

## COMPUTATIONAL ANALYSIS OF SERVICEABILITY LIMIT STATE OF BEAMS REINFORCED WITH FRP BARS

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**Abstract:** *Fiber Reinforced Polymers (FRP) reinforcement are increasingly common as internal reinforcement of Reinforced Concrete (RC) members, because of their excellent properties as high tensile strength, corrosion resistance and non-magnetization. However, the flexural and mechanical characteristics of FRP bars such as low modulus of elasticity or non-yielding characteristics, results in large values of deflection and wide crack of FRP RC element. This paper investigates and compares the Serviceability Limit State (ULS) of simply supported beam subjected to various values of flexural stresses. The beams reinforced with Carbon Fiber Reinforced Polymer (CFRP) and Aramid Fiber Reinforced Polymer (AFRP) bars were examined. The computational analysis of beams static work was based on American guide for the design (ACI 440.1R-06), with taking into consideration all reduction factors. The influence of various parameters as the reinforcement ratio or load level were analyzed.*

**Keywords:** FRP reinforcement, Beam, Deflection, Crack width, Flexural strength.

### 1. Introduction

The fibre reinforced polymers (FRP) bars offer an attractive alternative to conventional steel reinforcement. Their good physical and mechanical properties as low density and thermal expansion coefficient, high corrosion resistance in aggressive environments, high tension strength-to-weight ratio and very good fatigue properties cause the development of investigation of FRP reinforced concrete members. However, no discernible yield point, relatively low modulus of elasticity and different flexural strength depending on the FRP type result in large crack widths and deflections. These characteristics are the reason that the serviceability limit states, primarily deflection and cracking, is one of the critical issues necessary to be examined during design of concrete structural elements reinforced with FRP bars. In the investigation the beam under different level of live load, with various reinforcement ratio of two types of FRP bars was considering. This research evaluate the relation between deflection and crack width to nominal moment capacity of concrete member reinforced with FRP bars (Abdalla, 2002 and Gravina, 2008).

### 2. Available design recommendations

Many years of designers and researchers experience contributed to development of the guides based on among others design equations which predict flexural strength, crack widths and deflections of concrete beams reinforced with FRP bars. The design recommendations are available in four elaborations: American (ACI 440.1R-06), Canadian (CSA-S806-02, 2002), Japanese (JSCE No.23, 1997) and Italian (CNR-DT 203/2006).

The guidelines for cracking and deflection based on standards for typical reinforced concrete, because of significant differences in modulus of elasticity, tensile strength and deformation characteristics cannot be directly applied for FRP rebar (Toutanji, 2003). However, the recommendations for design FRP

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reinforced concrete (RC) include following assumptions (Bywalski, 2014): perfect adhesion between FRP bars and concrete surface, the applicable flat cross-section condition, linear relation  $\epsilon$ - $\sigma$  for FRP in tension until destruction of member and neglected expandable concrete zone.

### 3. Computational analysis

#### 3.1. Calculation model

The simply supported exterior beams with rectangular cross-section sized 350 x 180 mm are calculated. A superimposed service dead load of  $w_{SDL} = 3.0$  kN/m and three levels of service live load  $w_L$  (3.0, 4.5 and 6.0 kN/m) are assumed. Beams are reinforced with AFRP and CFRP bars, with various reinforcement ratio: 0.61 % (3 $\Phi$ 12), 0.42 % (3 $\Phi$ 10) and 0.24 % (3 $\Phi$ 7.5). The concrete is specified by class C20/25 ( $f'_c = 14.3$  MPa) and concrete cover thickness  $c = 35$  mm is assumed. The  $L_{eff}/240$  limitation for long-term deflection and maximum crack width of 0.4 mm are taken into consideration. The span of the beam is constant for each of the analyzed case ( $L_{eff} = 3.0$  m).

Design procedure follows the ultimate and serviceability limit states according to ACI 440.1R-06. The tensile strength  $f'_{fu}$  and modulus of elasticity of FRP  $E_f$  of the bars are reported by the manufacturers (Sireg Geotech S.r.l) - AFRP:  $f'_{fu} = 1400$  MPa,  $E_f = 60$  GPa, and CFRP:  $f'_{fu} = 2300$  MPa (CFRP),  $E_f = 130$  GPa.

#### 2.2. The results of computations

During analysis the three various reinforcement depend on the diameter of the bar were considered. Depend on levels of service live load  $w_L$  following values of the ultimate moments  $M_u$  were achieved: 12.01 kN·m for  $w_L = 3$  kN/m, 14.52 kN·m for 4.5 kN/m and 17.07 kN·m for 6 kN/m. The deflection limitation equal 12.5 mm and the limit of crack width equal 0.4 mm was controlled according to available standards for design RC (ACI 318M-05 and EN 1992-1-1:2004).

Results of computational analysis of flexural strength  $\Phi \cdot M_n$ , ( $\Phi$  – reduction factor) long-term deflection  $\Delta_{LT}$  and crack width  $w$  are presented in the Tabs. 1 – 2.

Tab. 1: Result of computational analysis of AFRP RC beam.

| Property                                  | 3 $\Phi$ 12 AFRP |      |      | 3 $\Phi$ 10 AFRP |      |      | 3 $\Phi$ 7.5 AFRP |      |      |
|---|------------------|------|------|------------------|------|------|-------------------|------|------|
| Flexural strength $\Phi \cdot M_n$ [kN·m] | 29.80            |      |      | 26.22            |      |      | 21.08             |      |      |
| Service live load level $w_{LL}$ [kN/m]   | 3.0              | 4.5  | 6.0  | 3.0              | 4.5  | 6.0  | 3.0               | 4.5  | 6.0  |
| deflection $\Delta_{LT}$ [mm]             | 0.98             | 1.82 | 2.97 | 1.41             | 2.62 | 4.24 | 2.52              | 4.64 | 7.46 |
| crack width $w$ [mm]                      | 0.30             | 0.36 | 0.42 | 0.42             | 0.51 | 0.59 | 0.74              | 0.88 | 1.03 |

Tab. 2: Result of computational analysis of CFRP RC beam.

| Property                                  | 3 $\Phi$ 12 CFRP |      |      | 3 $\Phi$ 10 CFRP |      |      | 3 $\Phi$ 7.5 CFRP |      |      |
|---|------------------|------|------|------------------|------|------|-------------------|------|------|
| Flexural strength $\Phi \cdot M_n$ [kN·m] | 38.35            |      |      | 34.42            |      |      | 28.40             |      |      |
| Service live load level $w_{LL}$ [kN/m]   | 3.0              | 4.5  | 6.0  | 3.0              | 4.5  | 6.0  | 3.0               | 4.5  | 6.0  |
| deflection $\Delta_{LT}$ [mm]             | 0.60             | 1.11 | 1.77 | 0.87             | 1.59 | 2.51 | 1.56              | 2.80 | 4.37 |
| crack width $w$ [mm]                      | 0.14             | 0.17 | 0.20 | 0.20             | 0.24 | 0.28 | 0.35              | 0.42 | 0.49 |

The results of analysis indicate that the effect of reducing modulus of elasticity is significant for Serviceability Limit State (SLS). The reserve of flexural strength is very large – in analyzed cases the ultimate moment did not exceed the nominal moment capacity with strength reduction factor  $\Phi \cdot M_n$ . In almost each of the analyzed cases variant  $\rho \geq 1.4 \cdot \rho_{fb}$  were found (where  $\rho_{fb}$  – FRP reinforcement ratio producing balanced strain conditions). Thus, there were a cases of damage mechanism as concrete crushing failure (ACI 440.1R-06). Therefore, the strength reduction factor  $\Phi$  decreased the nominal moment capacity of 35 %. The long-term deflection of analyzed beams are lower than limitation: 0.98 – 7.46 mm for AFRP RC and 0.60 – 4.37 mm for CFRP RC. However, the crack limit equal 0.4 mm is fulfilled in only two of analyzed cases of beams reinforced with AFRP bars and in seven cases of beams reinforced with CFRP bars. The results were presented as dependency graphs. The relation between level of the ultimate moment (thereby the level of live load) and long-term deflection is presented in Fig. 1, while the dependence of crack width – in Fig. 2.

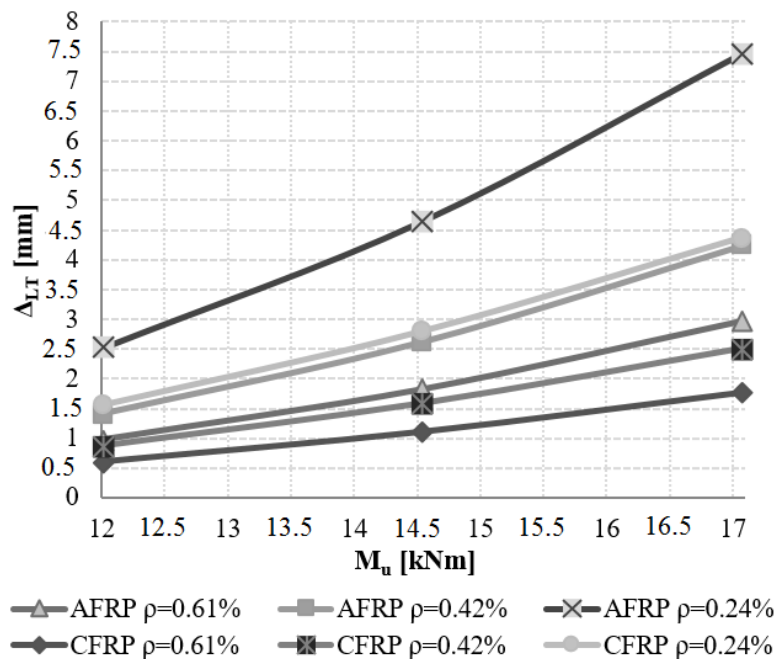


Fig. 1: Dependency graph of the long-term deflection and increasing ultimate moment.

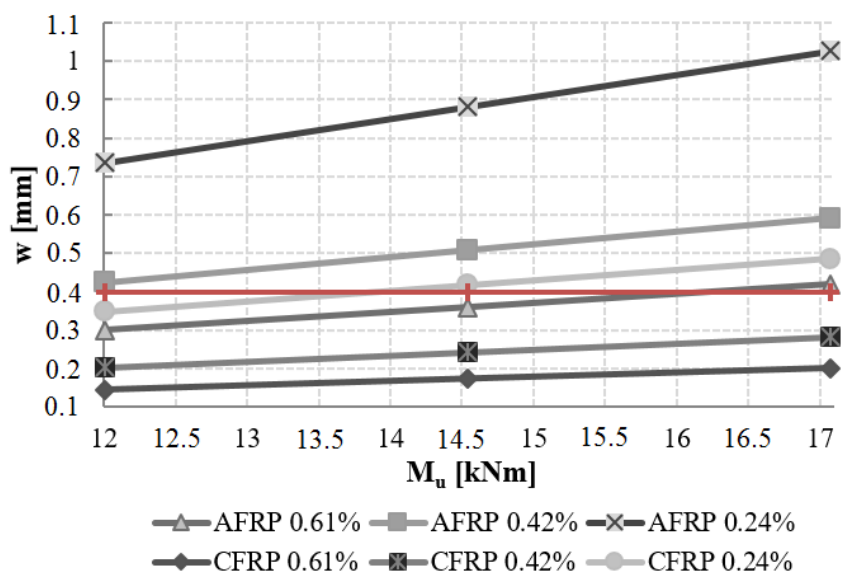


Fig. 2: Dependency graph of the crack width and increasing ultimate moment.

### 2.3. The analysis of the results

Based on the graph in Fig. 1, it can be estimated that depending on the level of the live load the values of long-term deflection increase exponentially. By estimating the trend line it is expected that limitation of

deflection of beam with the lowest load-capacity index (AFRP RC,  $\rho = 0.24\%$ ) can reach at the ultimate moment approximately equal to  $20 \text{ kN}\cdot\text{m}$  – which also indicates the fulfillment of the condition of flexural strength. However, in this case the crack width exceeds almost double limit value at the live load  $w_L = 3.0 \text{ kN/m}$  ( $M_u = 12.01 \text{ kN}\cdot\text{m}$ ). The crack width would be equal approximately  $1.2 \text{ mm}$  for  $M_u = 20 \text{ kN}\cdot\text{m}$  (it means triple exceeded condition).

In turn, based on the estimated trend line the beam with a highest load-capacity index (CFRP RC,  $\rho = 0.61\%$ ) can reach the deflection limitation in the value of ultimate moment nearly  $M_u = 47 \text{ kN}\cdot\text{m}$ . Whereas the crack width increases practically linear, it can be estimated that at a given  $M_u$  value, the crack width can reach slightly more than  $0.5 \text{ mm}$ .

Considering these conclusions the reduced value of modulus of elasticity makes the limit crack condition as crucial during designing the FRP RC members. At the assumed value of  $L_{\text{eff}}$ , achieving deflection limitation is practically proportional to achieving the maximum flexural strength. However, the assumed limit of cracking significantly limits the possibility of increasing the level of live load or length of beam span. Accordingly, during designing the FRP RC members it is important to consider that the influence of the relatively low modulus of elasticity is significant for Serviceability Limit State (SLS). Assuming that the modulus of elasticity of CFRP bars is about  $50\%$  higher than modulus of AFRP bars, the values of long-term deflection in CFRP RC beams are approximately  $40\%$  lower than in AFRP RC beams, while the cracks width of CFRP RC beams are lower by about  $50\%$ .

#### 4. Conclusions

The computational analysis of beams under static work, based on ACI 440.1R-06 guide for the design allowed the conclusion that influence of relatively low modulus of elasticity of FRP reinforcement is significant for Serviceability Limit State (SLS), especially crack width. Due to the very good tensile strength of FRP reinforcement, the flexural strength of beams had high load-capacity index in each of analyzed cases. However, the analysis of Serviceability Limit State is indispensable for faultless operation of beams. It is expected that in the analyzed cases under the assumed load level achieving the Ultimate Limit State for bending is almost synonymous with achieving the Serviceability Limit State for deflection. Despite this, the mechanical properties of FRP reinforcement have significantly influence on increasing the value of crack width. Having more than twice value of modulus of elasticity, the long-term deflections are reduced by almost  $40\%$ , and cracks width more than  $50\%$ .

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