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EVALUATION OF SAFETY OF MODELS FOR ASSESSMENT OF THE PUNCHING SHEAR RESISTANCE OF FLAT SLABS WITHOUT SHEAR REINFORCEMENT

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Abstract: *Reinforced concrete flat slabs are frequently used structural members in buildings construction. The most dangerous failure of these systems, as well as the most usual damage, is punching of the slab by column. Safety verification and avoidance of failure due to punching in the vicinity of a column is currently performed using empirical model which is introduced in Eurocode 2 (EC2 model). However, extensive discussions are held about safety of the EC2 model. The paper deals with statistical evaluation of EC2 model safety and with its comparison with the other relevant models for assessment of punching resistance (Model Code 2010, ČSN 731201, etc.). Database which includes results of more than 400 experimental tests of flat slab specimens has been used for the statistical evaluation*

Keywords: Punching Shear Resistance, Flat Slab, Safety, Reinforced Concrete

1. Introduction

Reinforced concrete slabs supported on columns are common in residential and commercial buildings. The most usual damage of these systems is punching of slab by support (column or corner walls). Punching of the slabs is caused by shear forces which are concentrated on the small area at the vicinity of a column. Such failures are usually associated with very small deformations and crack widths prior to failure and, as such, can occur without noticeable warning signs. Furthermore, the mechanism of punching failure is still poorly understood. The punching provisions in codes of practice are based on different theories or on empirical formulae, thus in some cases leading to very different strength predictions.
Design model for assessment of punching resistance in EC2 is model which was originally published in Model Code 1990. The model is very empirical because main parameters having influence on punching resistance were statistically determined using results of some experiments. The most of the experiments used for calibration were performed on the slab specimens with inner column. Therefore a question was raised if calibrated model is applicable also for slab in areas with edge or corner columns or for foundation slabs and footings.
Because database of experimental results has been significantly increased within last 20 years it is interesting to check reliability of currently used models and if necessary to recalibrate them. This is example of EC2 model update, which was released in January 2016 (*Hegger*, *Siburg*, *Kueres*) from RTWH Aachen. Besides empirical models there were developed new mechanical models within last decade. Among them is model based on the Critical Shear Crack Theory (CSCT), developed by *Muttoni and Schwartz* and updated by *Muttoni and Ruiz (2008, 2012)* or model of *A. Marí, A. Cladera* from Spain. Because many flat slabs were design using ČSN 731201 standard on the territory of Czech and Slovak republic the statistical evaluation has been also carried out for this model.

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2. Models for assessment of the shear resistance of flat slabs without shear reinforcement

As it has already been mentioned the statistical safety evaluation was carried out for several different models. The first one is ČSN model, the second one is current EC2 model, the third one is German update of EC2 model from 2016 and the others are models based on CSCT theory.

2.1. Control perimeter

The models for assessment of punching resistance differ each other with position of basic control perimeter and with the inclination of modeling shear crack. Current Eurocode has basic control perimeter at distance $2d$ from edge of a column, the others $0.5d$ and ČSN model $0.5h$ where d is average effective depth of a slab at column area and h is a thickness of slab, see fig. 1.

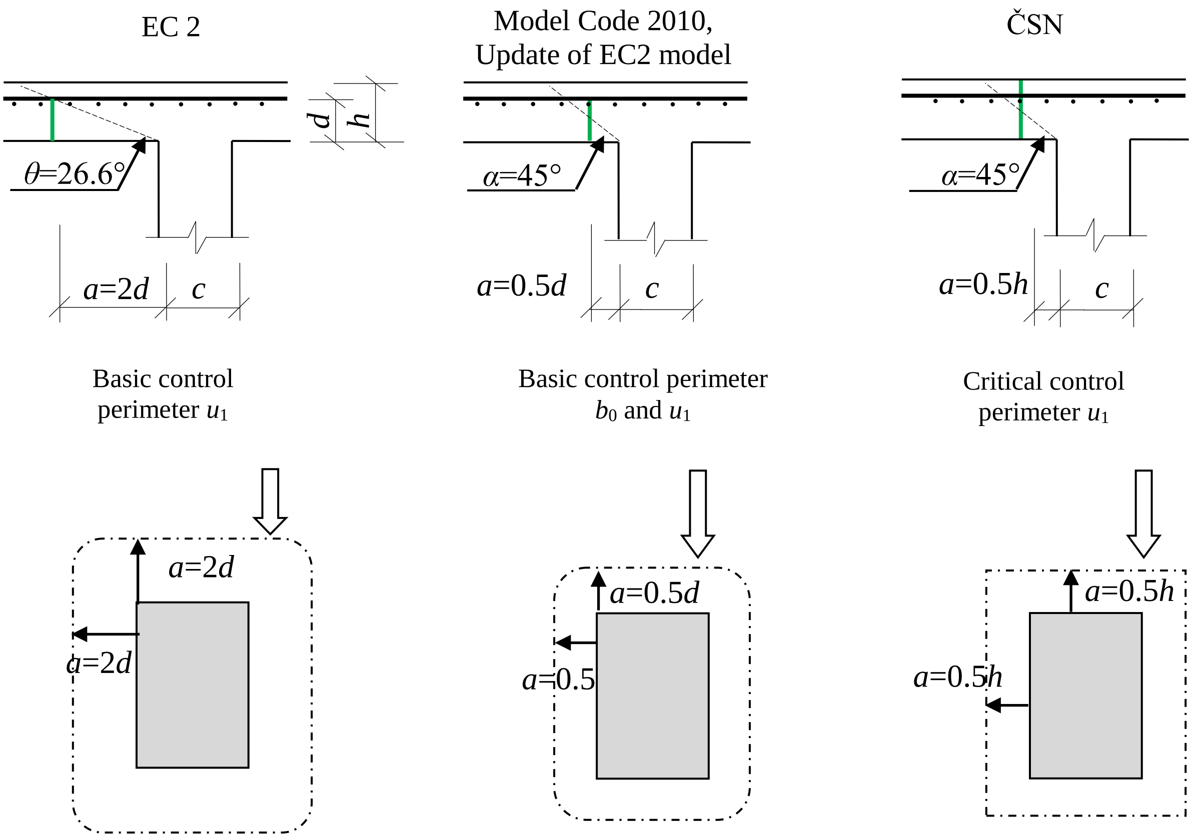

Fig. 1: Control perimeters for punching verification in codes of practice

2.2. Calculation of the shear resistance

In all major codes of practice, punching strength of flat slabs is verified by comparing the design shear strength $v_{Rd,c}$ of an element to a design shear stress v_{Ed} on a unit length of a control perimeter around a column or a loaded area. Effect of unbalanced moments is taking into account by factor β.

$$v_{Ed} = \beta * V_{Ed}/(u_1 * d) \leq v_{Rd,c} \quad (1)$$

The shear resistance $v_{Rd,c}$ generally depends on the following factors. Tensile strength of concrete, slab effective depth (size effect), maximum aggregate size $d_{g,max}$, slab rotation ψ, shear slenderness a_λ/d.

2.2.1 Empirical EC2 models and its update

Punching shear resistance without shear reinforcement can be determined using formula (2), according to current EC2 model, model “B”.

$$v_{Rd,c} = C_{Rd,c} * k * (100 * \rho_l * f_{ck})^{1/3} \text{ [MPa]} \quad (2)$$

Where design value of empirical factor $C_{Rd,c} = C_{Rk,c}/\gamma_C$ [MPa], with $C_{Rk,c} = 0.180$ MPa, ρ_l is reinforcement ratio [-] and f_{ck} is characteristic compressive strength of concrete [MPa].

German group proposed amendment of EC2 model. The model, released in January 2016 (*Hegger, Siburg, Kueres*), (model "B") is based on the similar formula with current EC2 model, see (3), where basic control perimeter u_1 has been moved closer to the column, at distance 0.5*d* from the edge of loaded area:

$$v_{Rd,c} = C_{Rd,c} * k_d * k_\lambda * (100 * \rho_l * f_{ck})^{1/3} \text{ [MPa]} \qquad (3)$$

Moreover, new factors were calibrated taking into account influence of shear slenderness and dimension of column periphery by coefficient $k_\lambda = [(a_\lambda/d)*(u_0/d)]^{-1/5}$, where a_λ is the distance between the edge of the loaded area and the line of contra flexure, u_0 is the perimeter of the loaded area. Coefficient taking into account size has been updated by formula $k_d = 1/[1+(d/200)]^{1/2}$. Empirical factor $C_{Rk,c}$ has a new value of $C_{Rk,c} = 1.80$ MPa.

2.2.2 Model of Critical Shear Crack Theory (CSCT)

CSCT is mechanical model for assessment of punching resistance. The principles of the theory came out from the assumption of critical crack development at the vicinity of a column. Punching resistance is ensured by aggregates interlocking in the critical crack and by tensile strength of the concrete. Shear resistance $v_{Rd,c}$ then depends on friction in the critical crack which is directly influenced by the crack width and by the maximum aggregate size d_g . The crack width is proportional to the slab rotation (ψ) at the vicinity of a column.

$$v_{Rd,c} = k_\psi * \sqrt{f_{ck}}/\gamma_C \quad \text{[MPa]} \qquad (4)$$

According to MC2010/CSCT $\quad k_\psi = 0.67/(1+0.6*k_{dg}*\psi*d) \leq 0.6 \qquad (5)$

According to EC2/CSCT $\quad k_\psi = 0.7/(1+0.45*k_{dg}*\psi*d) \leq 0.6 \qquad (6)$

Where coefficient k_{gd} depends on the maximum aggregate side d_g, $k_{dg}=32/(16+d_g)$ [mm].

Rotation of the slab ψ depends on several factors, mainly on strains in bending reinforcement crossing critical shear crack and on the shear slenderness r_s/d , where r_s is distance of zero radial bending moments with respect to support axis. The rotation can be calculated simply by assuming of steel yielding (7), level of approximation LoA I, e.g. for preliminary design, or by more precise procedure (8) LoA II and (9) LoAIII, where m_{Ed} is average design moment per unit length and m_{Rd} is bending resistance. Because all tests used for statistical evaluation have had concentric load, the m_{Ed} has been calculated by $m_{Ed} = V_{R,test}/8$.

$$\psi = 1.5*(r_s/d)*(f_{yd}/E_s) \qquad (7)$$

$$\psi = 1.5*(r_s/d)*(f_{yd}/E_s)*(m_{Ed}/m_{Rd})^{1.5} \qquad (8)$$

$$\psi = 1.2*(r_s/d)*(f_{yd}/E_s)*(m_{Ed}/m_{Rd})^{1.5} \qquad (9)$$

Together four different CSCT models were analyzed. The model "D" where k_ψ was determined by (5) and (7), model "E" using (5) and (8), model "F" using (5) and (9), and finally "G" with (6) and (9).

3. Statistical evaluation of the models for punching resistance

Statistical evaluation of the models for punching resistance without shear reinforcement has been carried out for six models with partial safety factor $\gamma_c = 1{,}0$. Cylinder strength of concrete f_{ck} introduced by authors of the experiments has been used. Control perimeters were assumed at distance 2*d* from the face of column for current EC2 model, *h*/2 for ČSN model, and *d*/2 for the other models. Main statistical variable in the evaluation was ratio $P_i = (V_{R,test}/V_{Rd,c})_i$, where "*i*" is number of a test, $V_{R,test}$ is a resistance obtained from an experimental test and $V_{Rd,c}$ (Q_{bu}) is punching resistance obtained from theoretical model. Only variables P_i which satisfy condition $0.5 < P_i < 2.0$ have been used in statistical evaluation. Mean value P_m was calculated using formula $P_m = (\Sigma P_i)/n$ where *n* is a number of assumed tests. Characteristic value was determined as 5% fractile for Gaussian distribution $P_{k,0.05} = P_m(1\text{-}1{,}645.V_p)$, where V_P is coefficient of variation $V_P = \sigma_P / P_m$ and σ_P is standard deviation $\sigma_{P2} = [\Sigma(P_i - P_m)^2]/(n-1)$. The target value of $P_{k,0.05}$ is 1.0 according to EN1990. However resistance models can be assumed reliable if $P_{k,0.05} > 0,85$, e.g. due to membrane forces which are present in real structures.

Tab. 1: Statistical evaluation of model safety

Models		Number of specimens [n]	Average value [P_m]	Variation coef. [V_p]	Characteristic value [$P_{k,0.05}$]
"A"	ČSN 731201	406	1.1219	0.2344	0.689
"B"	EC2 model	408	1.1679	0.2007	0.782
"C"	EC2 update 2016	389	1.1786	0.1614	0.866
"D"	MC2010 (LoA I)	127	1.6297	0.1607	1.199
"E"	MC 2010 (LoA II)	182	1.4109	0.2177	0.906
"F"	MC 2010 (LoA III)	192	1.3058	0.2130	0.848
"G"	CSCT/EC2	194	1.1213	0.1992	0.754

4. Conclusions

Technical committee CEN TC250/SC2/WG1/TG4 holds extensive discussions how to proceed with model for punching resistance in Eurocode 2 in connection with works on the second generation of Eurocodes. Based on our assessment the models with accepted level of reliability are German EC2 update (2016) and models that are based on CSCT theory from Model Code 2010. Current EC2 model "B" and model with EC2 format based on CSCT "G" does not have required safety. Opposite model "D" based on CSCT for level of approximation LoA I is pretty uneconomical and therefore does not suits much for standards. The best solution for Eurocode 2 represents empirical model "C" with $P_{k,0.05} = 0,87$ and physical model "F" with $P_{k,0.05} = 0,85$. The advantage of "C" model is similarity with current EC2 model, which is very convenient for engineers. However the model has some limitations in application for FRC, LWAC, members reinforced by FRP and for design of strengthening for punching. Physical models based on CSCT theory can be much simpler adapted for above mentioned issues.

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References

Muttoni A., Fernández Ruiz M. (2008) Shear strength of members without transverse reinforcement as function of critical shear crack width, ACI Structural Journal, V. 105, No 2, 2008, pp. 163-172

Muttoni, A., Fernández Ruiz, M., (2012) The levels-of-approximation approach in MC 2010: applications to punching shear provisions, Structural Concrete, Ernst & Sohn, Germany, Vol. 13, No. 1, 2012, pp. 32-41

Hegger J., Häusler F., Ricker M.(2008) Critical review of the punching shear provisions according to Eurocode 2 (in German: Zur Durchstanzbemessung von Flachdecken nach Eurocode 2), Beton- und Stahlbetonbau, Vol. 103, No. 2, 2008, pp. 93-102

Siburg C., Hausler F., Hegger,J. (2012) Flat slab punching design according to German annex of Eurocode 2, Bauinginieur, Volum 87,May 2012, pp.216-225

EN1992-1-1 Design of Concrete Structures, Part 1-1 General Rules and Rules for Buildings, May 2004

Fédération Internationale du Béton (fib), Model Code 2010 - Final draft, Vol. 1, fédération internationale du béton, Bulletin 65, Lausanne, Switzerland, 2012, Vol. 2

ČSN 731201/86 Navrhovaní betónových konstrukci