23rd International Conference
ENGINEERING MECHANICS 2017
Svratka, Czech Republic, 15 – 18 May 2017

THE INFLUENCE OF CONTACT REGION ON PROBABILITY OF CERAMICS FRACTURE

J. Kovář*

Abstract: *This paper focuses on assessment of probability of ceramic component fracture. The component is loaded by four point bending with different boundary conditions. The Weibull's weakest link theory, which includes the effect of first principal stress only, was chosen for calculation of probability of fracture. The stresses for calculations were evaluated numerically by finite element method. The component was discretizated by plane elements and constraints were considered in two variants. In the first variant, the constraints and loads are considered at the centerline of the rod (in accordance with rod theory). The second variant considers constraints and loads on the outer surface of the body. The contact between body and constraint (support) is considered in this variant. The influence of the radius of support on the value of probability of body fracture and the width of contact region is analyzed.*

Keywords: Weibull´s weakest link theory, Contact, Probability of fracture, Hertz´s theory, Ceramic.

1. Introduction

In the process of loading, ceramic materials have very small plastic deformation (Menčík, 1990). The fracture of this material is caused by initiation and growth of the crack, which was made in manufacturing process. The crack starts growing, when the stress intensity factor reaches its critical value. The value of stress intensity factor depends on the stress at the crack tip, length of the crack, the shape of the crack and the orientation in space. Therefore, the fracture mechanics is not used in this case. The statistic methods based on evaluating the probability of fracture are used, for example Weibull's weakest link theory (Weibull, 1939). The probability of fracture can be evaluated considering one or all of the three principal stresses. The following results were found with only the first principal stress considered. The probability of fracture of component, which has Weibull's distribution can be evaluated as:

$$P_f(V) = 1 - e^{-\frac{1}{V_0}\int_V \left(\frac{\sigma-\sigma_u}{\sigma_0}\right)^m dV} \tag{1}$$

where: V_0 is the volume of tested specimen, when the stress is lower than σ_u the component cannot crack, m is Weibull's modulus, which determines the width of probability density function of Weibull's distribution, and σ_0 is scale parameter, it is stress which makes 63 % specimens fractured.

The volume of specimen can be included in parameter σ_0, which will have the unit [MPa.mm$^{3/m}$], because the volume of specimen and σ_0 are constants. If the stress $\sigma_u = 0$, then we have the two-parameter Weibull's distribution and all tensile first principal stresses cause the process of body fracture. When the FEM is used to determine the stress in the component, the summation instead of integral can be used.

$$P_f(V) = 1 - e^{-\int_V \left(\frac{\sigma}{\sigma_0}\right)^m dV} = 1 - e^{-\sum_{i=1}^{N}\left(\left(\frac{\sigma_i}{\sigma_0}\right)^m \Delta V_i\right)} \tag{2}$$

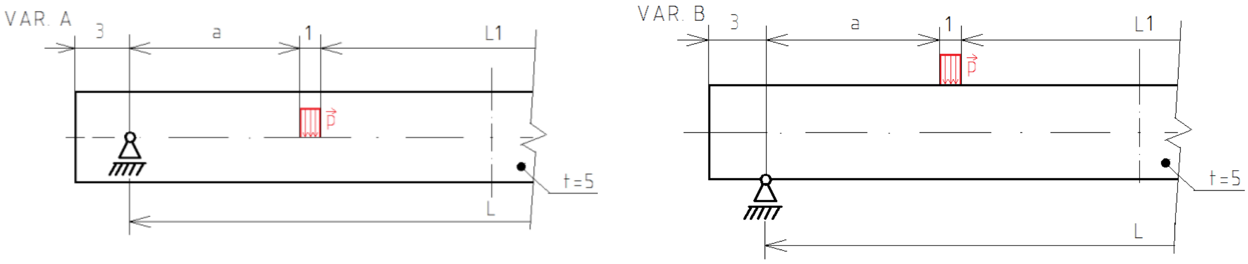
2. Computational model

The body, which is loaded by four point bending, is symmetrically supported by two supports, with distance L = 40 mm, forces (500 N) were replaced by pressure (p = 100 MPa, which acts at length 1 mm) at the distance L1 = 19 mm (Fig. 1). The body is discretized by plane elements PLANE 182. Two variants of boundary conditions are considered. In the first variant, the constraints are considered at the centerline

* Bc. Jaroslav Kovář.: Institute of Solid Mechanics, Mechatronics and Biomechanics, Faculty of Mechanical Engineering, Brno University of Technology; Technická 2896/2; 616 69, Brno; CZ, 160811@vutbr.cz

of the rod (in accordance with rod theory) and this variant is identified as VAR A. The second variant considers the constraints and the loads realistically, on the outer surface of body. This variant is identified as VAR B. At this variant the contact between the body and the constraint (support) is considered. Material of the body is bioceramics Al_2O_3, which is considered as linear isotropic continuum with elastic material parameters E = 390 GPa, μ = 0.24 (Fuis, 2004 and Fuis et al., 2001, 2009, 2010 and 2011) and parameters of Weibull's distribution σ_0 = 473.8 $MPa.mm^{3/7.19}$ and m = 7.19 (Málek, 2011; Fuis et al., 2006 and 2008).

Fig. 1: Geometry and the



constraints on the body: a) Constraints and the loads at the centerline of the rod (VAR. A); b) Constraints and the loads on the outer surface (VAR. B).

3. Results of computational modelling

3.1. Sensitivity analysis

The uniaxial stress is almost in the entire volume of the body, when the rod is loaded by four point bending. Exceptions are the positions of constraints (there is concentration of stress - in VAR. A. concentration is biggest, because the constraint is realized only in one node), loads (there is plane stress) and the area of overhanging end (there is zero stress). The first principal stress is illustrated in Fig. 2 and the values of probability of failure are in Tab. 1 (a = 9.5 mm). For small density of discretization of body (460 elements), the same value of probability of fracture was calculated, if the entire body is considered ($P_{f\ with\ constraint}$) or if the region near the constraints is not considered ($P_{f\ without\ constraint}$). With the increasing density of discretization, the difference between these probabilities is bigger.

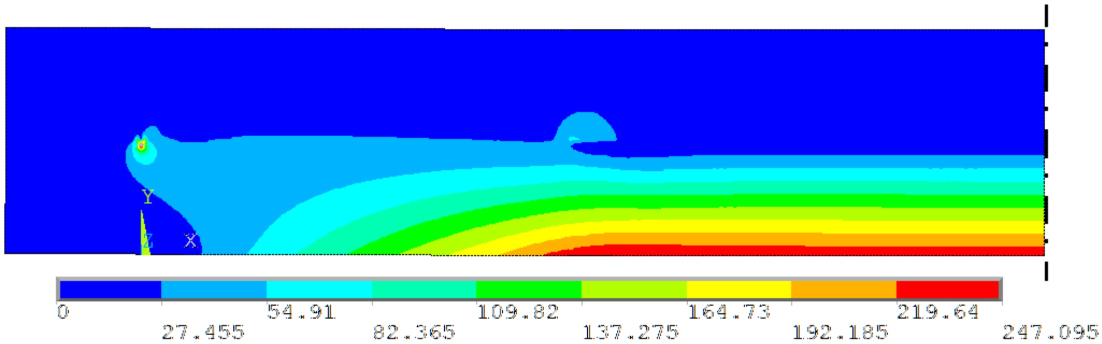


Fig. 2: First principal stress [MPa] in the body (max. stress in the constraint is 446 MPa).

Tab. 1: The influence of density of discretization at probability of fracture.

Size of element [mm]	**0.5**	**0.25**	**0.125**	**0.076**
Number of elements	460	1 840	7 360	20 064
$P_{f\ with\ constraint}$ **[%]**	27.5	24.7	34.6	90.7
$P_{f\ without\ constraint}$ **[%]**	27.5	24.4	23.6	23.4

The next analysis compares extreme values of stress in the areas of the loads for both variants, which have different positions of supports and loads (VAR. A and VAR. B.). The distances a, L and force F were changed, to cause constant bending moment between the loads at the length L1, which was constant (i.e. analytical calculated values of extreme stress were identical). The height of the body was constant. Fig. 3 shows extreme stresses at the body with different conditions. With the increasing slenderness of the rod, the numerically determined values approach analytical values. If we consider constraints at the centerline (VAR. A), the curve approaches from above and if we consider constraints on the outer surfaces (VAR. B), the curve approaches from the bottom.

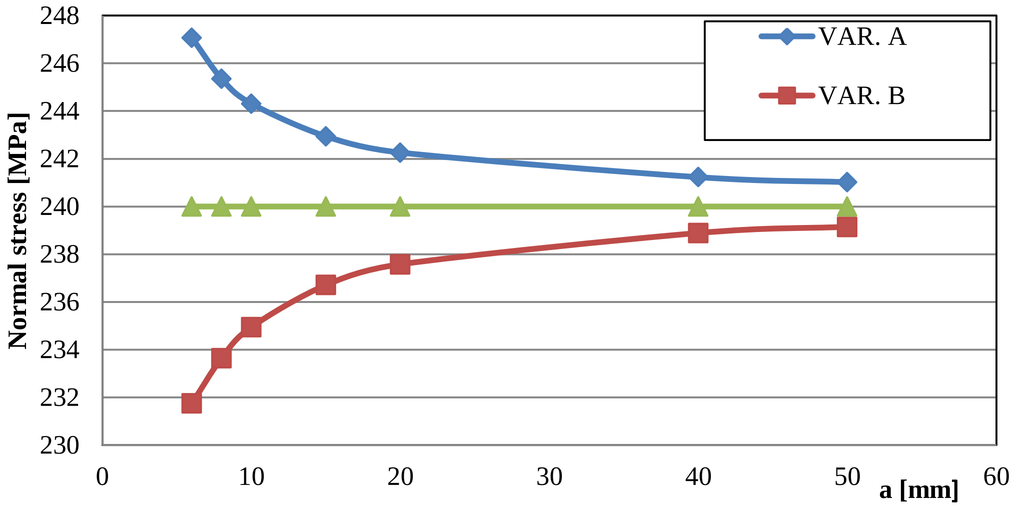


Fig. 3: Stress in depend on the distance between constraint and load (bending moment is constant).

3.2. Contact analysis

As it was said in the previous paragraph, the existence of constraint has an influence on probability of fracture. The task was modelled as contact, with considering real supports. At the place of the constraint, the ceramic component was supported by steel cylinder, which acts as the support. The component was loaded by moving steel cylinder, but loads in volume of body remained the same. The different radii of the supports were modelled (R = 0.5 mm, 1 mm, 2 mm and 4 mm). The contact stresses were compared with Hertz's theory for assessing the correctness. The mesh for this model was taken from the previous model (VAR. B size of element 0.076 mm), but the contact regions were refined. The results of modelling are in Tab. 2, where the results from numerical calculations are compared with the values evaluated by Hertz's theory (*b* is half width of contact region). The Hertz's theory shows that the stresses at *x* and *y* axis at contact region should be identical (Budynas et al., 2011). The stress at x axis is more different from theoretical results than stress at y axis, because its gradient at contact region is steeper. For more accurate results of this stress, the mesh would have to be more refined. The stresses in *x* and *y* axes are shown in Fig. 4. This error has no influence on probability of fracture, because the first principal stress in this region is compressive.

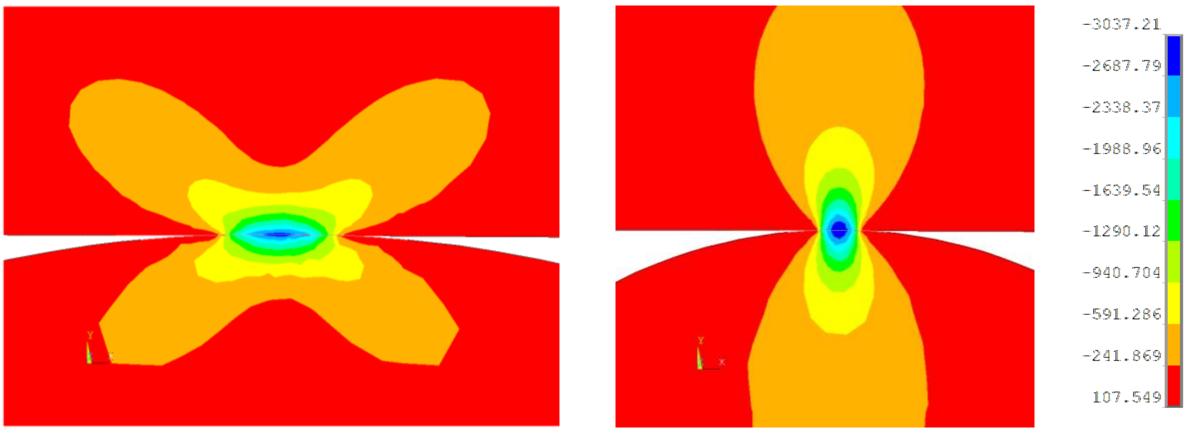


Fig. 4: Stress at x and y axis [MPa] for contact with R=0.5 mm.

In the terms of probability of fracture only the tensile first principal stresses σ_1 are significant. Near contact region, according to Hertz's theory σ_1 is compressive, which causes that this stress does not influence the probability of fracture. The results of probability confirmed this fact and Tab. 3 shows the values of probability of fracture for different radii of supports.

Tab. 2: Comparsion of theoretical and calculated values.

Radius of support [mm]	**0.5**	**1**	**2**	**4**
$p_{maxHERTZ}$ **[MPa]**	-3092	-2186	-1546	-1094
σ_{xmax} **[MPa]**	-2888	-2044	-1438	-1027
σ_{ymax} **[MPa]**	-3037	-2128	-1498	-1069
b_{theor} **[mm]**	0.026	0.029	0.041	0.058
b_{FEM} **[mm]**	0.023	0.026	0.038	0.061

Tab. 3: Comparsion of probabilities of failure.

	VAR. A	**VAR. B**			
Radius of support [mm]	**-**	**0.5**	**1**	**2**	**4**
σ_{1max} **[MPa]**		243.8	243.7	243.6	243.3
P_{fFEM}[%]	23.39	22.85	22.79	22.70	22.53

4. Conclusion

At the body loaded by four point bending, the extreme tension stresses approach analytical values with increasing slimness of the rod. In VAR. A (loads and constrains are at the centerline) values approach from above, in VAR. B (loads and constraints are on the outer surfaces of body) values approach form bottom. With the use of Weibull's theory, the probability of ceramic component fracture can be determined by FEM. The probability of fracture does not depend only on the stresses, but on the volume too, which causes that the probability of fracture is influenced by discretization. The probability of fracture of the model with loads and constraints at node is bigger than probability of fracture of the model with contact pair. The bigger radius of supports reduces the probability of fracture.

Acknowledgment

This work was supported by grant FSI-S-17-4004.

References

Budynas, R.G. et al. (2011) Shigley's mechanical engineering design McGraw-Hill Science.

Fuis, V. and Janicek, P. (2001) Stress and reliability analyses of ceramic femoral heads with axisymmetric production inaccuracies. in: Proc. 9th Mediterranean Conference on Medical and Biological Engineering and Computing, Pula, Croatia, IFMBE Proceedings Pts. 1 and 2, pp. 632-635.

Fuis, V. (2004) Stress and reliability analyses of ceramic femoral heads with 3D manufacturing inaccuracies, in: Proc. 11th World Congress in Mechanism and Machine Science, Tianjin, China pp. 2197-2201.

Fuis, V. et al. (2006) Reliability of the Ceramic Head of the Total Hip Joint Endoprosthesis Using Weibull's Weakest-link Theory. in: World Congress on Medical Physics and Biomedical Engineering, IFMBE Proc. Vol. 14, pp. 2941-2944.

Fuis, V, Janicek, P. and Houfek, L. (2008) Stress and Reliability Analyses of the Hip Joint Endoprosthesis Ceramic Head with Macro and Micro Shape Deviations, in: 13th Conf. ICBME, IFMBE Proc. Vol. 23, Iss. 1-3, pp: 1580-1583.

Fuis, V. and Varga, J. (2009) Stress Analyses of the Hip Joint Endoprosthesis Ceramic Head with Different Shapes of the Cone Opening. in: Proc. 13th International Conference on Biomedical Engineering, IFMBE Proceedings Vol. 23, Iss. 1-3, pp. 2012-2015.

Fuis, V., Navrat, T. and Vosynek, P. (2010) Analyses of the Shape Deviations of the Contact Cones of the Total Hip Joint Endoprostheses. in. Proc. 6th World Congress of Biomechanics (WCB 2010) Singapore, Series: IFMBE Proc. Vol. 31, pp. 1451-1454.

Fuis, V., Malek, M. and Janicek, P. (2011) Probability of destruction of Ceramics using Weibull's Theory, in: Proc. 17th International Conference on Engineering Mechanics, Svratka, Czech Republic, pp. 155-158.

Menčík, J. (1990) Strength and fracture glass and ceramic, SNTL Praha (in Czech).

Weibull, W. (1939) A statistical theory of the strength of materials, Generalstabens litografiska anstalts förlag, Stockholm.