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# PROBABILISTIC ESTIMATION OF A BRIDGE FATIGUE LIFE IN ACCORDANCE TO *fib* MODEL CODE 2010

**J. Krejsa*, M. Sýkora****

**Abstract:** *The service life of bridges is significantly affected by fatigue induced by crossings of heavy vehicles. Therefore, information of traffic flow, covering frequencies and vehicle weights, is of crucial importance for the calculation of fatigue damage and the prediction of the bridge lifetime. This paper investigates accuracy of fatigue life estimates depending on a length of traffic flow records. The presented data were obtained from the measurements carried out on a bridge of the Prague Highway Ring. The analysis reveals that the optimal length of records for fatigue life assessment is about 30 days.*

**Keywords: Length of records, Fatigue, Heavy traffic, Traffic flow, Palmgren-Miner rule.**

## 1. Introduction

The service life of a bridge depends on the effects of loadings imposed to a bridge. The *fib* Model Code 2010 introduces a fatigue limit state, which may be a critical criterion for road and railway bridges. Generally, the fatigue damage is caused by cyclic loading and depends on frequency and stress magnitude of loading cycles. Fatigue damage relates to the structures exposed to considerable dynamic stresses. The paper is focused on the effect of the length of traffic flow records on the accuracy of estimating road bridge fatigue life.

## 2. Fatigue

### 2.1. Material properties – Wöhler curve

Augustin Wöhler proposed in 1850 a theory of the fatigue failure of a material, describing the relationship between stress amplitudes and a number of loading cycles. This dependence is called the Wöhler fatigue curve (*S*-*N* curve) (fib, 2013), which is the most widely used tool for the assessment of fatigue life of concrete structures.

### 2.2. Palmgren-Miner rule of the cumulative damage

The Palmgren-Miner rule of the cumulative damage can be applied to account for different weights of heavy vehicles (fib, 2013):

$$D_{\mathrm{fat}} = \Sigma(n_i/N_i) \quad (1)$$

where $D_{\mathrm{fat}}$ is the fatigue damage, $n_i$ the number of recorded cycles and $N_i$ the number of cycles from the Wöhler curve. Variables $n_i$ and $N_i$ depend on the response of the structure induced primarily by weights of vehicles passing the bridge.

## 3. Data

The database contains measurements recorded in the period from January 2008 to January 2010 with the total number of 628 days. A number of recorded vehicles is over 1.8 million. The traffic flow data were

* Ing. Jan Krejsa: Klokner Institute: Department of Structural reliability, CTU in Prague, Solinova 7, 166 08, Prague; CZ, jan.krejsa@cvut.cz

** Assoc. Prof. Ing. Miroslav Sykora, PhD.: Klokner Institute: Department of Structural reliability, CTU in Prague, Solinova 7, 166 08, Prague; CZ, miroslav.sykora@cvut.cz

obtained on a bridge that is a part of the Prague Highway Ring (Polak et al., 2009, Polak et al., 2011 and Morales et al., 2014). The data contain real vehicle weights. Numbers of vehicles per day are divided into 32 categories:

- Light vehicles with weights from 0 to 10 tons,
- 30 categories in the range from 10 to 85 tons, with the difference of 2.5 ton between categories,
- One category for the range from 85 to 200 tons.

## 4. Procedure for data analysis

The main goal of this study is optimisation of the length of traffic flow records for a general design situation. The assessment is thus based on the Wöhler curve given by the *fib* Model Code 2010 (fib, 2013) for C35/45 concrete class and stress amplitudes caused by the real traffic spectrum. To provide a representative case study, the magnitude of stress amplitudes is chosen so that the partial factor method according to *fib* Model Code 2010 (fib, 2013) leads to cumulative damage obtained by the Palmgren-Miner rule slightly lower than unity for a reference period of 100 years. The calculation procedure can be summarised in the following steps:

- Development of sub-databases for different record lengths (1, 7, 30 and 120 days); the following steps are conducted for each of the sub-databases.
- Prediction of numbers of vehicles $n_i$ for each weight category, considering a reference period $t_{\mathrm{ref}}$ as a study parameter, i.e. by extrapolating the average number of vehicles given a particular length of record.
- Estimation of the stress amplitudes for vehicles $\sigma_{\mathrm{veh},i}$ in each weight category.
- Determination of the Wöhler curve according to *fib* Model Code 2010 (fib, 2013) considering the coefficient related to a concrete strength class as $s$ = 0.38, the concrete age during the first loading $t$ = 90 days, minimum compressive stress $\sigma_{\mathrm{c,min}}$ = 5 MPa and maximum compressive stress $\sigma_{\mathrm{c,max},i} = \sigma_{\mathrm{c,min}} + \sigma_{\mathrm{veh},i}$.
- Assessment of the cumulative damage by Palmgren-Miner rule (1).
- Statistical evaluation of $D_{\mathrm{fat}}$-characteristics.
- Probabilistic modelling of basic variables and reliability analysis.
- Estimation of fatigue life for a given target reliability level.

## 5. Statistical evaluation of $D_{\mathrm{fat}}$

The results of the statistical evaluation (the mean value $m_{D\mathrm{fat}}$, of the coefficient of variation $v_{D\mathrm{fat}}$ and the skewness $w_{D\mathrm{fat}}$) of fatigue damage for different lengths of record are reported in Tab. 1. Only the mean value $m_{D\mathrm{fat}}$ depends on $t_{\mathrm{ref}}$, other parameters are dependent only on database.

*Tab. 1: Statistical evaluation of fatigue damage for different lengths of record for $t_{\mathrm{ref}}$ = 100 years.*

| Length of record | Number of lengths of records | Total number of days under consideration | Total number of vehicles | $D_{\mathrm{fat}}$ | | |
|---|---|---|---|---|---|---|
| | | | | $m_{\mathrm{Dfat}}$ | $V_{\mathrm{Dfat}}$ | $w_{\mathrm{Dfat}}$ |
| Extrapolation from all data | 1 | 628 | 1864886 | 0.186 | - | - |
| 1 day | 628 | 628 | 1864886 | 0.186 | 2.17 | 3.19 |
| 7 days | 89 | 623 | 1852822 | 0.188 | 1.06 | 2.15 |
| 30 days | 20 | 600 | 1818270 | 0.194 | 0.69 | 0.69 |
| 120 days | 5 | 600 | 1818270 | 0.194 | 0.62 | 1.14 |

Tab. 1 shows the effect of different lengths of record on $D_{\mathrm{fat}}$ during the reference period $t_{\mathrm{ref}}$ = 100 years. Only complete measurements for a particular length of record are involved in the analysis, which causes minor differences in the total numbers of days and vehicles. Coefficient of variation significantly decreases with an increasing length of record while the mean value is nearly independent. When the length of record is 120 days, the coefficient of variation is 3.5 times lower than for daily records. The verification by the partial factor method according to the *fib* Model Code 2010 (fib, 2013) indicates that a fatigue life is longer than 100 year since the fatigue damage is less than unity, $D_{fat} = 0.85$.

## 6. Probabilistic modelling and estimation of fatigue life

Reliability indicators - failure probability $p_f(t_{ref})$ and reliability index $\beta(t_{ref})$ – are obtained by a probabilistic reliability analysis as follows:

$$p_f(t_{ref}) = P[\theta_D\ \theta_E\ D_{fat}(t_{ref}) > 1];\ \beta(t_{ref}) = -\Phi^{-1}[p_f(t_{ref})] \tag{2}$$

where $-\Phi^{-1}$ is the negative value of inverse function of general normal distribution; see EN 1990, $\theta_D$ the model uncertainties associated with Palmgren-Miner rule and $\theta_E$ the model uncertainty associated with load effects. The notation and probabilistic models for the basic variables are provided in Tab. 2.

*Tab. 2: Statistical parameters for uncertainties.*

| **Uncertainty** | ***m*** | ***V*** | ***w*** |
|---|---|---|---|
| $\theta_D$ | 1 | 0.5 | 1.63 |
| $\theta_E$ | 1 | 0.1 | 0.30 |

It is assumed that uncertainty related to material resistance (the application of Wöhler curves) is covered by $\theta_D$. The Wöhler curve applied strictly according to the *fib* Model Code and using a mean concrete compressive strength instead of its characteristic values leads to mean numbers of cycles to failure (fib, 2013).

Three-parametric lognormal distribution is chosen for $D_{fat}$, $\theta_D$ and $\theta_E$. The probabilistic lognormal model for $D_{fat}$ is based on the database and previous experience. Models for the uncertainty in load effect $\theta_E$ and the uncertainty related to the Palmgren-Miner rule $\theta_D$ are based on the information provided in the Probabilistic Model Code of the Joint Committee on Structural Safety (JCSS, 2006) (Tab. 2). Uncertainties in the load effect model take into account inaccuracies in determining the load effect, such as internal forces and stresses resulting from model simplifications in geometry, supporting and boundary conditions, redistribution of forces amongst structural members, uncertainties in specifying model parameters that are unknown, etc. Coefficient of variation of $\theta_D$ for structural steel is 0.3 (JCSS, 2006). It is estimated that uncertainty associated with the Palmgren-Miner rule for concrete is larger and coefficient of variation of 0.5 is thus taken into account.

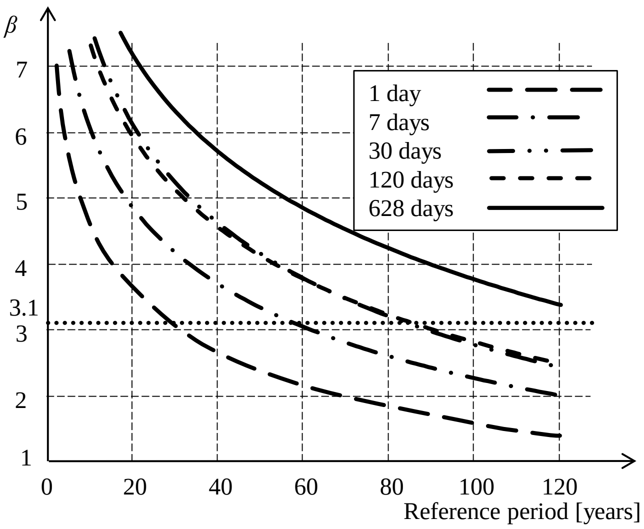


*Fig. 1: Reliability index as a function of reference period for different lengths of record.*

Fig. 1 shows the variability of reliability index with a reference period for different lengths of record and a target reliability level of 3.1 (fib, 2013). It appears that the optimal length of record is 30 days as a fatigue life for this length of record – 85 years – is more than three-times longer than daily records. Increasing a length of record above 30 days seems to improve fatigue life estimates insignificantly. For shorter lengths the effect of statistical uncertainty increases and the accuracy of the estimate of fatigue life decrease.

It is emphasised that changes in the traffic flow intensity are not considered; i.e. stationary conditions and ergodicity (Melchers, 2002) are assumed. Dynamic effects are to be considered in a fatigue analysis unless they are included in measurements (as is the case of this study). In further investigations sensitivity analysis will be conducted to identify basic variables, probabilistic models of which should be improved.

## 7. Conclusions

The paper investigates the effect of a length of traffic flow records on fatigue life estimates for road bridges. It appears that:

- With an increasing length of record, coefficient of variation of fatigue damage significantly decreases while the mean value changes insignificantly.
- Optimal length of record is 30 days; longer lengths of record improve fatigue life estimates insignificantly.
- The verification by the partial factor method indicates that a fatigue life is longer than 100 year while a fatigue life estimated by the probabilistic analysis is about 85 years for the optimum length of record; this is indicative of the need for updating the partial factor method procedure for fatigue verifications.

Within further studies, a more detailed analysis of traffic flow and an assessment of uncertainties will be conducted.

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