, R. Cajka^{****}, V. Krejsa^{*****}

Abstract: *The paper is focused on methods for the calculations of failure probabilities in structural failure and reliability analysis with special attention to newly developed probabilistic method: Direct Optimized Probabilistic Calculation (DOProC), which is highly efficient in terms of calculation time and the accuracy of the solution. The novelty of the proposed method lies in an optimized numerical integration that does not require any simulation technique. The algorithm of the DOProC based probabilistic calculation is easy to adjust for parallel computing on multiprocessor computers or supercomputers.*

Keywords: Parallel computation, Probabilistic Methods, Monte Carlo, Direct Optimized Probabilistic Calculation, DOProC.

1. Introduction to parallel computing

In recent years computers have become faster and faster due to series of improvements of their individual components. Operating speeds are approaching their physical limits. In spite of all the advantages, current computers still cannot solve many problems in reasonable time. One way to achieve further increase in performance is through the use of a collection of processors that cooperate in the solution process (Kindervater et al., 1989).

New computational techniques are still evolving. Along with powerful computing technology, it enables to solve complex engineering problems (e.g. Brozovsky & Pankaj, 2007; Cajka et al., 2014; Kormanikova & Kotrasova, 2014). Parallelization has found wide application for FEM based calculations and analysis (Kruis et al., 2002; Cho & Hall, 2012; Cermak et al., 2014) and last but not least in probabilistic calculations (Reh et al., 2006) and reliability assessments (Kralik, 2010).

2. Probabilistic methods

Probabilistic methods are used in engineering where a computational model contains random variables (Janas & Krejsa, 2012; Cajka & Krejsa, 2014; Janas et al., 2015). Some works deal with parallel process systems in which the input and output data are fuzzy (e.g. Lozano, 2013). For solution of the theoretical time-invariant structural reliability problem have been developed series of probabilistic methods (summary e.g. Krejsa & Kralik, 2015). The largest and most popular group of probabilistic approaches consists of techniques based on Monte Carlo simulation.

Crude Monte Carlo simulations are employed in several scientific domains due to their capacity to produce realistic results from repeated sampling of events generated by pseudo-random algorithms (Oran et al., 1998). Correct results require a large number of statistically independent events, which has a strong impact on the computing time of the simulation. Nevertheless, Monte Carlo simulations are easily

* Assoc. Prof. Ing. Martin Krejsa, Ph.D.: Department of Structural Mechanics, Faculty of Civil Engineering, VSB-Technical University of Ostrava, Ludvika Podeste 1875/17; 708 33, Ostrava - Poruba; CZ, martin.krejsa@vsb.cz

** Assoc. Prof. Ing. Jiri Brozovsky, Ph.D.: Department of Structural Mechanics, Faculty of Civil Engineering, VSB-Technical University of Ostrava, Ludvika Podeste 1875/17; 708 33, Ostrava - Poruba; CZ, jiri.brozovsky@vsb.cz

*** Assoc. Prof. Ing. Petr Janas, CSc.: Department of Structural Mechanics, Faculty of Civil Engineering, VSB-Technical University of Ostrava, Ludvika Podeste 1875/17; 708 33, Ostrava - Poruba; CZ, petr.janas@vsb.cz

**** Prof. Ing. Radim Cajka, CSc.: Department of Structures, Faculty of Civil Engineering, VSB-Technical University of Ostrava, Ludvika Podeste 1875/17; 708 33, Ostrava - Poruba; CZ, radim.cajka@vsb.cz

***** Ing. Vlastimil Krejsa: Department of Structural Mechanics, Faculty of Civil Engineering, VSB-Technical University of Ostrava, Ludvika Podeste 1875/17; 708 33, Ostrava - Poruba; CZ, vlast.krejsa@seznam.cz

parallelized (e.g. Camarasu-Pop et al., 2013), using various platforms, e.g. in MatLab (Vasek & Krejsa, 2014) or on supercomputer (Konecny et al., 2009). More advanced ways to increase parallel computing efficiency of this simulation technique are under development also (Qiu & Wang, 2011; Nishihara et al., 2014; Vrugt, 2016). As a very powerful tool for parallel programming seems to be currently MatLab included Parallel Computing Toolbox (Sharma & Martin, 2009).

3. Direct Optimized Probabilistic Calculation

The probabilistic method Direct Optimized Probabilistic Calculation – DOProC, is under development now. DOProC solves the probabilistic tasks in a purely numerical way that is based on basis of probability theory and does not require any simulation technique. This is highly effective way of probabilistic calculation in terms of computation time and accuracy of the solution for many probabilistic tasks. The novelty of the proposed method lies in an optimized numerical integration. Summary was published e.g. in (Janas et al., 2010). Application of the DOProC method in solving reliability problems described publications (Krejsa et al., 2013, 2014, 2016).

3.1. Optimization of basic calculation algorithm

Probabilistic calculations of especially complex tasks can be technically and temporally difficult. The computational complexity of the DOProC method is in particular due to:

- The number of random input variables $j = 1, \dots, n$.
- The number of classes (intervals) in histogram for each random input variables N_j .
- Difficulty of the solved task (computational model).
- Algorithm of the probabilistic calculation and the way how the calculation model is defined in the relevant software, e.g. in text mode or in machine code as a dynamic library.

Logically, there should get on optimization techniques that can be applied for two random and independent variables, and also for three or more independent random quantities.

In the probabilistic calculation with two statistically independent input variables come into consideration using of following optimizing steps:

- Interval optimization.
- Zone optimization.
- Simple trend optimization.

Three independent input variables come into consideration in addition to the above:

- Complex trend optimization,
- Grouping of input and output variables,
- Parallelization of probabilistic calculation.

The proposed optimization techniques have been published in details e.g. in (Janas et al., 2010) except the parallelization, which is aimed at further description.

3.2. Parallelization of probabilistic calculation

Probabilistic calculations by DOProC method can be very time consuming, but some parts of computing can proceed simultaneously. The calculation algorithm of DOProC method is advantageous for use on machines with two or more processing units, or their cores. All of computational operations of the DOProC basic algorithm can be divided to as many parts as there are available execution units. After partial calculations partial results can be summarized e.g. into the resulting histogram of safety margin when probabilistic assessment is performed.

Division of probabilistic calculation into sub-sections is appropriate to combine with grouping of input random variables, but this may not be required. Let the resulting random variable B is any function f of independent random variables A_j where j ranges from 1 to n . Then

$$B = f(A_1, A_2, A_3, \dots, A_j, \dots, A_n). \quad (1)$$

Relationship (1) is possible to adjust otherwise. Histogram of random variable B can be e.g. divided into parts:

$$B = B_1 + B_2 + B_3 + \dots + B_p + \dots + B_s, \quad (2)$$

where $B_p = f(A_{1,p}, A_2, \dots, A_j, \dots, A_n)$.

Parts of the histogram of random variable B_p are calculated from all histograms of random variables A_j for $j = 2$ to n , and from the part p of the histogram of random variable A_1 , which is marked as $A_{1,p}$. If the number of these parts is equal to s , then:

$$A_1 = A_{1,1} + A_{1,2} + \dots + A_{1,p} + \dots + A_{1,s}. \quad (3)$$

Parts of the histogram of random variable B in the equation (2) are not summed up by relation (1) but such that is selected width of the class $\Delta b_{1,p}$ for all parts the same, i.e. whether for the histogram of random variable B , and totaled probabilities in each class of all parts of histograms B_p of random variable B for $p = (1, \dots, p, \dots, s)$.

This procedure ensures that the result of parallel computing is theoretically identical with the results of non-parallel calculation. In case of computing with different widths of classes $\Delta b_{1,p}$ of all parts of histograms B_p under calculation the resulting histogram of random variable B according (2), the parallel computing of the histogram of the random variable B may vary somewhat from the non-parallel computing. The difference of results is not usually significant and even this method of calculation can be considered as correct.

4. Conclusions

This paper discussed possibilities of probabilistic calculation using parallel computing with special focus on new method under development – the Direct Optimized Probabilistic Calculation (“DOProC”). The highlight of the DOProC lies in an efficient and accurate optimized numerical integration and the possibility of straightforward implementation on parallel systems. Thanks to these properties DOProC method can be useful for solution of many probabilistic tasks and failure analysis.

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