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MODEL OF CAR SUSPENSION WITH PARAMETRIC UNCERTAINTY

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Abstract: *A quarter model of a car suspension is the simplest possible model for entry analysis of a suspension behavior. However, such a model is very rough typically without modelled suspension nonlinearities or other effects influencing the model dynamics significantly. The paper proposes approach where a given suspension parameter is modelled as uncertain. It is consequently investigated the amount of the uncertainty of a given parameter which covers the given nonlinearity. The advantage of the approach is that the model with parametric uncertainty has still very simple structure possibly in a form of linear state-space model with possible utilization in robust control design.*

Keywords: Multi-body model, Car suspension, Parametric uncertainty.

1. Introduction

Multi-body model of a car suspension is in its simplest form represented by a quarter model with two degrees of freedom (Fig. 1) which is constituted by two ordinary differential equations. Such a model can be considered as entry level for analysis of a car suspension behavior. However, it contains neither nonlinearities in the suspension nor other significant influences such as wear and tear of suspension elements projecting directly into the behavior of the system.

Introducing all of mentioned influences makes the model substantially more complicated which especially in a case of full suspension car model leads to increased computational demands and also to its lower readability.

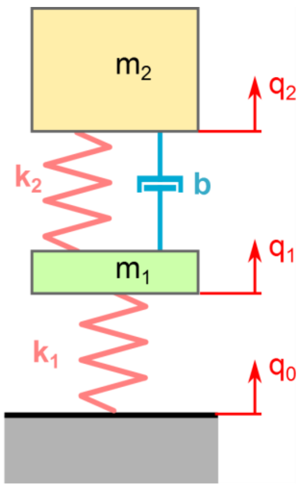

Fig. 1: Quarter model scheme.

The possible way how to overcome complicated introducing of details influencing the dynamics of the model and yet keeping it simple is to utilize apparatus of modeling with given amount of uncertainty (Green, 1994). The model containing an uncertainty (for example in some of parameters such as stiffness or mass) has still quite simple structure, possibly in a form of state linear time invariant model. This allows working with the model efficiently. Let's also note that models with uncertainty are generally utilized in robust control design (Gu et al., 2005).

The goal of the paper is to find out whether the simple quarter model with parametric uncertainty defined for a given suspension parameter is able to cover a behavior of more complicated suspension model with a nonlinearity. In the positive case will be investigated needed amount of uncertainty of the parameter.

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2. Suspension model

2.1. Quarter model

The quarter model of the car suspension (Fig. 1) generally describing oscillating system with two degrees of freedom is defined by two well-known ordinary differential equations

$$\begin{aligned} m_1\ddot{q}_1 &= b\left(\dot{q}_2-\dot{q}_1\right)+k_2\left(q_2-q_1\right)-k_1\left(q_1-q_0\right) \\ m_2\ddot{q}_2 &= -b\left(\dot{q}_2-\dot{q}_1\right)-k_2\left(q_2-q_1\right) \end{aligned}, \qquad (1)$$

where m_1 means unsprung mass, m_2 sprung mass, k_1 stiffness of the tire, k_2 stiffness of the coil spring, b damping of the damper, $\ddot{q}_1$, $\dot{q}_1$, q_1 are kinematics quantities of the unsprung mass, $\ddot{q}_2$, $\dot{q}_2$, q_2 are kinematics quantities of the sprung mass and finally q_0 is the excitation from the road surface.

2.2. Model of damping nonlinearity

The damping characteristic of a car damper is intentionally nonlinear. The main reason is the requirement of different intensity of the damping for the lifting and lowering of a wheel. Nonlinear damping characteristics used in the paper is obtained from (Pražák, 2006). It is linearized in the sense of least square method for obtaining of parameter b nominal value. Both, nonlinear and linear damping are shown in Fig. 2.

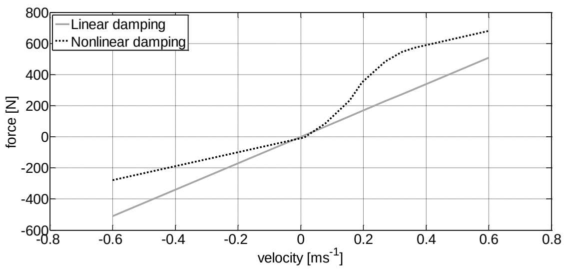


Fig. 2: Linear and nonlinear damping characteristics.

2.3. Quarter model with parametric uncertainty

For the sake of simplicity will be modeled as uncertain only parameter b, which means that

$$b_\Delta = b \pm \Delta , \qquad (2)$$

where b_Δ is the damping with parametric uncertainty, b represents the nominal value of the damping and Δ is the uncertainty value (in this case as percentage of b). In a same way can be defined uncertainties in other parameters.

For more comfortable work with the model, it was transformed into state-space form

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{Ax}+\mathbf{Bu} \\ \mathbf{y} &= \mathbf{Cx}+\mathbf{Du} \end{aligned}, \qquad (3)$$

where $\mathbf{x}=\left[q_2 \quad \dot{q}_2 \quad q_1 \quad \dot{q}_1\right]^T$ is vector of states, $\mathbf{u}=q_0$ is the input to the system and $y=\left[q_2 \quad q_1\right]^T$ represents outputs of the system (displacement of the both masses). Matrices **A**, **B**, **C**, **D** are defined as

$$\mathbf{A}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_2}{m_2} & -\frac{b_\Delta}{m_2} & \frac{k_2}{m_2} & \frac{b_\Delta}{m_2} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_1} & \frac{b_\Delta}{m_1} & \frac{-k_2+k_1}{m_1} & -\frac{b_\Delta}{m_1} \end{bmatrix}, \ \mathbf{B}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_1}{m_1} \end{bmatrix}, \ \mathbf{C}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \ \mathbf{D}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

2.4. Model of excitation from the road surface

Input to the quarter model as an excitation from the smooth terrain is generated using function (4). The function is defined by summing several sinusoids of appropriate amplitude and frequency as follows.

$$q_0 = (-0.143\sin(1.75(x+1.73)) - 0.180\sin(2.96(x+4.98)) - \\ -0.012\sin(6.23(x+3.17)) + 0.088\sin(8.07(x+4.63)))/5 \qquad (4)$$

3. Numerical experiments

The scope of numerical experiments was to compare output displacement of the sprung and unsprung mass of the linear quarter model (1) with the corresponding outputs of the model containing the nonlinear damping defined according to Fig. 2 (for results see Figs. 3 – 6) and consequently to compare outputs of the nonlinear model with outputs of the uncertain one.

The values of parameters for the linear model were used as follows: $m_1 = 33\ kg$, $m_2 = 400\ kg$, $k_1 = 125000\ N.m^{-1}$, $k_2 = 50000\ N.m^{-1}$ and from the linearization obtained $b = 849\ N.s.m^{-1}$.

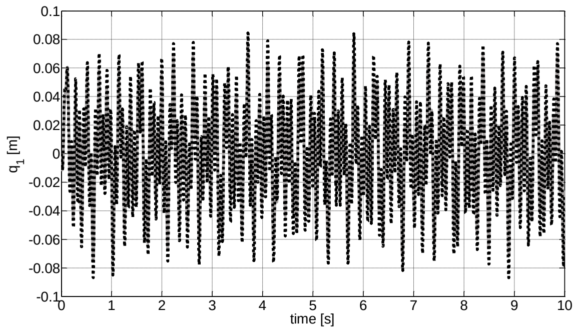


Fig. 3: Unsprung mass displacement, linear vs nonlinear (dashed) damping.

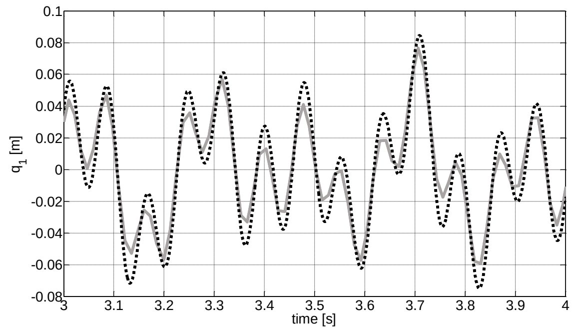


Fig. 4: Unsprung mass displacement, linear vs nonlinear (dashed) damping-detail.

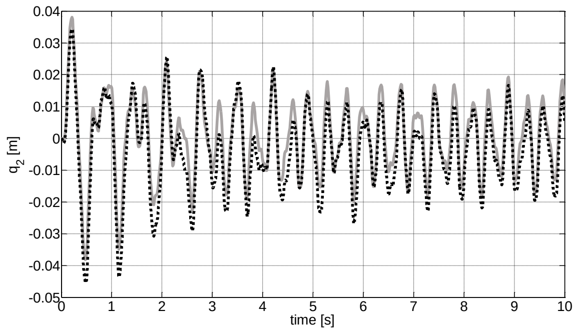


Fig. 5: Sprung mass displacement, linear vs nonlinear (dashed) damping.

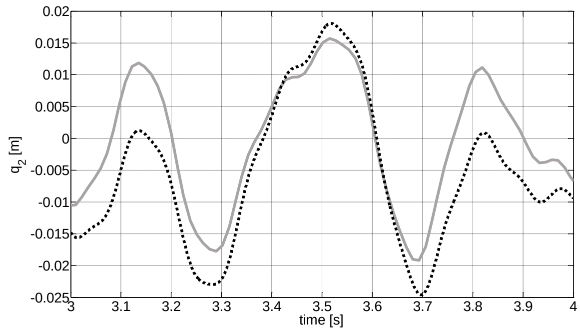
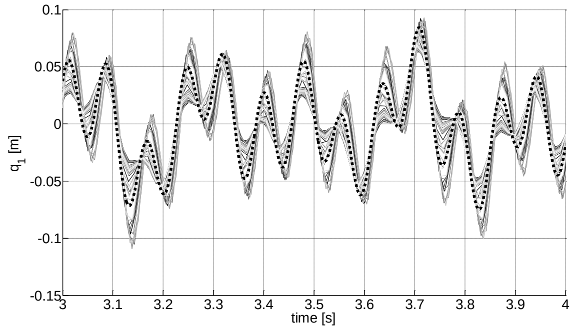


Fig. 6: Unsprung mass displacement, linear vs nonlinear (dashed) damping-detail.

The difference (according to Kullback-Leibler) between the outputs of the linear and nonlinear model is approximately 22.4 %, thus the linear model can be in some situations insufficient. The following simulation results (Figs. 7 and 8) compare outputs of the model with uncertainty on the damping according to (3) with the model containing the nonlinearity in the damping. The uncertainty was changed from ± 10 % to ± 100 % of *b*, the increment was 10 %.



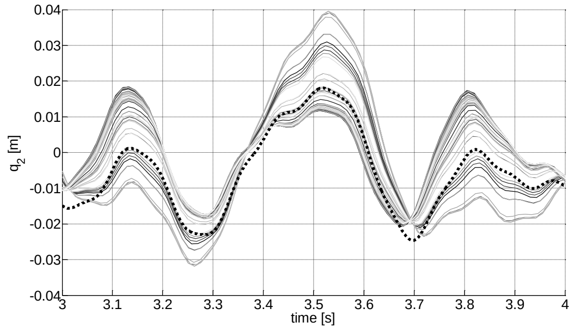
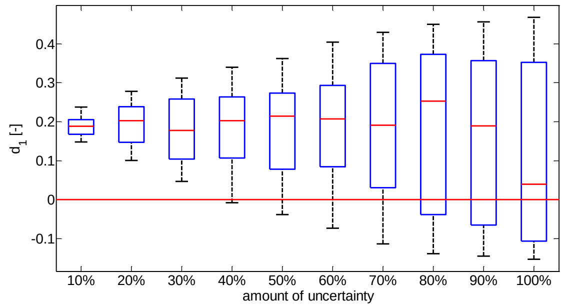


Fig. 7: Unsprung mass displacement, samples of the model with uncertainty ±100 % vs nonlinear (dashed)-detail.

Fig. 8: Sprung mass displacement, samples of the model with uncertainty ±100 % vs nonlinear (dashed)-detail.

The difference between outputs of the uncertain model samples (50 samples for every 10 %) and nonlinear model was evaluated by Kullback-Leibler method and it is represented by following boxplots (Figs. 9 and 10).



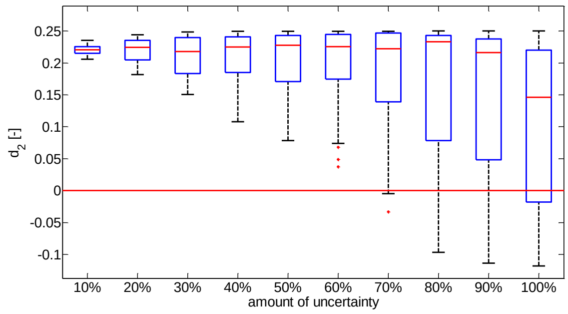


Fig. 9: Difference between q_1 of the uncertain model samples and nonlinear model.

Fig. 10: Difference between q_2 of the uncertain model samples and nonlinear model.

4. Conclusions

The difference between outputs of the linear quarter model and the model with nonlinear damping is approximately 22.4 % thus in some cases might be the linear model insufficient. The introduced parametric uncertainty of the damping was changed gradually by 10 % from ±10 % to ±100 % of *b*. It was investigated that outputs of the nonlinear model will be covered by outputs of the uncertain model for the uncertainty at least ± 40 % (for q_1), respectively at least ± 70 % (for q_2). Such a model with modeled uncertainty is able to cover behavior of the nonlinear model and due to its simple structure it is computationally efficient and suitable for consequent robust control design. In a similar way can be modelled different types of nonlinearities or other influences such as wear and tear of the elements which can be difficult to model or predict in classical way.

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