

## **AN EXAMPLE OF MARKOV MODEL OF TECHNICAL OBJECTS MAINTENANCE PROCESS**

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**Abstract:** *The paper presents the usefulness diagnosis method of the technical objects to realise the tasks received by the controlling subsystem. The investigation object being the basis to illustrate the considerations presented here in, is a municipal-transport bus maintenance system in a selected urban agglomeration. In order to solve the discussed problem it has been assumed that the mathematical model of the bus maintenance process is homogenous Markov process. As a criterion to select (purchase) a specific bus type to be maintained, the usefulness potential usage rate. The paper does not cover investigation of the economic aspects related to the purchase of a specific bus type.*

**Keywords:** diagnosis, homogenous Markov process, maintenance process.

### **1. Introduction**

The investigation object is the urban transport bus maintenance system in the chosen urban agglomeration. The effects resulting from the activity of the analysed system are precisely associated with efficiency of management, which is defined as an ability to control resources, processes and information for their optimal use in the business. There are various different technical objects on the market, and therefore it is useful to develop a tool, for decision-makers, which enables the rational (based on accepted criterion) selection of buses in the analysed system.

The primary aim of studied system is an effective and safe carriage of passengers by urban transport buses in defined quantitative and territorial range (Landowski B., Woropay M., Neubauer A., 2004, Woropay M., Knopik L., Landowski B., 2001).

An Example of Markov Model of Technical Objects Maintenance Process

A subsystem which is directly responsible for performing tasks of the system is the executive subsystem which contains basic subsystems of type <C – OT> (driver – bus), wherein a man is coupled by serial structure with a technical object. Reliability of operated technical objects is maintained at an appropriate level as a result of the operating processes in the subsystem ensuring roadworthiness.

The analysed system have two bus depots in which service stations are located.

The roadworthiness and diagnosis processes are ensured in the service stations and in particular: daily services, periodic maintenance, current repairs and technical inspections of the vehicles.

The subsystem ensuring roadworthiness also contains another subsystems so called mobile decentralised departments, the set of emergency technical service units. The main task of these departments is to restore as soon as possible the roadworthiness of buses which are outside the bus depots or, in case it is impossible to fix the buses outside the depots, to tow out-of-order buses to a service station.

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The result of assumption and limitation analysis shows that Markov process and the theory of the process analysis has been considered as the best tool, from research purpose point of view, for mathematical modelling of real maintenance process of the investigation object.

## 2. Selected results of the preliminary maintenance tests

The time analysis in selected operating states were carried out during the preliminary maintenance tests conducted in real urban transport bus maintenance system (Woropay M, Landowski B, Perczyński D., 2004). The tests were carried out in natural conditions of maintenance by using passive observation method.

The results presented in the following part of the paper refer to Mann (16 vehicles) and Volvo bus brand (90 vehicles) which are operated in the analysed system. Table 1 and Table 2 present the results of preliminary tests regarding selected time statistics of proper bus operation ( $T_1$ ), bus repair done by emergency technical service units ( $T_2$ ) and bus repair done by service stations ( $T_3$ ) (Woropay M, Landowski B, Perczyński D., 2004).

*Tab. 1. Values of selected statistics of examined characteristics for Volvo bus brand*

<b>Volvo</b>			
<b>Statistics</b>	<b>T<sub>1</sub></b>	<b>T<sub>2</sub></b>	<b>T<sub>3</sub></b>
Number of observations	823	474	407
Average value	242.5	1.13	1.67
Standard deviation	289.9	0.72	0.85
Minimum	0.34	0.03	0.47
Maximum	2237.67	4.88	7.23
Range	2237.33	4.85	6.76
Variance	84092.40	0.51	0.73
Median	143.65	143.65	1.40
Mode	24	1.00	1.07

*Tab. 2. Values of selected statistics of examined characteristics for Mann bus brand*

<b>Mann</b>			
<b>Statistics</b>	<b>T<sub>1</sub></b>	<b>T<sub>2</sub></b>	<b>T<sub>3</sub></b>
Number of observations	174	89	87
Average value	295.06	1.11	1.71
Standard deviation	353.74	0.54	1.15
Minimum	7.22	0.05	0.23
Maximum	1824	2.75	6.17
Range	1816.78	2.70	5.94
Variance	125131.0	0.29	1.32
Median	165.81	1.15	1.30
Mode	48	1.67	0.92

## 3. Maintenance process model carried out in the investigation object

The investigation object identification resulted in defining three, important for research purposes, maintenance states of the buses, i.e. (Woropay M, Landowski B, Perczyński D., 2004):

$S_1$  - operating state, realisation of transport tasks,

$S_2$  - repair state done by emergency technical service units,

$S_3$  - repair state done by service stations,

It is assumed that the preliminary mathematical model of bus maintenance process is stochastic process  $\{X(t), t \geq 0\}$ . Analysed stochastic process  $\{X(t), t \geq 0\}$  has finite phase space  $S$ ,  $S = \{S_1, S_2, S_3\}$ . It is

assumed that the theory of homogenous Markov processes is used for description of maintenance process of analysed technical objects. Bus state space and maintenance event analysis enables to create a digraph, presented in Fig. 1, that imitate the maintenance process carried out in the investigation object.

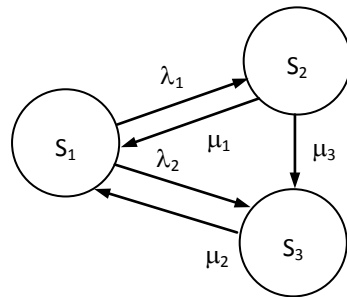


Fig. 1: Digraph of maintenance process state

Transition intensity of analysed process is included in so called transition intensity matrix  $\Lambda$ :

$$\Lambda = \begin{bmatrix} -(\lambda_1 + \lambda_2) & \lambda_1 & \lambda_2 \\ \mu_1 & -(\mu_1 + \mu_3) & \mu_3 \\ \mu_2 & 0 & -\mu_2 \end{bmatrix}$$

Through using the theory of Markov processes it possible to determine probabilities  $P_i(t)$ ,  $i = 1, 2, 3$ , of technical objects positions in selected maintenance states  $S_i$  at the moment  $t$  for maintenance process model. For that purpose it is needed to solve A. N. Kolmogorov system of differential equations:

$$P'(t) = P(t)\Lambda,$$

where:

$P'(t)$ -column vector composed of derivatives  $P_i'(t)$ ,

$P(t)$ -unconditional probability vector  $P_i(t)$ ,

$\Lambda$ -transition intensity matrix of process states.

A calculation algorithm has been developed to determine values  $P_i(t) = P(X(t)=S_i)$ ,  $i=1, 2, 3$ , and a computer programme has been designed (Knopik L., Landowski B., Perczyński D. 2002).

One of the research aims is to determine the distribution of random variables  $T_i$ ,  $i = 1, 2, 3$ , indicating duration of analysed maintenance states  $S_i \in S$  of Mann and Volvo bus brands. The null hypothesis  $H_0$ , stating that empirical distribution of random variable  $T_i$  complies with exponential distribution, is verified for each of the states  $S_i$  ( $i=1,2,3$ ). Compatibility test  $\chi^2$  (Pearson) is used to verify the hypothesis. There is no scientific basis to reject the verified hypothesis (level of significance  $\alpha=0.05$ ) for random variables  $T_i$ ,  $i=1,2,3$ , (Mann) and  $T_i=2,3$ , (Volvo). The verified hypothesis is rejected for random variable  $T_1$  (meaning time of proper operation state of Volvo bus brand). However, the positive result of compliance of the analysed random variable empirical distribution with Erlang distribution has been obtained. Then it is possible to present random variable  $T_1$  as follows (Buslenko N., Kałasznikow W., Kowalenko I., 1979):

$$T_1 = T_{11} + T_{12} + \dots + T_{1k},$$

where:  $T_{1i}$  - independent random variables with exponential distribution with parameter  $\lambda$ .

Transition from state  $S_{i1}$  ( $1 \leq i \leq k-1$ ) to state  $S_{i+1}$  is the only possible transition.

Minimum sum of squared deviations of Erlang distribution function  $F(x)$  from the value of empirical distribution  $F_e(x)$  is assumed as the criterion of optimal selection of parameters  $\lambda$  and  $k$ .

The calculation algorithm has been developed to estimate values for parameters  $k$  and  $\lambda$ .

The process  $\{X(t), t \geq 0\}$  with state space  $S_i$  has been transformed into the process  $\{Y(t), t \geq 0\}$  with state space  $ST$  for which time distributions of states are exponential distributions. on the cost of state space enlargement.

Various different technical objects are used in the urban transport maintenance system. When there is a need to purchase buses, the important problem for decision-makers is which brand should be chosen. In the paper, the roadworthiness potential is the criterion for buying specific technical object and can be written as:

$$W_u(t_a) = \frac{\int_0^{t_a} P_1(t) dt}{t_a},$$

where:

$t_a$ —analysis time,

$P_1(t)$ —probability of bus operating state.

The value of the indicator depends on the value of parameters which characterise the model (maintenance process of selected brand bus) and can be a diagnostic signal for decision-makers regarding usefulness of selected technical object for maintenance under operating conditions of the system.

The value of the indicator  $W_u(t_a)$  for two selected brands of buses has been determined to present the key idea of the paper.

The analysis of the indicator shows that purchase Volvo buses ( $W_u=0,9961$ ) is a better choice than Mann buses ( $W_u=0,9912$ ).

#### 4. Conclusions

The aim of the study was to present the possibilities of using Markov process model operation of technical facilities for pre-diagnose their suitability for the tasks adopted by the controlling subsystem, forecasting the state of the system operation after changing the input parameters of the model. By changing the input parameters there were simulated the impact of internal and external factors on the behaviour of the system.

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