22nd International Conference
ENGINEERING MECHANICS 2016
Svratka, Czech Republic, 9 – 12 May 2016

INVESTIGATION OF DYNAMIC BEHAVIOUR OF AN INVERTED PENDULUM DRIVEN BY CABLES REPRESENTED BY POINT-MASS MODEL

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Abstract: *The paper deals with point-mass modelling of cables, which are suitable for involving in dynamics of fibre mechanisms. The motivation is the development of a cable model, which could be efficient for the usage in a mechatronic model of a manipulator consisting of cables and an end-effector whose motion is driven by cables – particularly for the usage in the model of QuadroSphere. At previous research into the approach to the cable modelling, an inverted pendulum driven by two fibres (cables) attached to actuators was chosen as a suitable reference object. Its properties were investigated applying a calculation model. This paper is focused on using the point-mass model of cables, utilization of which generally proved to be very prospective for cable modelling. The effect of cable preload, the influence of the amplitude of the harmonic kinematic excitation of cables, the mass effect of cables and the influence of non-symmetric harmonic excitation on the pendulum motion were investigated. The influence of these crucial parameters of the system of an inverted pendulum driven by fibres (cables) on its dynamic behaviour is evaluated.*

Keywords: Cable, Inverted pendulum, Point-mass model, Multibody modelling, Vibration.

1. Introduction

Replacing rigid elements of manipulators or mechanisms (Chan, 2005) by flexible cables can be advantageous in the achievement of a lower moving inertia, which leads to a higher machine speed. Drawbacks can be associated with the fact that cables should be only in tension (Smrž & Valášek, 2009; Valášek & Karásek, 2009) in the course of a motion. The possible cable modelling approaches should be tested and their suitability verified in order to create efficient mathematical models of cable-based manipulators mainly intended for the control algorithm design.

At previous research into the approach to the cables modelling, an inverted pendulum driven by two fibres (cables) attached to actuators was chosen as a suitable reference object (see Fig. 1 left; e.g. Polach et al., 2012b), which is a simplified representation of a typical cable manipulator. Research into cable modelling and experimental verification of their usability has advanced (e.g. Polach & Hajžman, 2015; Dupal & Byrtus, 2015). Utilization of the point-mass model for cable modelling proved to be very prospective (Polach et al., 2015) in comparison with other approaches (see Chapter 2). Simulations with the inverted pendulum with the point-mass model of cables were applied in mapping the influence of some crucial parameters on dynamic behaviour of mechanical (or mechatronic) systems of this type. The effect of the cable preload (Polach & Hajžman, 2012b), the influence of the amplitude of the harmonic kinematic excitation of cables (Polach & Hajžman, 2012c), the effect of cable mass (Polach et al., 2012a) and the influence of non-symmetric harmonic excitation (Polach & Hajžman, 2012a) on the pendulum motion were investigated. In this paper, complete evaluation of the influence of crucial parameters on dynamic behaviour of the investigated system is given. This evaluation has not been presented yet.

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2. Possibilities of the cable modelling

The cable (fibre, wire etc.) modelling should be based on considering the cable flexibility and suitable approaches can be based on the flexible multibody dynamics (see Shabana, 1997; Gerstmayr et al., 2012). The simplest way how to incorporate cables in equations of motion of a mechanism is the force representation of a cable (e.g. Diao & Ma, 2009). It is assumed that the mass of cables is small to such an extent comparing to the other moving parts that the inertia of cables is negligible with respect to the other parts. The cable is represented by a force dependent on the cable deformation and its stiffness and damping properties. This way of the cable modelling is probably the most frequently used model in the cable-driven robot dynamics and control.

A more precise approach is based on the representation of the cable by a point-mass model (e.g. Kamman & Huston, 2001; Ottaviano et al., 2015). Point masses can be connected by forces or constraints. In order to represent bending behaviour of cables their discretization using the finite segment method (Shabana, 1997) or so called rigid finite elements (Wittbrodt et al., 2006) is possible. Other more complex approaches can utilize nonlinear three-dimensional finite elements (Freire & Negrão, 2006) or can employ absolute nodal coordinate formulation (ANCF) elements (Shabana, 1997; Gerstmayr et al., 2012; Liu et al., 2012).

In the case of the manipulator mechatronic model consisting of cables and an end-effector whose motion is driven by cables (particularly in the case of the QuadroSphere model) utilization of the point-mass model of a cable proved to be very prospective. The force model of a cable seems to be too simple, the sophisticated cable model created on the basis of the ANCF is dynamically correct but it is not usable for the calculations in a real time (Bulín et al., 2015). The cable models on the basis of the finite segment method or so called rigid finite elements can be supposed to be the same.

3. Inverted pendulum

The investigated inverted pendulum is driven by two fibres (cables) attached to actuators (see Fig. 1 left) and it is affected by a gravitational force.

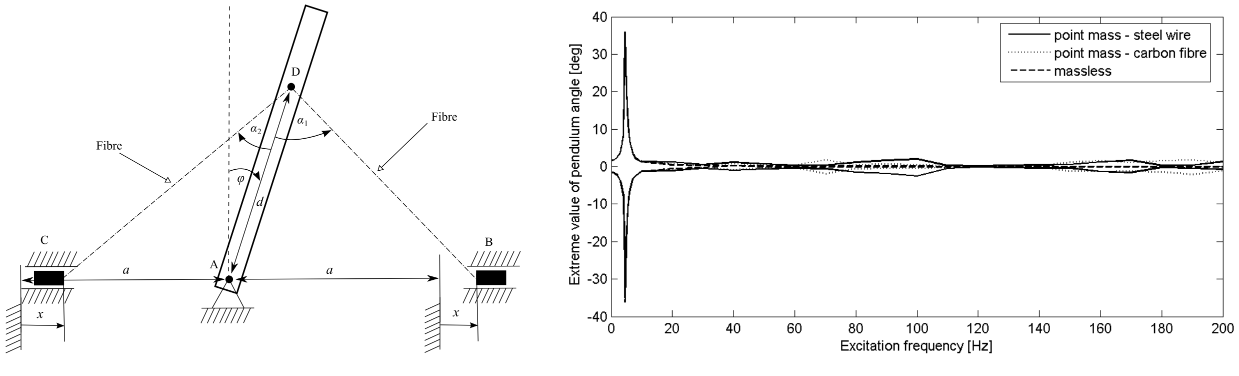

Fig. 1: Inverted pendulum actuated by the fibres (left) and time history of pendulum angle φ in dependence on the excitation frequencies (right).

When the pendulum is displaced from the equilibrium position, i.e. from the "upper" position, it is returned back to the equilibrium position by the tightened cable. As it has already been mentioned, this system was selected with respect to the fact that it is a simplification of possible cable-based manipulators.

The sophisticated point-mass cable model was validated on the basis of the results obtained using a massless cable model. In the cable model based on the point masses each cable is discretized using 10 point masses (e.g. Polach et al., 2012b). Each point mass is kinematically unconstrained (i.e. number of degrees of freedom is 3) in a two-dimensional model of the inverted pendulum system. The adjacent point masses are connected using spring-damper elements. Only axial (spring and damping) forces are considered in these spring-damper elements. The stiffness and the damping coefficients between the masses are determined in order to keep the global properties of the massless cable model. Validation of the point-mass model is given in Polach et al. (2012b). Correctness of the point-mass cable model was verified using another example in Polach et al. (2015). In order to investigate the pendulum motion the mentioned point-mass model of cables in the inverted pendulum models (Polach & Hajžman, 2012c) is used. For a better description of the solved problem a simple massless model is presented (the used model of the cable based on the point-mass model with lumped point masses corresponding to the mass of the cable is geometrically identical).

Kinematics of the system can be described by angle φ of the pendulum with respect to its vertical position (one degree of freedom), angular acceleration $\ddot{\varphi}$ and prescribed kinematic excitation $x(t) = x_0 \cdot \sin(2 \cdot \pi \cdot f \cdot t)$ (where x_0 is the chosen amplitude of motion, f is the excitation frequency and t is time). The equation of motion is of the form

$$\ddot{\varphi} = \frac{1}{I_\mathrm{A}} \cdot \left(F_{\mathrm{v}1} \cdot d \cdot \sin \alpha_1 - F_{\mathrm{v}2} \cdot d \cdot \sin \alpha_2 + m \cdot g \cdot \frac{l}{2} \cdot \sin \varphi \right), \qquad (1)$$

where I_A is the moment of inertia of the pendulum with respect to the axis in point A (see Fig. 1 left), α_1 and α_2 are the angles between the pendulum and the cables, m is the mass of the pendulum, $F_{\mathrm{v}1}$ and $F_{\mathrm{v}2}$ are the forces acting on the pendulum from the cables, g is the gravity acceleration, l is the length of the pendulum and d is the distance from the axis in point A to the position of attachment of cables to the pendulum (point D). Kinematic excitation acts in the points designated B and C (see Fig. 1 left).

The chosen model parameters are: $l = 1$ m, $a = 1.2$ m, $d = 0.75$ m, $I_\mathrm{A} = 3.288$ kg·m^2, $m = 9.864$ kg, $k_\mathrm{v} = 8.264 \cdot 10^4$ N/m (stiffness), $b_\mathrm{v} = 5 \cdot 10^{-4} \cdot k_\mathrm{v}$ N·s/m (damping coefficient). Excitation frequency f was considered in the range from 0.1 Hz to 200 Hz. Time histories and extreme values of pendulum angle φ and of the forces in the cables are the monitored quantities. Investigated parameters of the models of the inverted pendulum driven by two cables attached to actuators are given in Tab. 1.

Tab. 1: Investigated parameters of the cables.

Changed parameter	preload [N]	amplitude [m]	mass [g]	phase shift [deg]
The effect of the cables preload	0 to 8264	0.02	3.846	0
The influence of the amplitude of the harmonic kinematic excitation of cables	0	0.02 to 0.2	3.846	0
The effect of the cables mass	0	0.02	3.846 to 1269	0
The case of non-symmetric harmonic excitation	0	0.02	3.846	30

4. Conclusions

From comparing simulation results of inverted pendulum driven by two fibres (cables) attached to actuators it is evident that the greatest differences at utilizing the massless model and the point-mass model of cables are at investigating the influence of the cables mass (see Fig. 1 right). An unstable behaviour of the studied system was detected at investigating the cables preload influence on the pendulum vibration (Polach & Hajžman, 2012b) and at some combinations of the amplitude and the excitation frequency of the harmonic kinematic excitation of cables (Polach & Hajžman, 2012c). Changes in other investigated parameters of this system – i.e. the change in the cable mass (Polach et al., 2012) and non-symmetric harmonic excitation (Polach & Hajžman, 2012a) – do not cause the unstable behaviour of the pendulum.

Experimental verification of the cable dynamics within the manipulator systems is considered important in further research.

Acknowledgement

The paper has originated in the framework of the institutional support for the long-time conception development of the research institution provided by the Ministry of Industry and Trade of the Czech Republic to Research and Testing Institute Plzeň and solving project of the Czech Science Foundation No. 15-20134S entitled “Multi-level Light Mechanisms with Active Structures”.

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