23rd International Conference
ENGINEERING MECHANICS 2017
Svratka, Czech Republic, 15 – 18 May 2017

APPLICATIONS OF THE NEW METHOD OF THE LYAPUNOV EXPONENTS ESTIMATION

Dabrowski[*], M. Balcerzak[**], D. Pikunov[***]

Abstract: *In this article we show the new simple and effective method of Lyapunov Exponents (LE) estimation, based on the perturbation vector and its derivative dot product analysis. We show that presented method can be applied in different aspects of the nonlinear systems control. Moreover, our method is based on very simple computations, involving only basic mathematical operations, such as summing, subtracting, multiplying, dividing, thus it can be easier to apply than other methods. As the actual Lyapunov exponent value is calculated before the next integration step it does not involve an integration errors.*

Keywords: Stability, Lyapunov exponents, Nonlinear dynamics, Control system optimization.

1. Introduction

Controlling system dynamics with use of Lyapunov Exponents (LE) is employed in many different areas of the scientific research and is used in controlling dynamics of increasingly complex dynamical systems. LE are employed in scientific research of materials (Aniszewska, 2008), electric power systems (Wadduwage, 2013), non-continuous systems (Serweta, 2015), systems with time delay (Stefański, 2005), aerodynamics (Hu, 2012), time series analysis (Yang, 2012), optimal control (Zhu, 2004), chaotic encryption and secure communication (Chunbiao, 2012), multi-objective optimization (Fraga, 2014), parametric oscillations and fluctuating parameters (Stefanski, 2008), neuronal models investigations (Soriano, 2012). Thus, there is still need to elaborate fast and simple methods of LE calculation. The new method of LE estimation is presented in this paper.

2. Method

Generally presented method bases on the analysis of the disturbance changes d**z**(t) in the direction of the general disturbance vector **z**(t) (Fig. 1) and was discussed in (Dabrowski, 2012, 2012, 2014).

Fig. 1 shows two perturbations of **z** in the phase space. Assume that **z** for the dynamical state **x**(t) evolves according to linear transformation assigned U(**x**(t)).

Let $\mathbf{z}^*$ be the component of **z** in the direction of eigenvector $\boldsymbol{w}^*$ of linear transformation $\mathbf{U}(\mathbf{x}(t))$. In that case:

$$\frac{d\mathbf{z}^*}{dt} = \mathbf{U}(\mathbf{x}(t))\mathbf{z}^* = \boldsymbol{\lambda}^*\mathbf{z}^* \qquad (1)$$

where $\boldsymbol{\lambda}^*$ is the eigenvalue of $\mathbf{U}(\mathbf{x}(t))$, corresponding to eigenvector $\boldsymbol{w}^*$. After transformation of (1) to scalar form one can obtain:

$$\frac{|d\mathbf{z}^*|}{|\mathbf{z}^*|} = \boldsymbol{\lambda}^* dt \qquad (2)$$

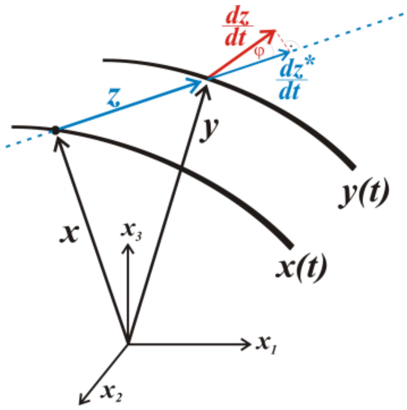


Fig. 1: Scheme of perturbation vector and its derivative.

[*] Assoc. Prof. Artur Dabrowski, PhD.: Division of Dynamics Dept of Mechanical Engineering, Lodz University of Technology, Stefanowskiego 1/15; 90-924, Lodz; Poland, Ar2rDe@p.lodz.pl

[**] Marek Balcerzak, MSc.: Division of Dynamics Dept of Mechanical Engineering, Lodz University of Technology, Stefanowskiego 1/15; 90-924, Lodz; Poland, marek.blck@gmail.com

[***] Danylo Pikunov, MSc.: Division of Dynamics Dept of Mechanical Engineering, Lodz University of Technology, Stefanowskiego 1/15; 90-924, Lodz; Poland, dpikunov@gmail.com

Integration of (2) yields:

$$|\mathbf{z}^*| = |\mathbf{z}_0^*|e^{\lambda^* t} \tag{3}$$

where $\mathbf{z}_0^*$ is the initial state of $\mathbf{z}^*$. Equation (3) describes evolution of the disturbance $\mathbf{z}^*$ in the direction of $\boldsymbol{w}^*$. One can notice that the formula (3) describes an averaged evolution of perturbation length. This shows a connection between largest eigenvalue of $\mathbf{U}(\mathbf{x}(t))$ and LLE of the considered dynamical system. Such connection stands the base of our new method and can be utilized further in estimation of LE spectrum.

3. Method applications

This section considers application of the method in estimation of the Lyapunov exponents in analysis of different types of dynamical systems. In the first case (Figs. 2a, 2b) perturbation **z**(t) was derived from the linearized equations and Largest Lyapunov exponent (LLE) was analysed. Mathematical model of the system is as follows:

$$\ddot{x} + \beta\dot{x} + \alpha x^3 = q\cos(\eta t) \qquad\qquad \ddot{z} + \beta\dot{z} + \alpha z^2 x = 0 \tag{4}$$

Application of the method in the bifurcational analysis can be seen in (Fig. 2a). One can see different types of system dynamics and values of LLE confirming chaotic, periodic dynamics existence. Zero LLE values which determine period doubling bifurcation points are visible as well. Comparison of the results with the Stefanski method (Stefanski, 2005) is shown in (Fig. 2b).

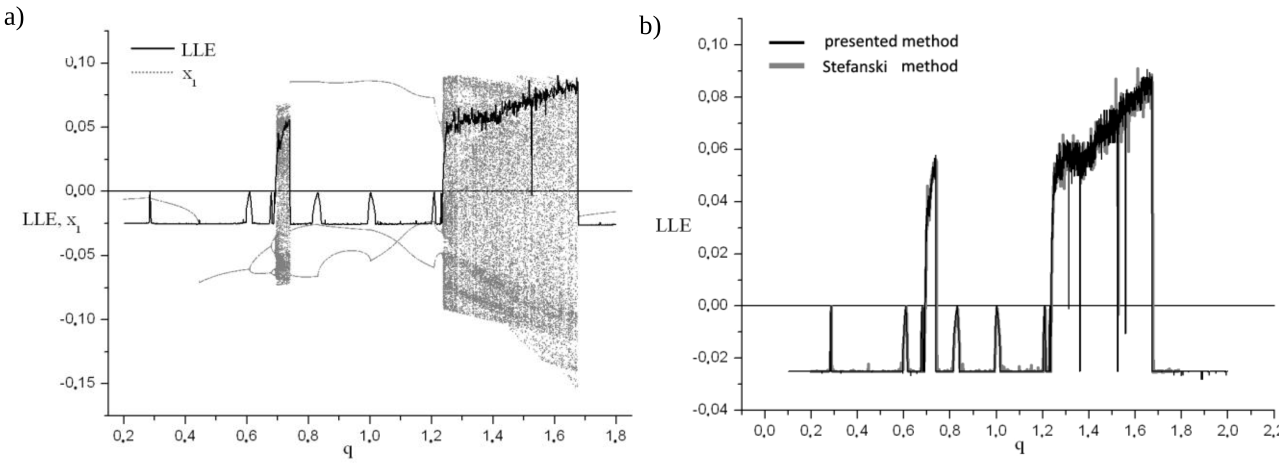


Fig. 2: Bifurcation diagrams for linearized equations of the perturbation analysis.

The next case considers studies of Duffing system dynamics where perturbation **z**(t) was derived from the differences of actual dynamical states of two identical systems with different initial states.

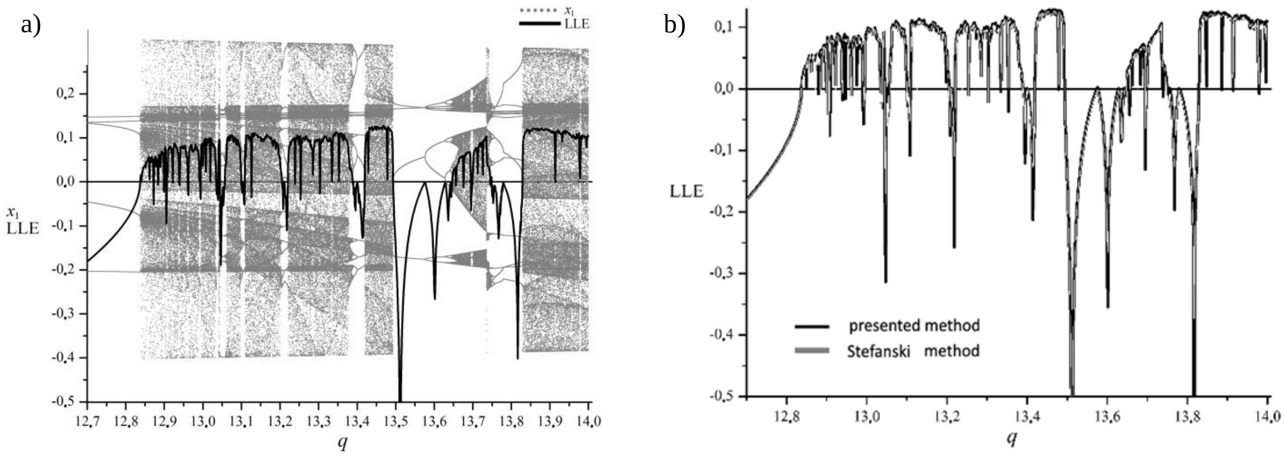


Fig. 3: Bifurcation diagrams for two systems perturbation analysis.

Similarly to the previous case application of the method in the bifurcational analysis has been presented. It can be seen in (Fig. 3a). One can see different types of the system dynamics and values of LLE confirming chaotic, periodic dynamics existence. Zero LLE values which determine period doubling bifurcation points are visible as well. Comparison of the results with the Stefanski method is shown in (Fig. 3b).

Application of the method in dynamical systems with desired controlled behaviour

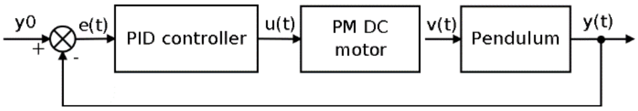
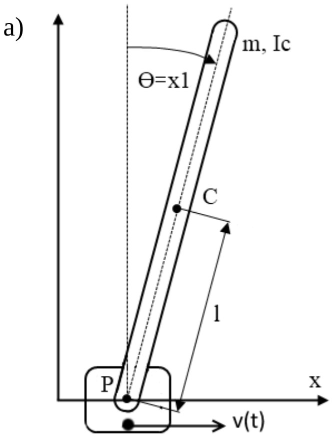


Fig. 4: Scheme of the control system.

In this section we investigate application of the method in analysis of dynamics of the controlled inverted pendulum (Fig. 5a) and optimization of its control systems parameters. Our method is applied to estimate the Largest Lyapunov Exponent (LLE) as a criterion for control performance assessment (CPA) in a simulated control system. As any control system can be analyzed as a dynamic system, disturbance acting on the system in time $t = 0$ can be treated as a change of initial conditions of the dynamic system. Thus behaviour of the error of regulation contains all the data necessary to the estimation of the Lyapunov exponents of the system. Scheme of the control system is presented in (Fig. 4).



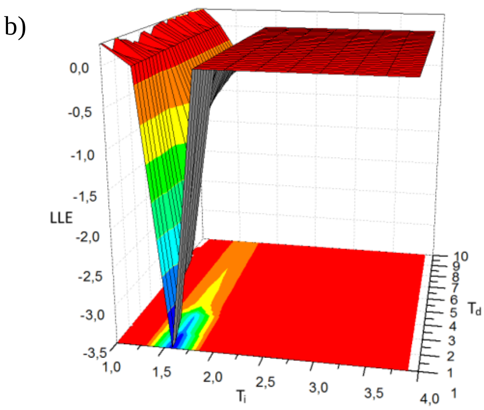


Fig. 5: a) Inverted pendulum; b) its bifurcational analysis.

During the experiment action of the control system was tested for different coefficients of PID regulator (k_p, T_I, T_D). For each combination of PID coefficients values of LLE were calculated. Based on that parameters ranges for each pendulum three dimensional bifurcational diagrams were obtained (Fig. 5b). It shows dependence of the LLE on the regulator parameters, and map of the parameters showing ranges of the LLE values in different colours. These investigations allowed us to choose optimal values of the regulator with taking into account the range of its best effectiveness. Finally values k_p, T_I and T_D were chosen for the lowest LLE values. Time series of the system regulation error *e(t)* for optimized parameters are presented in (Fig. 6). One can see fast decay of the oscillations proving big efficiency of optimized regulators.

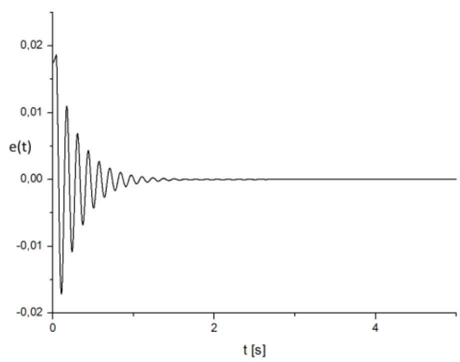


Fig. 6: Time series of the system regulation error e(t).

4. Conclusions

We have investigated the new method of LE estimation. We have presented theoretical description showing mathematical simplicity of the method and simplified background of the basic idea. We have introduced proofs of our method effectiveness based on results of simulations for different types of nonlinear dynamical systems. The next step of development of the method can be considered in estimation of LE from a real time series, systems with discontinuities, with time delay and others. It can be also extended onto multidimensional control.

Acknowledgement

This study has been supported by Polish Ministry of Science and Higher Education under the program Diamond Grant, project No. D/2013 019743.

References

Aniszewska, D. and Rybaczuk, M. (2008) Lyapunov type stability and Lyapunov exponent for exemplary multiplicative dynamical systems, Nonlinear Dyn vol. 54, pp. 345-354.

Chunbiao, L., Wang, J. and Wen, H. (2012) Absolute term introduced to rebuild the chaotic attractor with constant Lyapunov exponent spectrum, Nonlinear Dyn vol. 68, pp. 575-587.

Dabrowski, A. (2012) Estimation of the largest Lyapunov exponent from the perturbation vector and its derivative dot product, Nonlinear Dyn vol. 67, pp. 283-291.

Dabrowski, A. (2012) The largest Transversal Lyapunov Exponent and Master Stability Function from the perturbation Vector and its derivative Dot Product (TLEVDP), Nonlinear Dyn vol. 69, pp. 1225-1235.

Dabrowski, A. (2014) Estimation of the the Largest Lyapunov exponent-like (LLEL) stability measure parameter from the perturbation vector and its derivative dot product (part 2) experiment simulation, Nonlinear Dynamics. vol. 78, pp. 1601-1608.

Fraga, L.G. and Tlelo-Cuautle, E. (2014) Optimizing the maximum Lyapunov exponent and phase space portraits in multi-scroll chaotic oscillators, Nonlinear Dyn vol. 76, pp. 1503-1515.

Hu, D.L., Huang, Y. and Liu, X.B. (2012) Moment Lyapunov exponent and stochastic stability of binary airfoil driven by non-Gaussian colored noise, Nonlinear Dyn vol. 70, pp. 1847-1859.

Serweta, W., Okolewski, A., Blazejczyk-Okolewska, B., Czolczynski, K. and Kapitaniak, T. (2015) Mirror hysteresis and Lyapunov exponents of impact oscillator with symmetrical soft stops, International Journal of Mechanical Sciences vol. 101-102, pp. 89-98.

Soriano, D.C., Fazanaro, F.I., Suyama, R., Oliveira, J., Marconi, R.A. and Madrid, K. (2012) A method for Lyapunov spectrum estimation using cloned dynamics and its application to the discontinuously-excited FitzHugh–Nagumo model, Nonlinear Dyn vol. 67, pp. 413-424.

Stefanski, A. (2008) Lyapunov exponents of the systems with noise and fluctuating parameters, Journal of Theoretical and Applied Mechanics vol. 46(3), pp. 665-678.

Stefanski, A., Dabrowski, A. and Kapitaniak, T. (2005) Evaluation of the largest Lyapunov exponent in dynamical systems with time delay, Chaos, Solitons and Fractals vol. 23, pp. 1651-1659.

Wadduwage, D.P. and Annakkage, U.D. (2013) Power system transient stability analysis via the concept of Lyapunov Exponents, Electric Power Systems Research vol. 104, pp. 183-192.

Yang, C., Wu, C.Q. and Zhang, P. (2012) Estimation of Lyapunov exponents from a time series for n-dimensional state space using nonlinear mapping, Nonlinear Dyn vol. 69, pp. 1493-1507.

Zhu, W.Q. (2004) Feedback Stabilization of Quasi Nonintegrable Hamiltonian Systems by Using Lyapunov Exponent, Nonlinear Dynamics vol. 36, pp. 455-470.