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MITIGATION OF MESH DEPENDENCY IN PROBABILISTIC FINITE ELEMENT SIMULATION OF QUASIBRITTLE FRACTURE

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Abstract: *The contribution introduces probabilistic variant of the crack band model to mitigate mesh dependency issues in probabilistic finite element simulations of quasibrittle fracture. The probabilistic crack band model is limited to the element size larger than the localization band. It introduces transition from the weakest link theory (crack may choose freely where to initiate and propagate within the element) to standard crack band model with only one possible crack band position within the element. The transition is driven by level of localization reached by the element and its surroundings. The probability density function of the elemental strength and energy dissipation is modified accordingly. The model is applied in simulations of prisms loaded under tension, bending and three-point-bending. The obtained statistical characteristics of the results are nearly mesh independent proving applicability of the probabilistic crack band model.*

Keywords: Localization, Finite elements, Mesh dependency, Crack band, Spatial randomness.

1. Introduction

Concrete, composites, tough ceramics, etc. are examples of quasibrittle materials widely used in nowadays engineering structures. The tensile post-peak constitutive response of quasibrittle materials can be characterized by a gradual loss of load-carrying capacity. This phenomenon gives rise to the strain localization. Localization instability causes spurious mesh sensitivity in the finite element (FE) simulations, because the damage localizes into a single layer of elements and energy needed to cause material damage depends on the mesh discretization. Possible remedy is to introduce a material length scale into the model. These models with the internal length are usually referred to as the localization limiters. The simplest of them is the crack band model developed by Bažant and Oh (1983), in which the post-peak branch of the stress-strain curve is adjusted to keep constant fracture energy. Special care needs to be taken for the proper definition of the element size under a multi-axial stress (Jirásek and Bauer, 2012). The localization limiters were developed for deterministic analysis. The strain localization has, however, great importance in the reliability of structural design (Bažant at al. 2009, Le et al., 2011). So far, there is still a lack of understanding of the effect of the strain localization on the stochastic FE analysis of quasibrittle fracture (Strack et al., 2014).

The conventional crack band (CBM) model modifies the material constitutive relationship in order to preserve the fracture energy for localized damage. Following this concept, we attempt to investigate how to adjust the probability distributions of the constitutive parameters to ensure the mesh objectivity of probabilistic FE simulations. This contribution presents a probabilistic crack band model (PCBM) that involves a probabilistic treatment of damage evolution. It is limited to the case where the finite element size is larger than the crack band width. It is based on previously published paper of Le and Eliáš (2016).

2. Probabilistic crack band model

The fracturing occurs under tensile straining, where the stress-strain response is governed by three material parameters: the elastic modulus E, the tensile strength f_t and the energy dissipation density γ. In this study we consider that the tensile strength and the energy dissipation density are random and independent for each element and also mutually independent. The objective is to determine what

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modification of the probability distribution of f_t and γ would ensure independence of the simulation results on mesh discretization.

Tensile loading induces a damage band of fixed width h_0 (usually referred to as the crack band width) with size of approx. two to three times the maximum aggregate size. (Bažant and Planas, 1998, Bažant and Pang, 2007). In probabilistic analysis, another probabilistic length scale related to the spatial correlation of the material properties emerges. However, recent studies have shown that material properties of finite elements of size equal to the crack band width can be considered as statistically independent (Bažant and Pang, 2007). Assuming $h_e > h_0$, we can treat the material properties of each element as statistically independent, and omit completely the internal probabilistic length scale.

Three stages of fracture process can be distinguished: damage initiation, localization, and propagation. These three stages have very different implications on the regularization of energy dissipation as well as on the probabilistic treatment of localized damage. In the context of FE simulations, we propose a parameter for each Gauss point that measures the level of localization

$$\kappa_c = \begin{cases} \dfrac{1}{(n_i + n_o)} \left[\dfrac{(n_i + n_o + 1) \max\limits_{k \le n_o + n_i} (\phi_k)}{\sum\limits_{k=0}^{n_i + n_o} \phi_k} - 1 \right] & \text{if } \phi_0 > 0 \\ 0 & \text{if } \phi_0 = 0 \end{cases} \tag{1}$$

where n_i is the number of surrounding Gauss points within the element of interest (i.e. inner neighbors in Fig. 1b), n_o is the number of surrounding Gauss points within the neighboring elements (outer neighbors), ϕ_0 is the damage level of the Gauss point of interest, ϕ_k is the damage level of the kth surrounding Gauss point. In addition to the strain localization level of each Gauss point, it is also necessary to determine the localization level of the surrounding Gauss points, which is described by

$$\kappa_w = \frac{1}{n_o - 1} \left[\frac{n_o \cdot \max\limits_{k = n_i + 1}^{n_i + n_o} \phi_k}{\sum\limits_{k = n_i + 1}^{n_i + n_o} \phi_k} - 1 \right] \tag{2}$$

Since the damage localization is an irreversible process, both localization parameters κ_c and κ_w are treated as non-decreasing.

The modification of the energy dissipation density in conventional CBM assumes fully localized crack and yields: $\gamma\, h_e = G_f$ where G_f is the fracture energy. Let us define the reference energy dissipation density γ_0 that corresponds to element size equal to crack band width: $\gamma_0 = G_f/h_0$. The CBM does not explicitly address the transition from damage initiation to localization. At the damage initiation stage, in contrast to fully localized crack, the total energy dissipation of the material should be proportional to the element size because the entire material element would suffer distributed damage. We propose a smooth transition based on additional model parameter κ_{0c} using the localization parameter κ_c

$$\gamma = \gamma_0 f(\kappa_c) \quad \text{where} \quad f(\kappa_c) = \frac{h_0}{h_e} + \left(1 - \frac{h_0}{h_e}\right) \exp\left(-\frac{\kappa_c}{\kappa_{0c}}\right) \tag{3}$$

The random nature of considered solid implies variability of the crack band location within the element. We consider that the random onset of the damage band in the material element is reflected by the statistics of the tensile strength. Since independent strength at possible locations of the damage band are considered, one may use the weakest link model to describe the overall elemental strength distribution

$$F_{f_t}(\sigma) = 1 - [1 - P_1(\sigma)]^{n_e} \tag{4}$$

where n_e is the number of potential crack bands in one element and $P_1(x)$ is cumulative distribution function of strength of the single crack band. The number of potential crack bands is largely dependent on the localization level in the surrounding material elements. Fig. 1a shows two material elements, one with localized damage. Such situation would lead to stress concentration and would dictate the location of the localization band in the upper element. Therefore, there number of potential crack bands would be $n_e = 1$. The transition from unconstrained to fully constrained location of the crack band is expressed by an empirical function dependent on additional model parameter κ_{0w}

$$n_e = 1 + \left(\frac{h_e}{h_0} - 1 \right) \exp\left(-\frac{\kappa_w}{\kappa_{0w}} \right) \tag{5}$$

3. Simple examples

Concrete specimens with length L = 4 m, depth D = 0.5 m and thickness b = 0.1 m under three different loading configurations are simulated using the proposed PCBM, see Fig. 1c. The distribution of the maximal nominal stress, $\sigma_{N,\max}$, is calculated. The nominal stress is expressed as the maximum principal stress based on the elastic analysis $\sigma_N = P / bD$ for uniaxial tension, $6M / bD^2$ for pure bending and $3PL / 2bD^2$ for three-point-bending, where P and M are the applied force and moment. The performance of the PCBM is compared to (i) the conventional crack band model (CBM) without adjusting the probability distribution of tensile strength (i.e. $n_e = 1$ in Eq. 4) and (ii) the crack band model with considering the weakest link model of tensile strength (WLM) regardless of the localization level (i.e. $n_e = h_e / h_0$ in Eq. 4).

A simple isotropic damage model with Mazar's equivalent strain and linear softening is used. Specimens are discretized using linear quadrilateral elements with four integration points assuming a 2D plane stress condition. The solver is developed as a modification of the OOFEM software (Patzák, 2012 and Patzák and Rypl, 2012).

The material parameters are following: elastic modulus E = 30 GPa, Poisson's ratio ν = 0.2, mean tensile strength $\overline{f_\mathrm{t}}$ = 3 MPa, and mean fracture energy $\overline{G_\mathrm{f}}$ = 80 J/m^2. The probabilistic distribution for both strength and fracture energy is the Gaussian-Weibull grafted distribution (Bažant and Pang, 2007) with the coefficient variation 0.15, the grafting point P_{gr} = 10^{-3}, and the Weibull modulus m = 26. Three different mesh sizes are modeled: 50×50 mm, 100×50 mm, and 200×50 mm, where the first number is the width and the second the depth of the element, respectively. The crack band width h_0 is 50 mm. The PCBM parameters κ_{0c} = 0.19 and κ_{0w} = 0.283 are found by minimization of the difference between results computed on different mesh sizes.

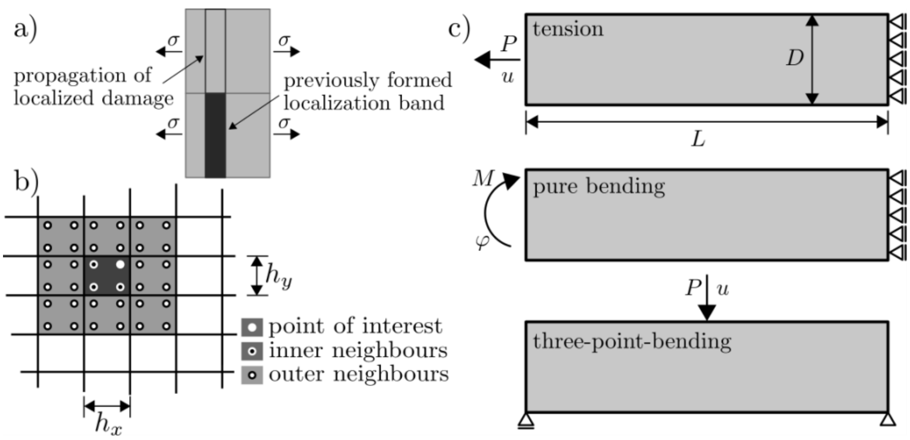

Fig. 1: a) Propagation of localized damage; b) determination of localization levels using information of neighboring Gauss points; c) loading configurations: uniaxial tension, pure bending, and three-point bending.

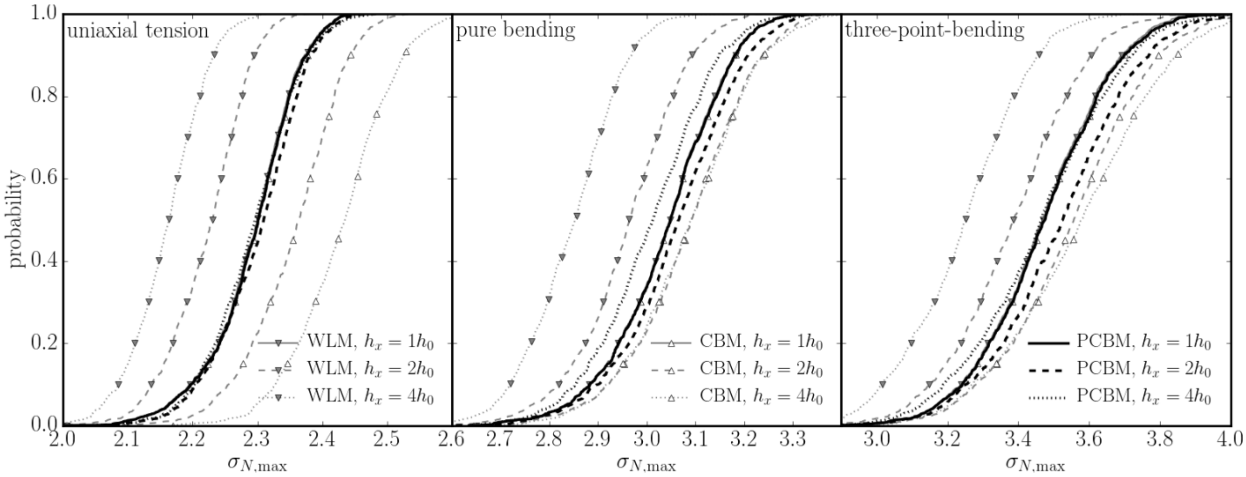

Fig. 2: Cumulative distribution function of the simulated maximal nominal stress.

4. Results and discussion

Fig. 2 presents the cumulative distribution function of the nominal strength obtained from 1000 realizations. For the reference size, $h_e = h_0$, all three methods yield the same result. For larger elements, the CBM overestimates the structural strength and the WLM underestimates it. Only the proposed PCBM shows reasonable mesh independence.

This study demonstrates that the conventional crack band model suffers from mesh sensitivity in the probabilistic simulations. The developed probabilistic crack band model improves the formulation by consideration of the random onset of localization band inside the material element. Besides, the regularization of fracture energy during the transition from damage initiation to localization is presented.

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