23rd International Conference
ENGINEERING MECHANICS 2017
Svratka, Czech Republic, 15 – 18 May 2017

SHOCK WAVES IN AN IDEAL GAS AND ENTROPY ANALYSIS

P. Šafařík*

Abstract: *In the paper, theories of normal and oblique shock waves will be presented. Principles of fluid mechanics and thermodynamics of an ideal gas are applied. Properties of shock waves will be discussed in connection with ambiguous solutions. Entropy analysis enables to determine conditions of occurrence of shock waves. An optimization task for shock wave parameters is formulated and solved.*

Keywords: Shock waves, Entropy analysis, Optimization task.

1. Introduction

Theory of shock waves is introduced in many textbooks of gas dynamics, for instance (Shapiro, 1953). Primary theoretical works on shock waves were achieved (Rankine, 1870 and Hugoniot, 1887). Experimental evidence of existence of shock waves was carried out in the year 1886. Mach and Salcher (1887) published the schlieren picture (see Fig. 1) where shock waves upstream and downstream of the projectile in flight are evident. Further experimental and theoretical investigations made possible development of theory of shock waves, for instance (Prandtl, 1907). The shock wave theory enables to solve parameters of fluid flow on shock waves. For numerical solutions in aerodynamic practice, auxiliary (subsidiary) tables (AMES Research Staff, 1948) and diagrams (AMES Research Staff, 1948) were prepared. Modern internet aids offer accessible calculators, for instance (Compressible Aerodynamic Calculator, 2014). A detail historical overview of investigations of shock waves is given in the book (Krehl, 2009).

The aim of this paper is to present theoretical approach and description of flow structure in experimental results at occurrence of shock waves.

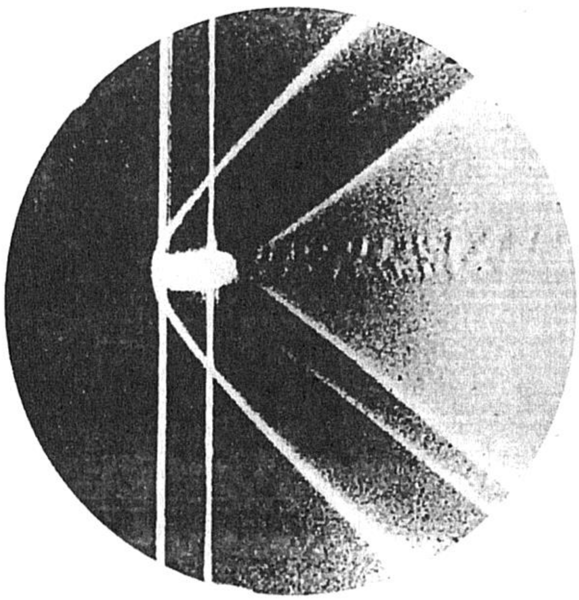

Fig. 1: Schlieren picture of the projectile in flight.

* Prof. Pavel Šafařík, PhD.: Department of Fluid Mechanics and Thermodynamics, Faculty of Mechanical Engineering, CTU in Prague, Technická 4; 166 07, Prague; CZ, pavel.safarik@fs.cvut.cz

2. Shock wave theory

Shock waves are adiabatic abrupt physical phenomena. They are surfaces with discontinuity of fluid flow parameters. Basic theoretical approach is based on balances of mass, momentum and energy fluxes. The condition of adiabatic process is defined by constant value of total specific enthalpy $h_{01} = h_{02} = h_0$. A control volume is chosen very thin in a part of the shock wave.

In following paragraphs, only substantial knowledge will be presented.

2.1. Normal shock wave theory

The scheme of control volume for normal shock wave is shown in Fig. 2. Following system of equations is based on balance of mass

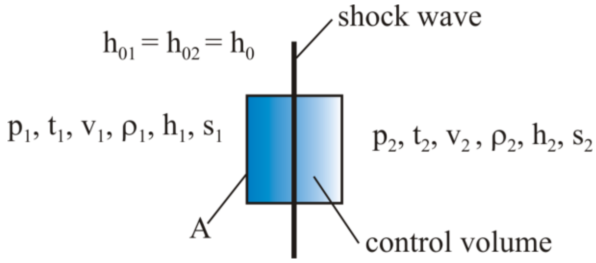

Fig. 2: The scheme of control volume for normal shock wave.

$$\frac{\dot{m}}{A} = \rho_1 v_1 = \rho_2 v_2 \,, \tag{1}$$

balance of momentum

$$p_1 - p_2 = \frac{\dot{m}}{A}\left(v_2 - v_1\right) \Rightarrow p_1 + \rho_1 v_1^2 = p_2 + \rho_2 v_2^2 \,, \tag{2}$$

balance of energy

$$h_1 + \frac{v_1^2}{2} = h_2 + \frac{v_2^2}{2} \tag{3}$$

and equation of state of an ideal gas

$$p = \frac{\kappa - 1}{\kappa} \rho h \,. \tag{4}$$

One of solutions of the system Eqs. (1 – 4) is Rankine-Hugoniot equation – relation between ratio of pressures downstream p_2 and upstream p_1 and ratio of densities downstream ρ_2 and upstream ρ_1 of the shock wave

$$\frac{p_2}{p_1} = \frac{\frac{\kappa + 1}{\kappa - 1} \cdot \frac{\rho_2}{\rho_1} - 1}{\frac{\kappa + 1}{\kappa - 1} - \frac{\rho_2}{\rho_1}} \,, \tag{5}$$

which is depicted in diagram in Fig. 3 in comparison with isentropic relation $\frac{p}{\rho^\kappa} = const.$ From the system of Eqs. (1 – 4) it is also possible to derive dependence of ratio of pressures downstream p_2 and upstream p_1 of shock wave on upstream Mach number M_1

$$\frac{p_2}{p_1} = \frac{2\kappa}{\kappa + 1} M_1^2 - \frac{\kappa - 1}{\kappa + 1} \,. \tag{6}$$

Prandtl relation for values of non-dimensional velocities upstream λ_1 and downstream λ_2 of a normal shock wave is well-known

$$\lambda_1 \cdot \lambda_2 = 1 \,. \tag{7}$$

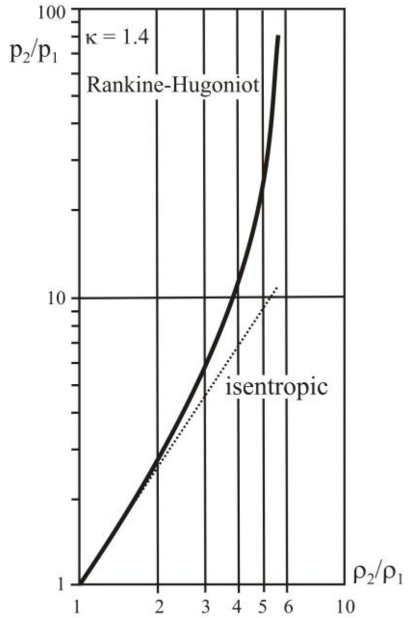

Fig. 3: Dependence of ratios of pressures and densities, Eq. (5).

2.2. Oblique shock wave theory

The scheme of control volume for oblique shock wave is shown in Fig. 4 where geometric parameters of the shock wave angle β and the deflection angle δ are depicted. Velocity vectors v_1 and v_2 are distinguished. Their normal v_n and tangential v_t components are used in balances of mass and momentum. The system of equations consists of balances mass, momentum in normal direction, momentum of tangential direction, energy and equation of state.

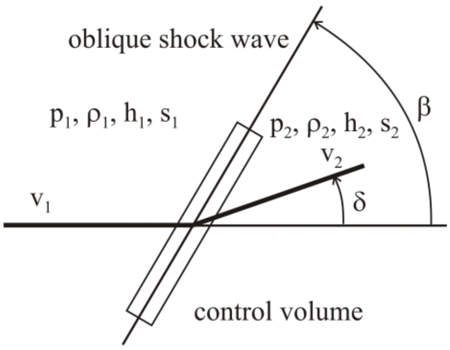

Fig. 4: The scheme of control volume for normal shock wave.

The solution of the system confirms that Rankine-Hugoniot equation, Eq. (5), holds also for oblique shocks. Ratio of pressures depends not only on upstream Mach number M_1 but also second independent variable – the shock angle β - appears

$$\frac{p_2}{p_1} = \frac{2\kappa}{\kappa + 1} M_1^2 \sin^2 \beta - \frac{\kappa - 1}{\kappa + 1}. \qquad (8)$$

Prandtl equation for oblique shock waves is not so easy as in the case of normal shock waves, Eq. (7), but beside normal velocity components v_{1n} and v_{2n} tangential velocity component $v_{t1} = v_{t2} = v_t$ takes place

$$v_{n1} \cdot v_{n2} = a_*^2 - \frac{\kappa - 1}{\kappa + 1} v_t^2, \qquad (9)$$

where a_* is critical speed of sound. For computation of the deflection angle δ, following equation was derived

$$tg\,\delta = 2 \cot g\,\beta \left[\frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\kappa + \cos 2\beta) + 2} \right]. \qquad (10)$$

From working formulas for parameters of oblique shock wave different tables and charts were prepared. The shock polars in hodograph plane proved to be very useful. In Fig. 5 shock wave polars are depicted and sonic conditions downstream of the oblique shock wave are indicated.

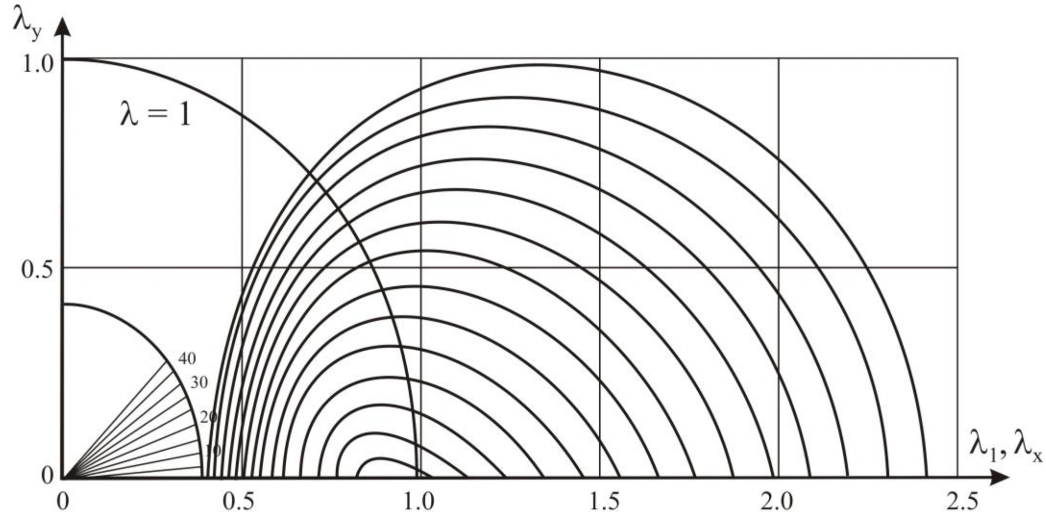

Fig. 5: Shock wave polars in hodograph plane,κ = 1.4.

3. Entropy analysis of shock waves

From aerodynamic equations for oblique shock waves and from thermodynamic relations it is possible to derive relation for entropy increase

$$\frac{s_2 - s_1}{r} = \ln\left\{\left(\frac{2\kappa}{\kappa+1}M_1^2\sin^2\beta - \frac{\kappa-1}{\kappa+1}\right)^{\frac{1}{\kappa-1}} \cdot \left[\frac{(\kappa-1)M_1^2\sin^2\beta + 2}{(\kappa+1)M_1^2\sin^2\beta}\right]^{\frac{\kappa}{\kappa-1}}\right\}. \quad (11)$$

The analysis of the Eq. (11) confirms that the existence of shock waves can be only in supersonic flow field of an ideal gas, $M_1 > 1$. For oblique shock waves the shock wave angle is in the region $\mu_1 = \arcsin\frac{1}{M_1} \le \beta \le \frac{\pi}{2}$. When β is positive the oblique shock wave is left-running. If it is negative the oblique shock wave is right-running. Maximum value of the deflection angle δ_{max} on oblique shock waves exists and can be derived as a function of Mach number M_1 upstream of the oblique shock wave $0 \le \delta \le \delta_{max}(M_1)$. For given value of the deflection angle δ two solutions of parameters of oblique shock wave exist. The dividing regime corresponds to the condition maximum of the deflection angle.

4. Optimal parameters of oblique shocks

Achieved knowledge on parameters of shock waves enables to formulate and solve optimization tasks. One of them has the criterion of optimum minimum entropy increase for given deflection angle δ = const., Šafařík (1993)

$$\left(\frac{d(s_2 - s_1)}{dM_1}\right)_{\delta = const} = 0. \quad (12)$$

The solution of the optimization task offered interesting results

the optimal Mach number M_{1opt} upstream of the oblique shock wave

$$M_{1opt} = \sqrt{\frac{2}{1 - \kappa \cdot \sin\delta}} \quad (13)$$

the optimal shock wave angle

$$\beta = \frac{\pi}{4} + \frac{\delta}{2} \tag{14}$$

the optimal ratio of static pressures

$$\left(\frac{p_2}{p_1}\right)_{opt} = \frac{1 + \kappa \cdot \sin\delta}{1 - \kappa \cdot \sin\delta} = M_{1opt}^2 - 1 \,. \tag{15}$$

In Fig. 6, parameters for maximum deflection angle and for sonic flow downstream of the oblique shock wave (see in book by Maršík, 2015) are compared with optimal parameters of oblique shock waves.

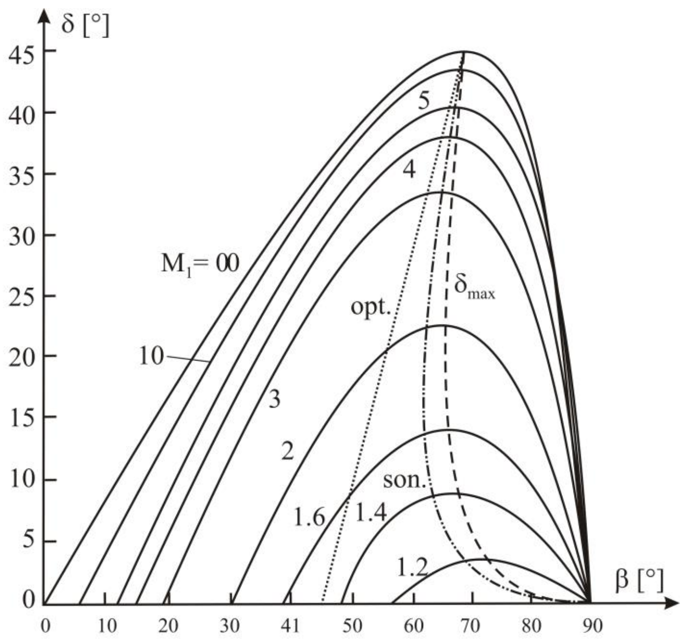

Fig. 6: Diagram the deflection angle δ versus the shock wave angle β, κ = 1.4.

5. Conclusions

High-speed flow investigations have to prepare new knowledge and data for reliable and effective design and operation of machines. Classical gas dynamics proposes important and interesting topics for study of compressible fluid flow. Their solutions can give new experience and instruments for understanding of complex processes in flow systems.

Acknowledgement

The support from the Technology Agency of the Czech Republic in the frame of the Competence Centre Advanced Technology of Heat and Electricity Output, No.TE01020036 is gratefully acknowledged.

References

AMES Research Staff (1948) Equations, Tables, and Charts for Compressible Flow, NACA Report 1135, Washington.

Compressible Aerodynamic Calculator (2014) http://www.dept.aoe.vt.edu/~devenpor/aoe3114/calc.html .

Hugoniot, H. (1887) Memoir on the Propagation of Movements in Bodies, Especially Perfect Gases, Journal de l´Ecole Polytechnique, Vol. 57, pp. 3-97, (in French).

Krehl, P.O.K. (2009) History of Shock Waves, Explosions and Impacts, Springer-Verlag, Berlin.

Mach, E. and Salcher, P. (1887) Photographical fixation of the processes introduced by projectile in the air, Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften. Mathematisch-Naturwissenschaftliche Classe, Vol. 95, Abteilung II, pp. 764-780, (in German).

Maršík, F. (2015) Propagation of Waves and Nonlinear Phenomena in Dissipative Systems, VŠB-TU, Ostrava (in Czech).

Prandtl, L. (1907) New study on the flow of gases and vapors, Physicalische Zeitschrift, Vol. 8, No. 1, pp. 23-30, (in German).

Rankine, W.J.M. (1870) On the Thermodynamic Theory of Waves of Finite Longitudinal Disturbances, Philosophical Transactions of the Royal Society of London, Vol. 160, pp. 277-288.

Rosenhead, L. (1954) A Selection of Graphs for Use in Calculations of Compressible Airflow, The Claderon Press, Oxford.

Šafařík, P. (1993) On Optimal Shock Wave Parameters, IT NEWS, Vol. 2, No. 3, pp. 9-14.

Shapiro, A.H. (1953) The Dynamics and Thermodynamics of Compressible Fluid Flow, Ronald Press, New York.