

A CONTROL WITH THE USE OF LQR MODIFIED METHOD IN THE GYROSCOPE SYSTEM OF TARGET TRACKING

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Abstract: An algorithm of the gyroscope system control in the seeker with the use of LQR (linear quadratic regulator) modified method was presented in this paper. The modification of the method consists in applying Jacobian J derived from the equations of the non-linear motion of the gyroscope system in the place of state matrix A with constant values. It makes it possible to determine the motion equations of the matrix system of K reinforcements of the closed control system with a square quality indicator in every step of integration. The research results showed that this type of control provides greater precision and effectiveness of control of the gyroscope system of tracking a moveable target in space. Some research results were presented graphically.

Keywords: LQR control, Jacobian, Target tracking, Gyroscope system.

1. Introduction

Gyroscope systems (GS) are widely applied in systems of observation and target tracking situated on the board of moveable objects (Koruba, 2010 and Dziopa et al., 2010). Due to the fact that high precision is required from them, stabilizing and tracking control should be carefully selected (Gapiński et al., 2014 and Gapiński, 2014). The classic method of optimal control of the gyroscope axis motion with a square quality indicator seems to be unsatisfactory as far as the influence of external disturbances in the form of kinematic base influence as well as the parameters of the gyroscope itself changeable over time (friction in frame bearings, rotor imbalance, a situation when the mass centre does not overlap with frame rotation centre, etc.) are concerned. LQR method assumes that the linear system and state matrix A with constant parameters are in question (Grzyb, 2016, Krzysztofik, 2014 and Koruba, 2013). Such simplification results in the fact that after relatively short time, tracking is not precise enough. In order to prevent this, Jacobian should be determined on the basis of the nonlinear equations describing the gyroscope system motion (Kim, 2010) in the following way:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial \mathcal{G}_g} & \frac{\partial f_1}{\partial \dot{\mathcal{G}}_g} & \frac{\partial f_1}{\partial \psi_g} & \frac{\partial f_1}{\partial \dot{\psi}_g} \\ \frac{\partial f_2}{\partial \mathcal{G}_g} & \frac{\partial f_2}{\partial \dot{\mathcal{G}}_g} & \frac{\partial f_2}{\partial \psi_g} & \frac{\partial f_2}{\partial \dot{\psi}_g} \\ \frac{\partial f_3}{\partial \mathcal{G}_g} & \frac{\partial f_3}{\partial \dot{\mathcal{G}}_g} & \frac{\partial f_3}{\partial \psi_g} & \frac{\partial f_3}{\partial \dot{\psi}_g} \\ \frac{\partial f_4}{\partial \mathcal{G}_g} & \frac{\partial f_4}{\partial \dot{\mathcal{G}}_g} & \frac{\partial f_4}{\partial \psi_g} & \frac{\partial f_4}{\partial \dot{\psi}_g} \end{bmatrix} \quad (1)$$

where: f_1, f_2, f_3, f_4 – non-linear functions of right sides of the motion equations of the gyroscope system; \mathcal{G}_g, ψ_g – deviation angles of the internal and external frames of the gyroscope (angles defining

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the position of the gyroscope axis in space); $\dot{\mathcal{G}}_g, \dot{\psi}_g$ – angular velocity of the deviation of the internal and external gyroscope frames.

Controlling the gyroscope system consists in tracking the set direction in space by its axis, e.g. the line of sight (LOS) connecting the centre mass of the optical system of the gyroscope system with the moveable point in space being tracked. The angular position of the line of sight in comparison with the axis of the non-moveable reference system will be marked with angles: ε i σ , which specify the so called desired motion of the gyroscope axis. During the operation of the gyroscope, the rotor axis adopts the positions determined with angles: \mathcal{G}_g i ψ_g , which define the so called performed motion of the gyroscope axis. The line of sight might be then determined incorrectly, on the basis of which the missile is guided towards the target in homing systems. With too excessive deviations of the line of sight from the set position, the sight of the target image might be lost.

Deviation of the performed motion from the desired one is determined in the following way:

$$\Delta = \sqrt{(\mathcal{G}_g - \varepsilon)^2 + (\psi_g - \sigma)^2} \quad (2)$$

Quantity Δ is called an actual control deviation. Apart from that, the notion of partial control deviations as the differences of angles is used:

$$e_g = \mathcal{G}_g - \varepsilon, \quad e_\psi = \psi_g - \sigma \quad (3)$$

The task of the tracking system is to minimize those deviations to zero. In order to perform the control in question, control with the use of LQR modified method might be applied.

2. An algorithm of optimal control of the gyroscope system

In order to determine the optimal control, the following quality criterion (target function) is formulated (Lewis et al., 2012 and Takosoglu, 2016):

$$I(u) = \frac{1}{2} \int_0^{t_f} (e^T Q e + u^T R u) dt \quad (4)$$

where e is a vector, whose components are the deviations described with an eq. (3) together with their derivatives in relation to time; u is a vector of the controlling moments put against the gyroscope frames; Q is a positive semi-definite square, symmetric matrix called the state weighting matrix; R is a positive definite square, symmetric matrix called the control cost matrix.

The optimal control problem consists in solving the feedback gain matrix, K , so that the scalar objective function, $I(u)$, is minimized if all state variables can be measured.

One of the important properties of LQR is that provided certain conditions are met, they guarantee nominally stable closed-loop system. The conditions for achieving a stable linear quadratic system are as follows (Tewari, 2002):

$$R > 0, \quad Q \geq 0, \quad (A, B) \text{ – controllable} \quad (5)$$

where A, B – matrices of state and control of the gyroscope system, respectively (as described in Koruba et al., 2010).

The reinforcement matrix might be determined with the use of Matlab function:

$$K = \text{lqr}(J, B, Q, R) \quad (6)$$

It should be highlighted that in LQR function described with eq. (6), the state matrix A state of the gyroscope system was replaced with Jacobian J which was presented with eq. (1). It constitutes an essence of the modified control of the gyroscope system. Matrix of reinforcements K becomes non-stationary and is calculated at each stage of determining the control signals $u = -Kx$ (x – state vector with components: $\mathcal{G}_g, \dot{\mathcal{G}}_g, \psi_g, \dot{\psi}_g$). Choosing the weight matrices Q and R usually involves some kind of trial and error.

3. Numerical example and the results

Let us consider the following issue. A moveable point in space is observed from the Earth by the seeker. The control task consists in the fact that the gyroscope axis is to overlap with the line of sight. Thus, the moveable target is being tracked. In the example, the results of which were presented in Figs. 1 – 4, the moveable point (target) determines the angles which are the desired signals, on the basis of the following equations:

$$\varepsilon(\tau) = a + 0.2 \cdot v \cdot \tau^2, \quad \sigma(\tau) = b + v \cdot \tau \quad (7)$$

where:

$a = 0.1; b = -0.2; v = 1.5$ – motion parameters of the point in space (set motion).

The research was conducted with an integration step amounting to $dt = 0.00001$ (Baranowski, 2013).

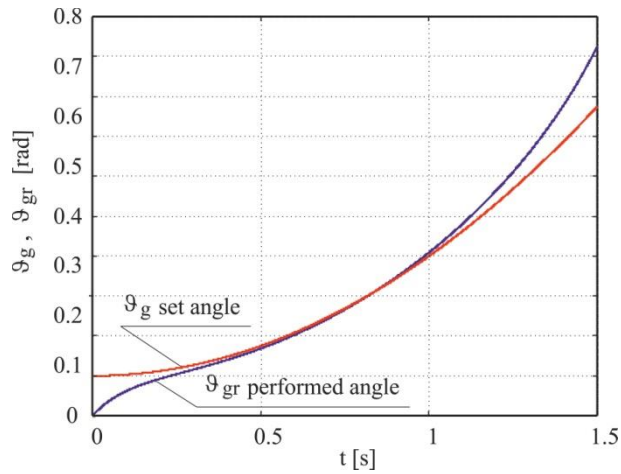


Fig. 1: Changes of performed and set angles in the time function with the use of matrix A with constant parameters.

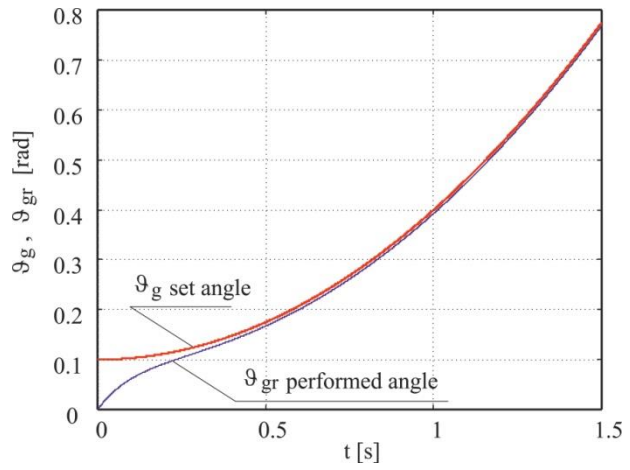


Fig. 2: Changes of performed and set angles in the time function with the use of Jacobian J.

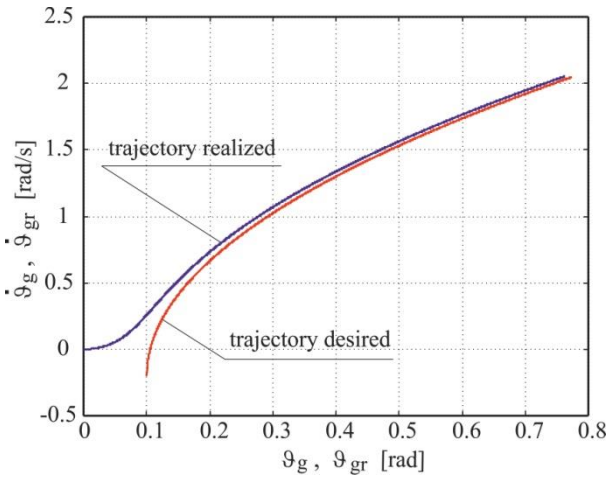


Fig. 3: Trajectories of the performed and desired motion with the use of matrix A with constant parameters.

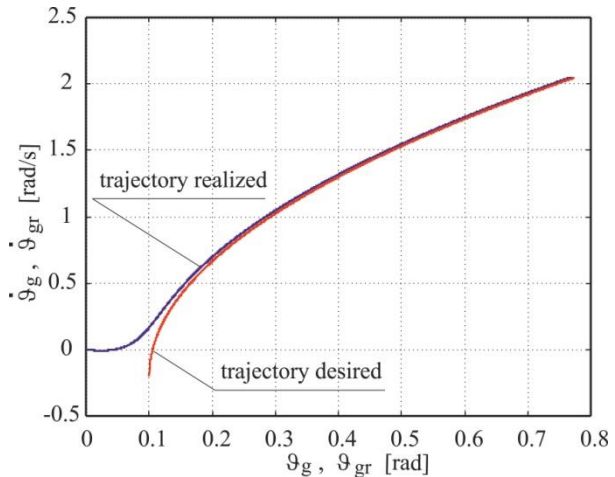


Fig. 4: Trajectories of the performed and desired motion with the use of Jacobian J.

The following matrix values were adopted:

$$A = [0 \ 1 \ 0 \ 0; 0 \ -200 \ 0 \ -1200; 0 \ 0 \ 0 \ 1; 0 \ 1200 \ 0 \ -200];$$

$$B = [0 \ 0; 4000 \ 0; 0 \ 0; 0 \ 4000];$$

$$Q = [1000 \ 0 \ 0 \ 0; 0 \ 10 \ 0 \ 0; 0 \ 0 \ 1000 \ 0; 0 \ 0 \ 0 \ 10];$$

$$R = [0.25 \ 0; 0 \ 0.25].$$

Taking the above charts showed in Figs. 1 – 4 into consideration, it is clear that control with the use of LQR modified method using Jacobian J operates in a better way – the gyroscope system axis reflects more faithfully the set motion along the observation line of the moveable point in space described with eq. (7).

4. Conclusions

The control algorithm presented in this paper makes it possible to control the gyroscope system more precisely also in the case of changing its parameters over time. The example analyzed in the article shows that tracking the moveable material point in space by the gyroscope system with the use of Jacobian in a closed control loop is more effective than with the use classical LQR control method. As the research results showed, the improvement of control precision of the gyroscope system of 10 % might be of crucial importance as far as the target achievement in system of missile homing using the gyroscope system of this type is concerned. Further studies will refer to the influence of random disturbances in conditions with no measurement data of some state variables of the gyroscope system in homing systems.

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