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ROBOT CONTROL IN TERMS OF HAMILTONIAN MECHANICS

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Abstract: *The paper deals with a mathematical modeling of robot motion and control. Instead of frequently used Lagrangian formulation of robot dynamics, this paper presents robot dynamics by Hamiltonian formulation. This formulation leads to different physical descriptive quantities considered for control design. In the paper, as a comparative control approach, PD control with gravity compensation is considered. The control approach considering Hamiltonian formulation is demonstrated for simplicity on two-mass robot-arm system. However, the explained modeling approach is general and it can be applied, e.g., to usual industrial articulated robots-manipulators with multiple degrees of freedom.*

Keywords: Robot-manipulator, Hamiltonian formalism, Modeling, Robot control, PD control.

1. Introduction

Mechanical engineers usually prefer conventional Newton mechanics in their works. However, for robot control, there are preferred Lagrange's equations (Siciliano, 2008; Samson, 1991). In solutions of robot control, there exist some limits for positions, velocities or accelerations as well as some limits for control torques, respectively. The limits of velocities are usually constant for all configurations of robots without respecting the fact that inertia moments are discrepant for different configurations. The Lagrangian formalism is based on kinetic and potential energies and on a phase space formed by the positions and velocities. In the robot-manipulator dynamics, all momentums change very quickly, often in the rate 1/10 or more (Arimoto, 1996). Hence, it is interesting and useful to study control methods based not only on Lagrange formalism, but also on Hamiltonian one. It was investigated as the property of passivity of the robot (Landau, 1988).

Described approach can modify the natural energy of the robot so that it can satisfy the desired objectives (position or tracking control). Hamiltonian formalism with using a modified Hamiltonian (Takegaki, 1981) was used as new function there. Various choices are possible for the desired potential energy function (Wen, 1988). An alternative approach for potential function is in (Takegaki, 1981). In this contribution, we investigate and show differences in key features of Lagrangian and Hamiltonian formalism applied to robot control. Hence, we shall omit such changes as (Takegaki, 1981), but shall compare almost the same algorithms on the same problems of robot controls defined in both Lagrangian and Hamiltonian configuration spaces.

2. Lagrangian and Hamiltonian Formalism

The momentums and moments of momentums are very different in arbitrary configurations of robots. The classical methods of robot control use information on positions and velocities. It predetermines, that control methods based on feedback of positions and generalized momentums, will be different in results. In robotics, the generalized momentum is really momentum or moment of momentum, respectively. Hence, the Hamiltonian formalism may be better for aims of robot control than the Lagrangian one. In the following part we develop analogical differential equations of robot dynamics with using Hamilton's equations.

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2.1. Lagrange's equations of robot motion

Lagrange's equations of classical mechanics (Fasano, 2002) are frequently used for description of non-trivial mechanical systems. These equations are usually defined as follows

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = F_j, \quad j = 1, 2, \cdots, n \tag{1}$$

where n represents degrees of freedom (DOF); $L = K - V$ is Lagrange's function expressing subtraction of kinetic K and potential V energies; F_j are generalized forces and q_j generalized coordinates. For technical applications, the generalized forces F_j represent only a sum of non-conservative forces and complementarily conservative forces are represented by the potential energy function V.

Then, Lagrange's equations of robot motion are written in the following form (Arimoto, 1996)

$$\mathbf{M}(\mathbf{q})\,\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\,\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{u} \tag{2}$$

2.2. Hamilton's equations of robot motion

The Hamilton function is defined by (Fasano, 2002)

$$H = \sum_{i=1}^{n} p_i\,\dot{q}_i - L \tag{3}$$

where generalized momentums are defined by

$$p_j = \frac{\partial L}{\partial \dot{q}_j}, \quad j = 1, 2, \cdots, n\,. \tag{4}$$

The Hamilton's equations can be written as

$$\dot{q}_j = \frac{\partial H}{\partial p_j}, \quad p_j = F_j - \frac{\partial H}{\partial q_j} \tag{5}$$

These equations can be rewritten in the matrix form

$$\dot{\mathbf{q}} = \left(\frac{\partial H}{\partial \mathbf{p}}\right)^T, \quad \dot{\mathbf{p}} = \mathbf{F} - \left(\frac{\partial H}{\partial \mathbf{q}}\right)^T \tag{6}$$

Arbitrary robot may be considered as the time invariant system. Then, the Hamiltonian (3) is total energy: the sum of kinetic and potential energies. The Lagrangian L depends on positions and velocities, but the Hamiltonian depends on positions and generalized momentums, so we can write

$$H(\mathbf{q},\mathbf{p}) = K(\mathbf{q},\mathbf{p}) + V(\mathbf{q}) \tag{7}$$

The kinetic energy in coordinates **p** and **q** has the following form

$$K(\mathbf{q},\mathbf{p}) = \frac{1}{2}\mathbf{p}^T\mathbf{M}^{-1}(\mathbf{q})\,\mathbf{p} \tag{8}$$

Let us define the gradient of potential energy

$$\frac{\partial V}{\partial \mathbf{q}} = \mathbf{g}^T(\mathbf{q}) \tag{9}$$

The derivation with respect vector **q** is different in Lagrange space and in Hamilton space

$$\frac{\partial K(\mathbf{q},\mathbf{p})}{\partial \mathbf{q}} = -\frac{\partial K(\mathbf{q},\dot{\mathbf{q}})}{\partial \mathbf{q}} \tag{10}$$

Hence, from the equation (6) we can obtain

$$\dot{\mathbf{p}} = \mathbf{F} - \mathbf{g}(\mathbf{q}, t) - \left(\frac{\partial K(\mathbf{q}, \mathbf{p})}{\partial \mathbf{q}} \right)^T . \tag{11}$$

Let a skew symmetric matrix **S** be defined as follows

$$S_{ij} = \frac{1}{2} \sum_{k=1}^{n} \dot{q}_k \left(\frac{\partial M_{ik}}{\partial q_j} - \frac{\partial M_{jk}}{\partial q_i} \right). \tag{12}$$

It can be derived that mentioned matrix **S** holds

$$\mathbf{S}\dot{\mathbf{q}} = \frac{1}{2}\dot{\mathbf{M}}\dot{\mathbf{q}} - \frac{1}{2}\left(\frac{\partial}{\partial \mathbf{q}} \left(\dot{\mathbf{q}}^T \mathbf{M} \dot{\mathbf{q}} \right) \right) \tag{13}$$

and hence (11) can be rewritten in the final form

$$\dot{\mathbf{p}} = \left(\frac{1}{2}\dot{\mathbf{M}} - \mathbf{S} \right) \mathbf{M}^{-1}\mathbf{p} - \mathbf{g}(\mathbf{q}) + \mathbf{u} \tag{14}$$

From the first equation of (5) follows the second vector equation

$$\dot{\mathbf{q}} = \mathbf{M}^{-1}\mathbf{p} . \tag{15}$$

The equations (14) and (15) represent the robot motion. Remember, these equations are described in Hamilton's phase space **p**, **q**.

3. Robot Control

For the sake of brevity, the simplest control algorithm often called *Position control* is studied. Other methods as tracking control or force control will be omitted. Recall, the Hamiltonian phase space is represented by coordinates (**q**, **p**). For simplicity, in this space, the control will be called simply *control in Hamilton space*. On the other hand, the control in Lagrangian phase space, which is represented by coordinates $(\mathbf{q}, \dot{\mathbf{q}})$, will be simply called *control in Lagrange space*.

The controlled system (robot) is described by eqs. (14) and (15). Let the controller be described as

$$\mathbf{u} = \mathbf{g} + \mathbf{A}\mathbf{e} - \mathbf{B}\,\mathbf{p} . \tag{16}$$

where **A** and **B** are positive definite diagonal matrices, **g** is gravity compensation for the robot and $\mathbf{e} = \mathbf{q_d} - \mathbf{q}$. This approach may be called *PD control with full gravity compensation*. The analogical versions for robot control described by Lagrange's equations are in (Siciliano, 2008; Arimoto, 1996) etc. The target position $\mathbf{q_d}$ in terms of joint coordinates is fixed. Consider a set point control problem, in which the posture of the robot arm is allowed to asymptotically approach towards the target position state $(\mathbf{q}, \mathbf{p}) = (\mathbf{q_d}, \mathbf{0})$. Substitution of the control law (16) into (14) yields

$$\dot{\mathbf{p}} = \left(\frac{1}{2}\dot{\mathbf{M}} - \mathbf{S} \right) \mathbf{M}^{-1}\mathbf{p} + \mathbf{A}\mathbf{e} - \mathbf{B}\,\mathbf{p} \tag{17}$$

Consider the following Lyapunov function

$$W_L = \frac{1}{2}\mathbf{p}^T \mathbf{M}^{-1}(\mathbf{q})\mathbf{p} + \frac{1}{2}\mathbf{e}^T \mathbf{A}\mathbf{e} \tag{18}$$

where its time derivation along the trajectory given by (17) is

$$\dot{W}_L = -\,\mathbf{p}^T \mathbf{M}^{-1}\mathbf{B}\,\mathbf{p} \tag{19}$$

If **B** is a diagonal positive definite matrix and the inverse of matrix **M** is positive definite, then the multiplication of these matrices in (19) is positive definite. Thus, the quadratic form in (19) is negative semi-definite and control process is stable according to Lyapunov theory of stability. We have now to prove that if **p** = **0**, the robot does not reach a position $\mathbf{q} \neq \mathbf{q_d}$. This can be done by the La Salle invariant set theorem (LaSalle, 1960). The set **S** of points in the neighborhood of the equilibrium, that satisfies $\dot{W}_L = 0$, is such that **p** = **0** and $\dot{\mathbf{p}} = \mathbf{0}$. From (17) follows **e** = **0**. Hence, the equilibrium point **e** = **0**, **p** = **0** is the only possible equilibrium for the controlled system and is the largest invariant set in **S**. Hence, the equilibrium point is asymptotically stable. If we compare the similar method for control in Lagrangian space, we can obtain instead of (19) the following result

$$\dot{V}_L = -\dot{\mathbf{q}}^T \mathbf{B}\, \dot{\mathbf{q}} \tag{20}$$

and so from (19) and (20) we obtain the following criterion

$$\dot{W}_L \le \dot{V}_L \Leftrightarrow \dot{\mathbf{q}}^T \mathbf{B} \left(\mathbf{M} - \mathbf{E}\right) \dot{\mathbf{q}} \ge 0\,. \tag{21}$$

Since **B** is positive definite, hence $\dot{W}_L \le \dot{V}_L$ holds if and only if the matrix **M** – **E** is positive semidefinite (**E** is identity matrix). If this matrix is positive definite, the trajectory of W_L is under the trajectory of V_L in time.

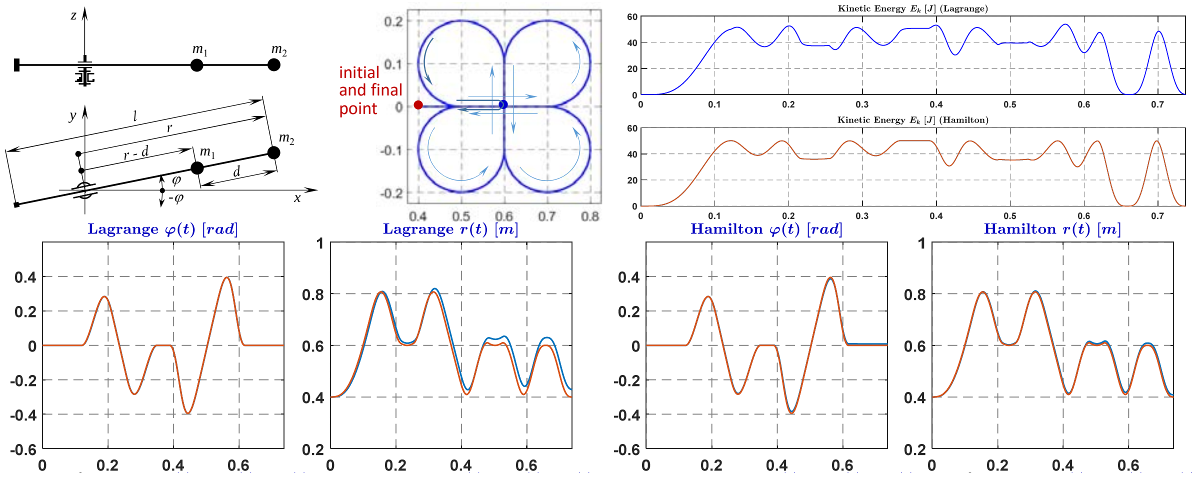

Fig. 1: Comparative examples of the PD control of the robot system shown at left top corner.

4. Conclusion

The developed theory and simulated examples show promising behavior of the control in Hamiltonian space against Lagrangian one, although generalized momentums cannot be directly measured but only computed from velocities and positions of the robot links. The difference of a quality of control increases for increased velocities of robot motion. The Fig. 1 is for max. tangential velocity 5 ms^{-1} of point m_2.

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