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OPTIMAL REPAIR AND REINFORCEMENT OF BAR STRUCTURES USING FINITE TOPOLOGY VARIATIONS

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Abstract: *The problem of optimal repair and/or optimal reinforcement of bar structures by introduction of new elements is considered in this paper. The corresponding optimization problem is formulated as the maximization of the global stiffness increment induced by repair (reinforcement) under the cost constraint. The potential energy is assumed as the measure of the global structure stiffness, while the cost constraint corresponds to condition imposed on the maximal cost of the repair and/or reinforcement. The method of determination of the structure global stiffness increment induced by finite topology modification is proposed and the optimization algorithm is presented. Numerical examples of optimal repair and/or reinforcement of some structures illustrate the theoretical considerations.*

Keywords: **Bar Structures, Optimal Repair, Optimal Reinforcement, Finite Topology Variations, Global Stiffness.**

1. Introduction

Global stiffness and load capacity of a structure due to operation of certain factors, for example atmospheric, biological, chemical, etc., may significantly decrease. Moreover, need of taking into account additional loads sometimes occurs. In these and similar situations appears necessity of repair and/or reinforcement of the structure in the current state.

In the present paper, at first general approach for determination of finite increments of the global stiffness caused by finite topology modifications (cf. Mróz and Bojczuk, 2003) is adjusted to the bar structures. Next, assuming that the structure is weakened or damaged, the problem of optimal repair or reinforcement is formulated. The method of solving this problem using finite topology modifications corresponding to introduction of substructures or structural elements is presented.

The hitherto existing papers usually apply probabilistic approach to the damage processes and focus attention on optimal inspection and maintenance (cf. Jido et al., 2008 and Ortega-Estrada et al. 2013), in contrary to this paper, which is devoted to optimal repair or optimal reinforcement of weakened structures.

2. Formulation of problem of optimal repair (reinforcement)

The problem of optimal repair and/or optimal reinforcement of the fixed primary structure in order to maximize global stiffness, admitting finite topology modifications, can be presented in the form

$$\max \Delta\Pi \quad \text{subject to} \quad C \le C_0, \qquad (1)$$

where $\Delta\Pi$ denotes increment of the potential energy caused by introduction of stiffening substructures or structural elements subjected to optimal design, C is the global cost of the structure after this finite modification and C_0 denotes the upper bound on the global cost. Here, the potential energy Π is assumed as the measure of the structure global stiffness.

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3. Determination of finite changes of global stiffness

Let us consider, on the example of a frame, determination of finite increment of the potential energy induced by finite topology modification. We assume that the primary structure 1 will be connected with a new additional substructure 2 (Fig. 1), which position and stiffness parameters are the design variables. In view of Betti's reciprocity theorem separately for the structures 1 and 2, before and after connection, we get

$$\begin{aligned} \mathbf{P}_1 \cdot \mathbf{u}_1 &= \mathbf{P}_1 \cdot \mathbf{u}_1^0 + T_C w_{1C}^0 + N_C u_{1C}^0 + M_C \theta_{1C}^0, \\ \mathbf{P}_2 \cdot \mathbf{u}_2 &= \mathbf{P}_2 \cdot \mathbf{u}_2^0 - T_C w_{2C}^0 - N_C u_{2C}^0 - M_C \theta_{2C}^0, \end{aligned} \tag{2}$$

where $\mathbf{P}_1$, $\mathbf{P}_2$ are the vectors of generalized loads, $\mathbf{u}_1^0$, $\mathbf{u}_2^0$ and $\mathbf{u}_1$, $\mathbf{u}_2$ denote the vectors of generalized displacements before and after connection, while $\mathbf{P}_C(N_C, T_C, M_C)$, $\mathbf{u}_{1C}^0\left(u_{1C}^0, w_{1C}^0, \theta_{1C}^0\right)$, $\mathbf{u}_{2C}^0\left(u_{2C}^0, w_{2C}^0, \theta_{2C}^0\right)$ are the vectors of internal forces and generalized displacements of the structures at the connection point. Now, the increment of the potential energy can be presented as follows

$$\begin{aligned} \Delta\Pi = \Pi - \Pi^0 &= -\frac{1}{2}(\mathbf{P}_1 \cdot \mathbf{u}_1 + \mathbf{P}_2 \cdot \mathbf{u}_2) + \frac{1}{2}\left(\mathbf{P}_1 \cdot \mathbf{u}_1^0 + \mathbf{P}_2 \cdot \mathbf{u}_2^0\right) = \\ &= \frac{1}{2}\left[N_C\left(u_{2C}^0 - u_{1C}^0\right) + T_C\left(w_{2C}^0 - w_{1C}^0\right) + M_C\left(\theta_{2C}^0 - \theta_{1C}^0\right)\right] = \\ &= \frac{1}{2}\mathbf{P}_C \cdot \left(\mathbf{u}_{2C}^0 - \mathbf{u}_{1C}^0\right) = \frac{1}{2}\mathbf{P}_C \cdot [\mathbf{u}_C^0], \end{aligned} \tag{3}$$

where $[\mathbf{u}_C^0]$ denotes discontinuity of generalized displacement vectors at the connection point. Taking into account that $[\mathbf{u}_C] = \mathbf{u}_{2C} - \mathbf{u}_{1C} = \mathbf{0}$ and $\mathbf{u}_{1C} = \mathbf{u}_{1C}^0 + \mathbf{D}_{1C}\mathbf{P}_C$, $\mathbf{u}_{2C} = \mathbf{u}_{2C}^0 - \mathbf{D}_{2C}\mathbf{P}_C$, we get that $[\mathbf{u}_C^0] = \langle \mathbf{D}_C \rangle \mathbf{P}_C$, where $\langle \mathbf{D}_C \rangle = \mathbf{D}_{1C} + \mathbf{D}_{2C}$ is the sum of the local compliance matrices related to the interface. Therefore, finally the equation (3) can be rewritten as follows (cf. Mróz and Bojczuk, 2003)

$$\Delta\Pi = \frac{1}{2}\mathbf{P}_C \cdot [\mathbf{u}_C^0] = \frac{1}{2}\mathbf{P}_C \cdot \langle \mathbf{D}_C \rangle \mathbf{P}_C = \frac{1}{2}[\mathbf{u}_C^0] \cdot \langle \mathbf{D}_C \rangle^{-1} [\mathbf{u}_C^0]. \tag{4}$$

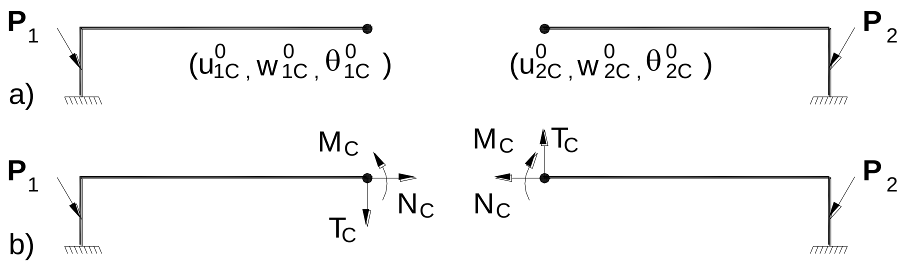

Fig. 1: Connection of a primary structure 1 with reinforcing substructure 2: a) the structures before connection; b) the structures after connection.

So, the stiffness increment depends on, easy to determine, vector of initial displacements discontinuity $[\mathbf{u}_C^0]$ and inverse of the matrix $\langle \mathbf{D}_C \rangle$. In order to calculate $\langle \mathbf{D}_C \rangle$, at first stiffness matrices of the primary structure and additional substructure $\mathbf{K}_1$, $\mathbf{K}_2$ and their inverses $\mathbf{D}_1 = \mathbf{K}_1^{-1}$, $\mathbf{D}_2 = \mathbf{K}_2^{-1}$ are determined. Now $\mathbf{D}_{1C}$, $\mathbf{D}_{2C}$ can be easily specified as the submatrices of $\mathbf{D}_1$, $\mathbf{D}_2$ of considerably smaller dimensions. It is important to notice that during optimization matrices $\mathbf{K}_1$, $\mathbf{D}_1$ do not change, so only $\mathbf{K}_2$ and $\mathbf{D}_2$ should be separately calculated in each step of optimization.

4. Optimal design algorithm

In order to determine optimal repair or reinforcement by introduction of an additional substructure the following algorithm, which combines FEM analysis and any gradient optimization method, can be used:

1° Determine plan of the primary structure repair or reinforcement.

2º Using information from the topological derivative (cf. Bojczuk and Mróz, 1998; Mróz and Bojczuk, 2003) or by direct decision of designer, choose, under the constraint of the maximal cost, initial location and stiffness of the finite reinforcing modification.

3º Applying any gradient optimization method, where $\Delta\Pi$ is calculated from (4), as it was described in Section 3, determine optimal values of the design parameters for the substructure.

4º Terminate the procedure or propose another method of repair or reinforcement and return to 1º.

5. Illustrative examples

Numerical examples of optimal repair and reinforcement of bar structures presented in this Section illustrate applicability and usefulness of the proposed approach.

5.1. Example 1: Optimal repair (reinforcement) of symmetric truss

The symmetric truss made of steel, which global stiffness is not sufficient, should be reinforced. All bars of the truss have cross-section of circular tube shape and their areas respectively are $A_1 = 8 \cdot 10^{-4}\,\mathrm{m}^2$, $A_2 = 5 \cdot 10^{-4}\,\mathrm{m}^2$, $A_3 = 3 \cdot 10^{-4}\,\mathrm{m}^2$ (Fig. 2). It is assumed that reinforcement can be performed by adding symmetrically located one or two pairs of new bars, but only in tension.

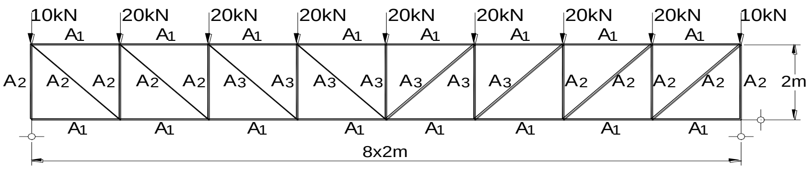

Fig. 2: Static scheme of the truss.

The problem of optimal design has the form (1), where it is assumed that the cost is proportional to the volume and the volume of the additional bars should not exceed $V_0 = 10^{-2}\,\mathrm{m}^3$. Due to symmetry only half of the truss is analyzed. Possible initial locations of new bars are chosen, as these lines between two nodes, so far not connected by bars, where virtual strains are positive (because of tension) and the biggest (cf. Mróz and Bojczuk, 2003). During optimization optimal pairs (pair) of bars are selected and their cross-section areas are determined.

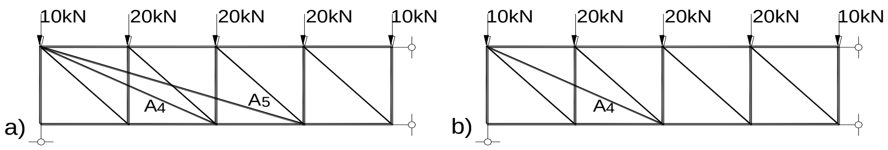

Fig. 3: The optimal designs.

The primary structure (Fig. 2) has the potential energy $\Pi = -2.200\,\mathrm{kJ}$. The optimal designs are shown in Fig.3. In the case of introduction of two pairs of bars (Fig. 3a) the potential energy increased to $\Pi = -1.929\,\mathrm{kJ}$ and the cross-section areas of the bars are $A_4 = 6.24 \cdot 10^{-4}\,\mathrm{m}^2$ and $A_5 = 3.49 \cdot 10^{-4}\,\mathrm{m}^2$ respectively. When only the one pair of bars is introduced optimal solution is shown in Fig. 3b. In this case the potential energy of the truss increased to $\Pi = -1.948\,\mathrm{kJ}$. The cross-section areas of the pair of bars are $A_4 = 11.18 \cdot 10^{-4}\,\mathrm{m}^2$.

5.2. Example 2: Optimal repair (reinforcement) of frame

Now, let us consider frame structure made of steel, which is shown in Fig. 4. Both, columns and cross-beam are double-T bars of cross-section areas $A = 10^{-2}\,\mathrm{m}^2$. It is assumed that the frame can be reinforced by adding two or one column. The optimization problem consists in maximization of the total potential energy increment under cost constraint. Here we assume that the cost is proportional to material volume and the maximal volume of additional columns is $V_0 = 5 \cdot 10^{-2}\,\mathrm{m}^3$. The design parameters are locations of columns s_i and their cross-section areas A_i. Here, two types of connections of additional columns to foundation and to the cross-bar denoted by k are considered, namely either pinned or fixed.

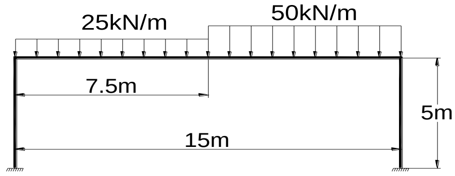

Fig. 4: Static scheme of the frame

The primary structure (Fig. 4) has the potential energy $\Pi = -25.45\,\text{kJ}$. For the case of introduction of two columns, these columns are initially located symmetrically in relation to the point of cross-beam maximal deflection, while the optimal solution is shown in Fig.5a. Tab.1 contains optimal values of the design parameters and potential energy for three types of optimization problems considered here, namely introduced columns are simply supported, clamped or clamped but with equal cross-section areas. In the case of introduction of one column (Fig. 5b), this column is initially located at the point of the maximal deflection of cross-beam (cf. Bojczuk and Mróz, 1998). Optimal values of design parameters are in the fixed case: $s = 8.22\,\text{m}$, $\Pi = -0.8335\text{kJ}$, and in the pinned case: $s = 8.35\,\text{m}$, $\Pi = -0.8489\text{kJ}$.

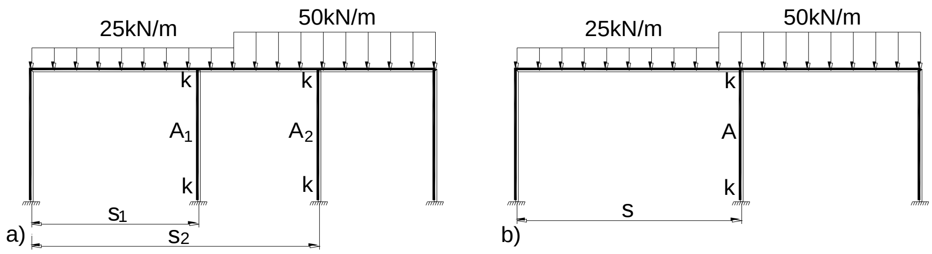

Fig. 5: Optimal design of the frame: a) the case with two additional columns,
b) the case with one additional column

Tab. 1: Optimal values of design parameters

Connection type	Design parameters				
	s_1[m]	s_2[m]	$A_1 \cdot 10^{-4}$[m^2]	$A_2 \cdot 10^{-4}$[m^2]	Poten. en.[kJ]
fixed	6.21	10.69	43.1	56.9	-0.3349
pinned	6.18	10.68	42.8	57.2	-0.3353
fixed, constant A_1, A_2	6.40	10.82	50	50	-0.3375

6. Conclusions

The problem of optimal repair and/or reinforcement of bar structures in order to maximize global stiffness under cost constraint is analyzed in this paper. The algorithm of optimization, in which, initially, finite topology variations corresponding to new elements and substructures are introduced into the structure, is presented and successfully applied in some illustrative examples. The formulated approach can be also used for other types of finite modifications like replacement or removal of structural elements.

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